Maximum Likelihood Reliability Estimation from Subsystem and Full-System Tests: Method Overview and Illustrative Examples

Coire J. Maranzano and James C. Spall

ABSTRACT

This article provides an overview and examples of a novel and practical method for estimating the reliability of a complex system, with confidence regions, based on a combination of full-system and subsystem (and/or component or other) tests. It is assumed that the system is composed of multiple processes, where the subsystems may be arranged in series, parallel, combination series/parallel, or other mode. Maximum likelihood estimation (MLE) is used to estimate the overall system reliability based on the fusion of system and subsystem test data. The method is illustrated on two real-world systems: an aircraft-missile system and a highly reliable low-pressure coolant injection system in a commercial nuclear-power reactor. The examples demonstrate the following properties of the method: (1) Increasing the number of full-system tests improves the confidence in the full-system reliability estimate. (2) Increasing the number of tests of one subsystem stabilizes the subsystem reliability estimate. (3) The likelihood function and optimization constraints can readily be modified to handle systems consisting of repeated components in a mixed series/parallel configuration. (4) A normal distribution approximation for computing confidence intervals is not always appropriate, especially for highly reliable systems. (5) Performing a mixture of full-system and subsystem tests is important when the model that relates the subsystem reliability to the full-system reliability is uncertain.

INTRODUCTION

System, subsystem, component, interface, and other tests are often carried out on complex systems to ensure that an operational reliability requirement is satisfied. (Note: To avoid the need to repeatedly refer to tests on subsystems, components, processes, and other aspects of the system as the key source of information other than information obtained from full-system tests, we will generally refer to only subsystem tests; *subsystem tests* in this context should be considered a proxy for all possible test information short of that obtained from full-system tests.) Using a combination of full-system and subsystem test data to evaluate the reliability of a com-

Note: This work is an updated version of an earlier work (Ref. 1): C. J. Maranzano and J. C. Spall, "Implementation and application of maximum likelihood reliability estimation from subsystem and full system tests," in *PerMIS'10, Proc. Performance Metrics for Intelligent Systems Workshop*, Baltimore, MD, 2010, pp. 146–153, http://doi.acm.org/10.1145/2377576.2377604. © ACM, 2010.

plex system is desirable when full-system testing can be costly or dangerous or when it requires destruction of the system itself. Additionally, it is desirable to include full-system testing in an overall reliability assessment to help guard against possible mis-modeling of the relationships between the subsystems and the full system in calculating overall system reliability.

One method of combining full-system and subsystem reliability test data to form a full-system estimate of reliability is the method of maximum likelihood (Ref. 2). This general maximum likelihood formulation for the combination of reliability test data applies across all system configurations (series, parallel, etc.); only the optimization constraints change, leading to an appropriate maximum likelihood estimate (MLE). The method of maximum likelihood provides a characterization of the estimation uncertainty—statistical uncertainty about the model parameters—through the Fisher information on the parameters of the system reliability model.

The general maximum likelihood method of reliability estimation combines data from subsystem tests and full-system tests via a model that reflects the constraints associated with the operation of the full system. If the reliability of the system must be known within a specified confidence interval or if the test plan is limited by cost, there is an inherent trade-off between performing full-system tests or subsystem tests. However, the model is often subject to error, leading to an inaccurate system reliability estimate when subsystem tests alone are performed. Performing full-system tests guards against error in the system reliability estimate due to an imperfect model. The general method is extended in Ref. 3 to include a robust test planning capability to simultaneously minimize estimation uncertainty and the effect of modeling error on the full-system reliability estimate for a series system. The capability enables a planner to determine the optimal number of system and subsystem tests to include in an experiment.

Certainly, other approaches exist for estimating system reliability when the subsystems are independent (see Ref. 2 for a complete review). Of note is the Bayesian approach, developed in Ref. 4, to combining subsystem and full-system test data for estimating reliability. While prior information may be useful and appropriate in some situations, the MLE approach offers a prior-free alternative to the Bayesian approach that is parsimonious in the model construction (no priors or hyperpriors) and in the subjective input (no prior parameters or hyperparameters). Also, the Bayesian estimation approach described in Ref. 4 is ultimately one of numerical integration, where MLE is ultimately a problem of function maximization, which is typically less computationally demanding. Another approach, which allows for statistically dependent data, is the inequality-based reliability approach described in Ref. 5; this approach is based on probability inequalities and provides a conservative confidence interval for reliability.

This article provides an overview and examples of a novel and practical method for estimating the reliability of a complex system, with confidence regions, based on a combination of full-system and subsystem tests. It is assumed that the system is composed of multiple processes, where the subsystems may be arranged in series, parallel (i.e., redundant), combination series/parallel, or other mode. The general MLE method described in Ref. 2 is used to estimate the overall system reliability. The MLE approach provides asymptotic or finite-sample confidence bounds through the use of Fisher information or Monte Carlo sampling (bootstrap). The examples demonstrate the need for developing a robust test plan that includes a mixture of full-system and subsystem tests to reduce the influence of model error on the system reliability estimate and to minimize testing costs.

The method is illustrated on three systems. First, a hypothetical system is used to demonstrate the value of combining full-system and subsystem test data for reducing the uncertainty in the full-system reliability estimate even with model error. Second, the MLE method is used to form estimates of system and subsystem reliability on the series aircraft-missile system described in Ref. 6. The example demonstrates that increasing the number of full-system tests improves the confidence in the full-system reliability estimate and that increasing the sample size of one of the subsystems stabilizes the subsystem reliability estimate but only slightly improves confidence in the full-system reliability estimate. The asymptotic and Monte Carlo (bootstrap) confidence intervals are computed and compared. The system reliability MLE and 90% confidence interval is also compared with the Bayesian posterior distribution on the system reliability computed in Ref. 6. The comparison shows that prior information significantly influences the full-system reliability estimate. Also, prior information on the subsystem reliabilities is used to determine a minimum cost test program for achieving a specified mean square error (MSE) given that the system reliability model is in error. Third, the MLE method is used to form estimates of system and subsystem reliability on a highly reliable low-pressure coolant injection (LPCI) system in a commercial nuclear-power reactor described in Ref. 7. The example shows that likelihood function and optimization constraints can readily be modified to handle systems consisting of repeated components in a mixed series/parallel configuration. Through the presentation of the empirical distribution of the bootstrap sample used to determine the confidence interval, the example also shows that the asymptotic normal assumption for computing confidence intervals is not always appropriate, especially for highly reliable systems.

MLE APPROACH

Background

Consider a system composed of p subsystems. The general estimation formulation involves a parameter vector $\boldsymbol{\theta}$, representing the parameters to be estimated. Let ρ and ρ_j represent the reliabilities (success probabilities) for the full system and for subsystem j, respectively, $j = 1, 2,\ldots,p$. The vector $\boldsymbol{\theta} = [\rho_1, \rho_2,\ldots,\rho_p]^T$, where superscript T represents matrix or vector transpose. Let Θ represent the feasible region for the elements of $\boldsymbol{\theta}$. To ensure that relevant logarithms are defined and that the appropriate derivatives exist, it is assumed, at a minimum, that the feasible region Θ includes the restriction that $0 < \rho_j < 1$ for all j. The system reliability ρ is not included in $\boldsymbol{\theta}$ because it is uniquely determined (or bounded) by the subsystem reliabilities ρ_j for $j = 1, 2,\ldots,p$ and possibly other information via relevant constraints. Herein, the relation is restricted such that ρ is uniquely determined by a function $h(\boldsymbol{\theta})$, i.e., $\rho = h(\boldsymbol{\theta})$. The mapping, h, between $\boldsymbol{\theta}$ and ρ dictates the arrangement of the system, which may be configured in series $\left(\rho = h(\boldsymbol{\theta}) = \prod_{i=1}^{p} \rho_i\right)$, parallel $\left(\rho = h(\boldsymbol{\theta}) = 1 - \prod_{i=1}^{p} (1-\rho_i)\right)$, combination series/parallel, or some other configuration, and it is analogous to a model of system reliability in terms of its subsystems. To mirror the lexicon commonly used in the literature, h will be referred to as the model for the system reliability. Thus, an estimate of the system reliability $\hat{\rho}$ is found by evaluating $h(\cdot)$ at the estimate $\hat{\boldsymbol{\theta}}$.

MLE Formulation

Let us now describe the relationship between the reliability MLE and the data. Let Y be the number of successes in n independent identically distributed tests of the system, and let X_j be the number of successes in n_j independent identically distributed tests of the j^{th} subsystem, for $j = 1,\ldots,p$. Let $\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}(\mathbf{Z})$ be a function that produces an estimate of $\boldsymbol{\theta}$, where $\mathbf{Z}$ is the complete set of test data on the full system and its subsystems $\{Y, X_1,\ldots,X_p\}$. Let $p(\mathbf{Z}|\boldsymbol{\theta}, \rho)$ be the probability mass function for the vector $\mathbf{Z}$ conditional on specified values for $\boldsymbol{\theta}$ and ρ. Consider the following general maximum likelihood estimator of the parameter vector $\boldsymbol{\theta}$,

$$\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}(\mathbf{Z}) \equiv \underset{\boldsymbol{\theta}\in\Theta}{\operatorname{argmax}}\, \mathfrak{L}(\boldsymbol{\theta}) \quad \text{subject to } \rho = h(\boldsymbol{\theta}), \tag{1}$$

where $\mathfrak{L}(\boldsymbol{\theta}) \equiv \log(p(\mathbf{Z}|\boldsymbol{\theta}, \rho))$ and the argmax operator returns the vector $\boldsymbol{\theta}$ that maximizes the likelihood function $\mathfrak{L}(\boldsymbol{\theta})$ (Ref. 2). Given both system and subsystem test data, $\mathbf{Z}$, the estimate of ρ is derived from the MLE for $\boldsymbol{\theta}$ through the model for the system, h. In particular, the MLE for ρ is $\hat{\rho} = h(\hat{\boldsymbol{\theta}})$. (Note that a more general formulation of the MLE expression above, allowing for a non-discrete distribution for Y and/or X_j, is given in Refs. 8–10.)

The model dictates how the subsystems are arranged in the full system ($\mathfrak{L}(\boldsymbol{\theta})$ is the same regardless of whether the subsystems are in series, parallel, or some other arrangement). For a given parameter vector $\boldsymbol{\theta}$, the definition of $\mathfrak{L}(\boldsymbol{\theta})$ does *not* depend on the model for the system. However, the MLE does change as a function of the model for the system. This is a consequence of the system model being used as a constraint in the optimization problem that is solved to produce the MLE. Given that the test data are independent, the probability mass function is:

$$p(\mathbf{Z}|\boldsymbol{\theta}, \rho) = \underbrace{\binom{n}{Y}\rho^{Y}(1-\rho)^{(n-Y)}}_{\text{system}} \times \underbrace{\binom{n_1}{X_1}\rho_1^{X_1}(1-\rho_1)^{(n_1-X_1)}\cdots\binom{n_p}{X_p}\rho_p^{X_p}(1-\rho_p)^{(n_p-X_p)}}_{p\text{ subsystems}}, \tag{2}$$

leading to the log-likelihood function:

$$\mathfrak{L}(\boldsymbol{\theta}) = Y\log\rho + (n-Y)\log(1-\rho) + \sum_{j=1}^{p}[X_j\log\rho_j + (n_j - X_j)\log(1-\rho_j)] + \text{constant}, \tag{3}$$

where the constant does not depend on $\boldsymbol{\theta}$. The MLE for $\boldsymbol{\theta}$ is determined by finding a root of the score equation $\partial\mathfrak{L}(\boldsymbol{\theta})/\partial\boldsymbol{\theta} = 0$. The solution to $\partial\mathfrak{L}(\boldsymbol{\theta})/\partial\boldsymbol{\theta} = 0$ must generally be found by numerical search methods.

Theoretical Properties

Except in trivial cases, the analytic expression for the variance of the general MLE for system reliability is not easily found. However, Ref. 2 showed that the Fisher information, $\boldsymbol{F}(\boldsymbol{\theta})$, is easily obtained for the general maximum likelihood estimator of the parameter vector $\boldsymbol{\theta}$. Further, Ref. 11 showed that the general MLE of system and subsystem reliability has a strong convergence property and that the rate of convergence for the system reliability estimate to the true reliability ρ^* is

$$O\left(\sqrt{\frac{\log\log(n+n_s)}{n+n_s}}\right), \tag{4}$$

where s is the index of the slowest increasing subsystem sample size, and $O(\cdot)$ is the standard order notation. Invoking the Cramér–Rao inequality (Ref. 12, p. 357), the inverse of the Fisher information is a lower bound on the variance of the MLE (for an unbiased estimator). The general theory supporting that the MLE is asymptotically normal is given in Ref. 8. Therefore, the variance of the general MLE, $\hat{\boldsymbol{\theta}}$, is approximated with the inverse Fisher information, and the variance of the general MLE of the full-system reliability, $\hat{\rho}$, is approximated by

$$h'(\boldsymbol{\theta})^T \boldsymbol{F}(\boldsymbol{\theta})^{-1} h'(\boldsymbol{\theta}), \tag{5}$$

where $h'(\boldsymbol{\theta})$ is the $p \times 1$ gradient vector of $h(\boldsymbol{\theta})$ and superscript T denotes transpose.

Aside from being used to form approximate confidence regions for the MLE when the sample size is sufficiently large, the Fisher information is helpful in determining when the estimation problem in the section on MLE formulation is well posed (i.e., when ρ and/or $\boldsymbol{\theta}$ is identifiable) through an evaluation of the conditions ensuring that the information matrix is positive definite (e.g., Ref. 13, pp. 104 and 139; and Refs. 14 and 15). Further, the Fisher information matrix (FIM) is used in determining the optimal combination of subsystem and full-system tests for estimating reliability when performing test design (Ref. 3).

Confidence Bounds

There are two general methods for constructing confidence bounds for the estimate $\hat{\rho}$, a large-sample approach based on an asymptotic normal distribution (and accompanying inverse FIM for variance calculation) and a finite-sample approach based on Monte Carlo (bootstrap sampling) methods. The discussion below summarizes key aspects of both of these general methods.

For the large-sample approach, the asymptotic distribution provides an approximate probability distribution for $\hat{\rho}$ for use in finite-sample (practical) analysis. Herein, the inverse average information matrix for $\boldsymbol{\theta}$ (or information number for ρ) is used as the covariance matrix (or variance) appearing in the asymptotic distribution of the appropriately normalized MLE (see the section on theoretical properties).

There are, however, potential problems in the use of the asymptotic approach in practical reliability settings. The problem is especially acute when sample sizes are too small to justify the asymptotic normality and/or when confidence intervals from the asymptotic normality fall outside of the interval [0, 1] as a consequence of the need to approximate the true asymmetric distribution with the symmetric normal distribution. The latter factor is exacerbated by the fact that practical reliability estimates are often very near unity. Therefore, we summarize a bootstrap (Monte Carlo)–based method below. Bootstrap methods are well-known Monte Carlo procedures for approximating important statistical quantities of interest when analytical methods are infeasible (e.g., Ref. 16; Ref. 17, pp. 304 and 334; and Ref. 18). The bootstrap-based method for constructing confidence intervals for the full-system reliability estimate $\hat{\rho}$ relies on the assumption that $\hat{\rho}$ is uniquely determined from $\hat{\boldsymbol{\theta}}$. References 1 and 8 present sufficient conditions for such a function via the implicit function theorem.

The steps below describe a parametric bootstrap approach to constructing confidence intervals for $\hat{\rho}$. Parametric bootstrap methods rely on many Monte Carlo samples from the distribution associated with the likelihood function, where the unknown parameters in the distribution are replaced by their estimated values (in contrast, a standard bootstrap method uses Monte Carlo samples from the raw data, typically from a histogram of the raw data). The bootstrap approach performance is compared with the asymptotic approach in example 2 below (see the section describing example 2).

Bootstrap Method for Computing Confidence Intervals for $\hat{\rho}$:

Step 0. Treat the MLE $\hat{\boldsymbol{\theta}}$, and associated $\hat{\rho} = h(\hat{\boldsymbol{\theta}})$, as the true value of $\boldsymbol{\theta}$ and ρ.

Step 1. Generate (by Monte Carlo) a set of bootstrap data of the same collective sample size $\{n, n_1, n_2, \ldots, n_p\}$ as the real data $\boldsymbol{Z}$ using the assumed probability mass function in Eq. 2 and the value of $\boldsymbol{\theta}$ and ρ from step 0.

Step 2. Calculate the MLE of $\boldsymbol{\theta}$, say $\hat{\boldsymbol{\theta}}_{\text{boot}}$, from the bootstrap data $\boldsymbol{Z}$ in step 1, and then calculate the corresponding full-system reliability MLE, $\hat{\rho}_{\text{boot}}$.

Step 3. Repeat steps 1 and 2 a large number of times (perhaps 1,000) and rank order the resulting values; one- or two-sided confidence intervals are available by determining the appropriate quantiles from the ranked sample of $\hat{\rho}_{\text{boot}}$ values.

Estimator Uncertainty with Mis-Modeling in the Reliability Model

The system reliability model, h, is a function that relates the subsystem reliabilities to the system reliability based on the layout of the subsystems. As h is a mathematical representation of the true relation among reliabilities, it is imperfect. For example, a subsystem may be neglected and not included in the model, or two subsystems assumed to be stochastically independent in the model may have a subtle dependence. An imperfect mathematical model results in a true system reliability that is inaccurate. Reference 3 develops a method for test planning that minimizes the MSE of the MLE using a local design for a series system assuming that the system reliability is uniquely determined from the subsystem reliability and the model error $\beta(\boldsymbol{\theta})$; that is,

$$\rho = h(\boldsymbol{\theta}) + \beta(\boldsymbol{\theta}). \tag{6}$$

The expression for the MSE is formed to explicitly include a contribution from the modeling error. The result is summarized next for completeness.

Given the relation in Eq. 6, the estimator of system reliability, $\hat{\rho} = h(\hat{\boldsymbol{\theta}})$ no longer satisfies the conditions for convergence to ρ^* as described in Refs. 11 or 8, and the optimal test plan depends on $\boldsymbol{\theta}$, h, and β. However, a test planner does *not* know $\boldsymbol{\theta}$ or β before testing the system (and determining β may be intractable). To cope with the optimal test plan's dependence on $\boldsymbol{\theta}$, test planners might assume a nominal value of $\boldsymbol{\theta}$ and develop an optimal test plan based on this fixed value. For a fixed value of $\boldsymbol{\theta}$, the function β becomes a constant, and only a single value of the model error needs to be determined. Hence, to cope with the optimal test plan's dependence on β, test planners can also assume a maximum value for the model error, $\tilde{\beta}$, and develop a test plan based on the fixed maximum value. Using this approach, a test planner can avoid explicitly determining the function $\beta(\boldsymbol{\theta})$ (analogous to a local design; see Ref. 12, section 17.4) and develop a test plan so that the estimate of system reliability is robust to modeling errors (i.e., worst-case analysis).

The MLE of $\boldsymbol{\theta}$ with a fixed model error, $\tilde{\beta}$, is assessed by modifying the constraint in Eq. 1 such that it is $\rho = h_{\tilde{\beta}}(\boldsymbol{\theta}, \tilde{\beta}) \equiv h(\boldsymbol{\theta}) + \tilde{\beta}$. The addition of the modeling error to the constraint does *not* change the log-likelihood function of the general MLE. However, the relationship between ρ and the ρ_j differs, and so the MLE of $\boldsymbol{\theta}$ differs from what it would be if there were no modeling error. The MSE of the general maximum likelihood estimator is composed of the asymptotic variance of the estimate from Eq. 5 and the approximate expected bias of the estimate. The expression for the MSE is

$$E\left[(h(\boldsymbol{\theta})-\rho)^2\right] \approx h'(\boldsymbol{\theta})^T \boldsymbol{F}(\boldsymbol{\theta})^{-1} h'(\boldsymbol{\theta}) + (E[\hat{\rho}_{\tilde{\beta}} - \hat{\rho}])^2. \tag{7}$$

Practically, the outcome of the tests is unknown before a testing regime must be developed. Thus, the expectation of the quantity $\hat{\rho}_{\tilde{\beta}} - \hat{\rho}$, where $\hat{\rho}_{\tilde{\beta}}$ is the MLE of ρ given the deterministic error $\tilde{\beta}$, is useful for test sizing and evaluating estimator accuracy.

$$E(\hat{\rho}_{\tilde{\beta}} - \hat{\rho}) \approx \tilde{\beta} \left.\frac{\partial h_{\tilde{\beta}}}{\partial \boldsymbol{\theta}}\right|_{E[\hat{\boldsymbol{\theta}}]}^{T} \boldsymbol{F}(\boldsymbol{\theta})^{-1} E\left[\frac{\partial g_{\mathrm{MLE}}(\boldsymbol{\theta}, \tilde{\beta})}{\partial \tilde{\beta}}\right]_{\tilde{\beta}=0} + \tilde{\beta}, \tag{8}$$

where the function $g_{\mathrm{MLE}}(\boldsymbol{\theta}, \tilde{\beta})$ is the first derivative of the likelihood function with the function h replaced with $h_{\tilde{\beta}}$ (Ref. 19). Note that for implementing a local design, $\boldsymbol{\theta}$ and $E[\hat{\boldsymbol{\theta}}]$ are replaced with a nominal estimate of the parameter vector. Equation 8 is an approximation for the bias in the MLE, for a specific test plan $n, n_1, \ldots n_p$, given h, a nominal estimate of $\boldsymbol{\theta}$, and the maximum model error, $\tilde{\beta}$. When *only* full-system tests are planned, model error does not contribute any bias to the full-system reliability MLE. When full-system tests are *not* planned, $E\left[\hat{\rho}_{\tilde{\beta}} - \hat{\rho}\right] = \tilde{\beta}$ and $\tilde{\beta}^2$ is the bias squared term of the MSE for the MLE.

EXAMPLE 1: UNCERTAINTY REDUCTION FROM COMBINING TEST DATA

For an asymptotically efficient MLE, increasing the estimation sample size reduces the asymptotic uncertainty (increases the statistical information) about the variate being estimated. This example was designed to demonstrate that the general MLE method, described

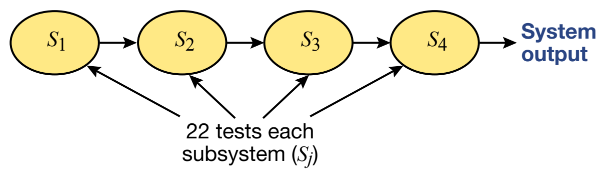

Figure 1. A simple series system with four independent subsystems.

in the section on MLE formulation and Ref. 2, decreases the asymptotic uncertainty about the system reliability estimate when subsystem reliability test data are added to full-system reliability test data. A variation on this example demonstrates that the benefit, in terms of decreased uncertainty in the full-system reliability estimate, can still be realized if there is error in the model that relates the subsystem reliabilities to the full-system reliability.

Consider the system with four independent subsystems in series depicted in Figure 1. Assume that each subsystem is tested 22 times. Further assume that the true reliability of each subsystem is 0.987. This implies a full-system reliability of $0.987^4 = 0.95$. This formulation assumes no uncertainty about the model specification [i.e., $\rho = h(\boldsymbol{\theta}) = \rho_1\rho_2\rho_3\rho_4$ and $\tilde{\beta} = 0$ in the section on estimator uncertainty with mis-modeling].

The system reliability and the asymptotic 90% lower confidence limit about the true system reliability are plotted as a function of the number of full-system tests in Figure 2. The lower confidence limit is computed using two different samples of data. First, the lower confidence limit is computed using only full-system test data. Second, the lower confidence limit is computed using full-system test data and all available subsystem test data (22 tests for each subsystem). As expected, the lower confidence limit computed from full-system test data alone is below the lower confidence limit from the combined full-system and subsystem data samples. This indicates that the estimation uncertainty about the full-system reliability estimate is decreased by adding the available subsystem test data to the full-system test data. Also, the difference between the two confidence intervals depicted in Figure 2 is greatest when the number of full-system tests is small. Thus, in this example, the greatest potential for decreasing the estimation uncertainty exists when adding the subsystem test data to a few tests of the full system. The example also illustrates that by combining the system and subsystem test data via MLE, test planners require fewer full-system tests to meet evaluation objectives in terms of statistical confidence.

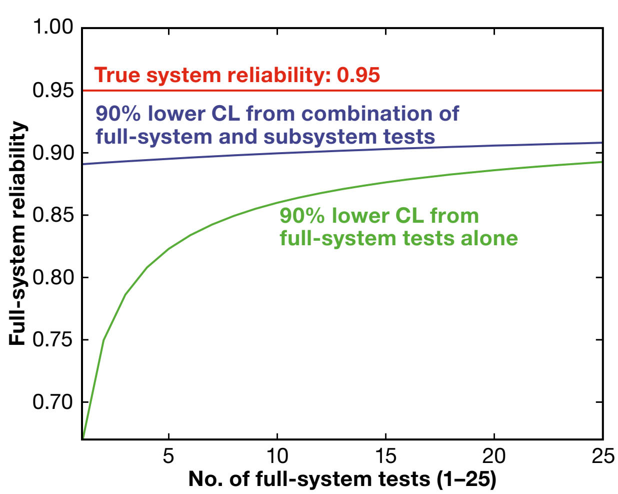

Figure 2. The 90% asymptotic lower confidence limits (CLs) about the true reliability of a hypothetical series system. (It is assumed that there is no model error; i.e., $\tilde{\beta} = 0$ in the section on estimator uncertainty with mis-modeling.)

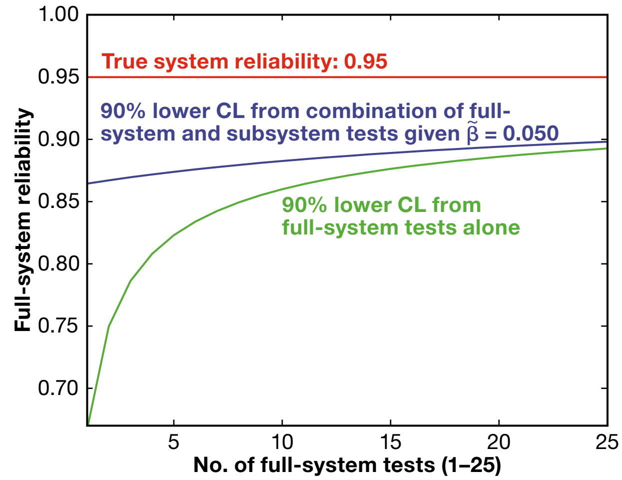

Figure 3. The 90% lower confidence limits (CLs) about the true reliability of a hypothetical series system given a maximum model error $\tilde{\beta} = 0.050$.

As discussed in the section on estimator uncertainty with mis-modeling, the model that relates the subsystem reliabilities to the system reliability may be in error. Because the subsystem test data are combined with full-system test data that are not subject to model error, the addition of a deterministic model error to the model does not uniformly increase the uncertainty in the full-system reliability estimate as the number of full-system tests is increased. To illustrate this property, the 90% lower confidence limit about the true system reliability is modified. Specifically, the asymptotic variance used in the confidence bound computation is replaced with the MSE, computed via Eq. 7 with a maximum model error of $\tilde{\beta} = 0.050$. The resulting bound is plotted in Figure 3 and represents the uncertainty in the full-system reliability estimate assuming a maximum model error of $\tilde{\beta} = 0.050$. The resulting bound is not a simple translation of the bound in Figure 2. In addition, the figure shows that, even under assumption of modest model error, there is advantage, in terms of reducing the estimate uncertainty, to combining the subsystem and full-system reliability test data using the MLE method, especially when there are few full-system tests available.

EXAMPLE 2: AIRCRAFT-MISSILE SERIES SYSTEM

The second example consists of three parts. First, the MLE method described in the section on MLE formulation and Ref. 2 is applied to test data from a certain series air-to-air heat-seeking missile system described in Ref. 6. Second, the test data from Ref. 6 are modified to illustrate the effect on the MLE of increasing system and subsystem sample sizes. Third, the prior information from Ref. 6 is used as the basis for designing a hypothetical robust test program for the example system using the method described in the section on estimator uncertainty with mis-modeling and Ref. 3. In particular, comparisons are drawn between the reliability estimates from the general MLE method here and the naive maximum likelihood system and subsystem reliability estimates of the form (number of successes/total number of tests). To avoid confusion, these estimates of the individual subsystems or full system are referred to as ratio estimates.

Aircraft-Missile System Reliability

For simplicity, the aircraft-missile system example in Ref. 6 is restricted to the aircraft and aircraft-to-missile interface, which consists of nine subsystems in series. The subsystems, the binomial test data, and the ratio estimates for the aircraft system and its subsystems are listed in Table 1. The product of the subsystem ratio estimates is 0.954, which is larger than the full-system ratio estimate. The aircraft system and subsystem MLEs are also listed in Table 1. The subsystem MLEs are slightly smaller than the ratio estimates, and the aircraft system MLE is larger than the system-level ratio estimate. The MLEs represent a compromise between the ratio estimates from subsystem and system test data. The MLEs reflect increased information about the subsystem and system reliability as a result of combining the test data; the degree of change in the estimates depends on the statistical information in the data samples used for estimation (represented by the Fisher information; see the section on theoretical properties).

The two-sided 90% confidence interval on the aircraft system reliability is computed using the large-sample approach (the inverse Fisher information in Ref. 8, Corollary 4.1, is used as the asymptotic variance) and using the bootstrap method described in the section on confidence bounds. The asymptotic 90% confidence interval on the MLE of system reliability is (0.921, 0,962), and the 90% bootstrap confidence interval from 500 Monte Carlo trials is (0.920, 0.961). In this case, the two confidence intervals are very similar. The MLE of aircraft system reliability and 90% confidence interval are much different from the aircraft system reliability estimate and 90% credible interval derived from the posterior distribution presented in Ref. 6. The posterior mean and 90% credible interval derived from the posterior distribution are 0.927, (0.925, 0.928). The Bayesian aircraft system reliability estimate is significantly lower than the MLE and ratio estimate because the Bayesian reliability estimate is heavily influenced by the prior distributions selected for the subsystem and system reliabilities (prior means are listed in Table 4). The total number of prior parameters needed to form the Bayesian estimate is 20. The Bayesian estimate also provides a 90% credible interval on the system reliability estimate that is much smaller than the 90% confidence interval provided by the MLE approach because the additional prior information narrows the distribution about the posterior mean (i.e., reduces the estimate uncertainty).

MLE Estimate Sensitivity to Sample Size

The aircraft-missile system example described above is modified to illustrate the effect of (1) increased full-system testing and (2) increased subsystem testing on the MLEs of system and subsystem reliability.

Table 1. Subsystem and system test data and MLE reliability estimates for the aircraft system

Subsystem	No. of Tests[a]	No. of Successes[a]	No. of Successes / No. of Tests	MLE
1. Flight structure	130	129	0.992	0.990
2. Avionics	130	130	1.000	1.000
3. Power	130	129	0.992	0.990
4. Flight control	130	129	0.992	0.990
5. Environmental	130	130	1.000	1.000
6. Acquisition/fire control	250	247	0.988	0.986
7. Launching	130	129	0.992	0.990
8. Missile interface	250	249	0.996	0.995
9. Human intervention	130	130	1.000	1.000
Aircraft system	205	191	0.932	0.941

[a]Data are from Ref. 6.

Table 2. Modified subsystem and system test data and MLE reliability estimates for the aircraft system demonstrating the effect of increased system testing

Subsystem[a]	No. of Tests	No. of Successes	No. Successes / No. Tests	MLE
1. Flight structure	130	129	0.992	0.988
2. Avionics	130	130	1.000	1.000
4. Flight control	130	129	0.992	0.988
6. Acquisition/fire control	250	247	0.988	0.985
Aircraft system	**1,000**	**932**	0.932	0.935

[a]Four of nine shown.

Bold denotes the change in the data for the purpose of example. See the associated text for details.

To demonstrate the sensitivity of the MLEs to full-system testing, the number of full-system tests is increased to 1,000 and the total number of successful tests is increased such that the system reliability ratio estimate remains 0.932 (the same as in the original example). The resulting MLEs of the aircraft system and its subsystems are in Table 2 (four of nine subsystems shown to save space). The 90% confidence interval on the system estimate is (0.923, 0.947). The aircraft system MLE is much closer to the system reliability ratio estimate; it reflects the additional information from increased full-system testing. The system reliability confidence interval width is significantly decreased from the original example (the confidence interval shrinks from 0.041 to 0.024 as the full-system sample size increases from 205 to 1,000), representing more certainty about the system reliability estimate. The additional information from increased system testing also decreases the MLEs of the subsystem reliabilities. They are smaller than the subsystem ratio estimates and MLE estimates in the original example (see Table 1).

To demonstrate the MLEs' sensitivity to subsystem testing, the number of subsystem tests about the acquisition/fire control subsystem is increased to 1,000 and the total number of successful tests is increased such that the subsystem reliability ratio estimate remains 0.988 (the same as in the original example). The resulting MLEs of the aircraft system and its subsystems are in Table 3 (four of nine subsystems shown to save space). The 90% confidence interval on the system estimate is (0.922, 0.962). The aircraft system MLE is slightly larger (but virtually unchanged) from the original example; it reflects the additional information from increased subsystem testing. The increased subsystem testing results in a very stable estimate of the acquisition/fire control subsystem. It is identical to the subsystem ratio estimate, and the MLE-derived 90% confidence interval about the acquisition/fire control subsystem is (0.982, 0.993) versus (0.975, 1.004) for the flight structures subsystem. The MLE system reliability confidence interval width is virtually unchanged from the original example (it shrinks from 0.041 to 0.040 as the subsystem sample size increases from 205 to 1,000), indicating that increasing the testing about one subsystem does not greatly improve confidence in the system reliability estimate. The subsystem MLEs are slightly increased to reflect the additional information about the acquisition/fire control subsystem (compare Tables 1 and 3).

Optimum Test Planning

In this section, the optimal combination of system and sets of subsystem tests, in terms of total test plan cost, is determined for the aircraft series system using the methodology described in the section on estimator uncertainty with mis-modeling. (The robust test planning method of Ref. 3 is not restricted to optimizing the number of sets of subsystem tests for test sizing. The number of sets of subsystem tests are optimized herein to simplify the presentation of the approach.) Let the pre-

Table 3. Modified subsystem and system test data and MLE reliability estimates for the aircraft system demonstrating the effect of increased subsystem testing

Subsystem[a]	No. of Tests	No. of Successes	No. of Successes / No. of Tests	MLE
1. Flight structure	130	129	0.992	0.989
2. Avionics	130	130	1.000	1.000
4. Flight control	130	129	0.992	0.989
6. Acquisition/fire control	**1,000**	**988**	0.988	0.988
Aircraft system	205	191	0.932	0.942

[a]Four of nine shown.

Bold denotes the change in the data for the purpose of example. See the associated text for details.

Table 4. Initial reliability estimates for aircraft subsystems derived from subsystem prior distributions in Ref. 6

Subsystem	Initial Reliability Estimate
1. Flight structure	0.989
2. Avionics	0.984
3. Power	0.992
4. Flight control	0.989
5. Environmental	0.994
6. Acquisition/fire control	0.992
7. Launching	0.996
8. Missile interface	0.996
9. Human intervention	0.971
Aircraft system[a]	0.907

[a]Product of subsystem reliabilities.

sumed reliabilities of the nine subsystems be equal to the prior means (see Table 4). These estimates represent the best knowledge, information, and experience about the system before testing has begun. Among other reasons, model error may arise because some of the subsystems are dependent or because a subsystem is left out of the subsystem definitions or test plan. The methodology described in the section on estimator uncertainty with mis-modeling and Ref. 3 allows a test planner to assume that the system reliability model may be incorrect and to supply a maximum model error, $\tilde{\beta}$. The model error contributes a bias to the MSE of the general maximum likelihood estimator based on the number of full-system/subsystems tests planned. Loosely, full-system tests contribute unbiased information to the general maximum likelihood estimator. Thus, as the number of full-system tests increases relative to the number of sets of subsystem tests, the model error contributes less to the bias term of the MSE.

To achieve an MSE of 0.003 or less (root mean squared error 0.050 or less), many different test plans can be devised. Thus, the design of a test plan should also account for the cost of the tests. To illustrate the effect of cost on the test plan design, assume that a set of subsystem tests costs one fourth as much as one full-system test. The cost benefit, relative to only performing full-

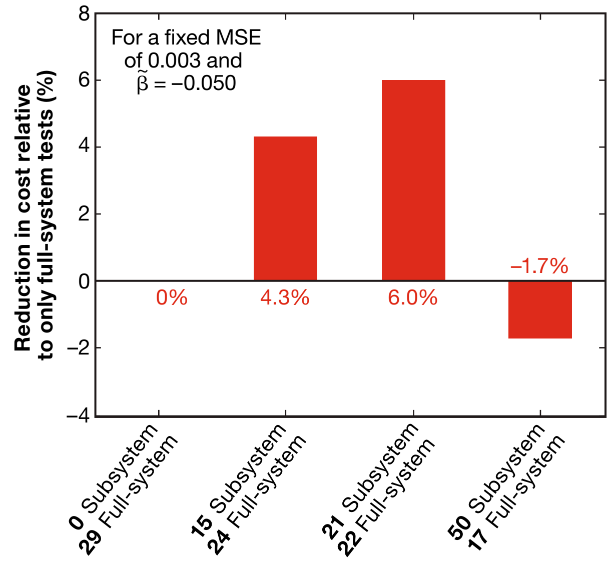

Figure 4. The potential cost reduction from performing a mixture of full-system and subsystem tests instead of performing only full-system tests; numerical values in the labels are the number of sets of subsystem tests and number of full-system tests.

system tests, is depicted in Figure 4. Four test plans are listed, each having an MSE of 0.003, given $\tilde{\beta} = -0.050$. The baseline test plan consists of performing only full-system tests. The other three test plans consist of a mixture of full-system and subsystem tests. The potential cost reduction from performing one of these three test plans instead of performing only full-system tests is plotted as a percentage. For $\tilde{\beta} = -0.050$, the least costly test plan of the three consists of 21 sets of subsystem tests and 22 full-system tests. If several other test plans have the same total cost, it is optimal to perform the maximum number of full-system tests that can be performed while achieving the desired MSE for the least cost.

EXAMPLE 3: LPCI SYSTEM

A third example is carried out on a highly reliable LPCI system in a commercial nuclear-power reactor. The system provides coolant to the reactor vessel during

Table 5. Subsystem and system test data and MLE reliability estimates for the LPCI system

Subsystem	No. of Tests	No. of Successes	No. of Successes / No. of Tests	MLE
1. Pump 1	240	236	0.98333	0.98341
2. Pump 2	240	238	0.99167	0.99168
3. Pump 3	190	189	0.99474	0.99478
4. Check valve (CV) 1	14,232	14,231	0.99993	0.99993
5. CV 2	240	240	1.00000	1.00000
6. Motor-operated valve (MOV)	470	469	0.99787	0.99790
LPCI system	200	200	1.00000	1.00000

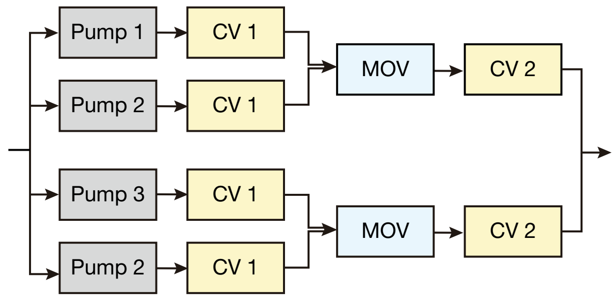

Figure 5. LPCI system block diagram. (Modified from Ref. 7.)

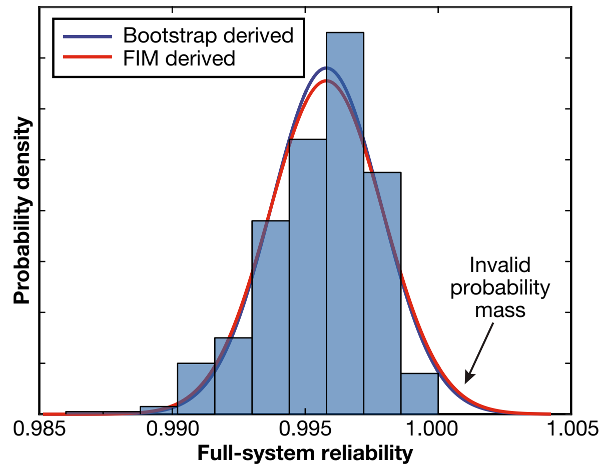

Figure 6. Histogram of the bootstrap MLE reliability estimates, the Fisher information–derived asymptotic density function about the MLE (FIM derived), and the normal density function estimated from the sample of bootstrap MLEs (bootstrap derived); the Fisher information–derived asymptotic density function and the bootstrap-derived normal density function incorrectly distribute a significant portion of the probability density above 1.0.

accidents in which the vessel pressure is low. The system consists of six subsystems (listed in Table 5), some of which are repeated in a mixed series/parallel system configuration (see Figure 5). The system is designed with redundancy to achieve extremely high reliability, as the system is critical to avoiding catastrophic failure in the event of an emergency (because the system reliability is so close to 1.0, to aid interpretation, some reliability estimates are also given in terms of the probability of failure). The system was used in Ref. 7 to illustrate Bayesian reliability estimation methods.

The LPCI system reliability estimate from subsystem data alone is 1.0 to five decimal places. With 200 successful tests of the full system, the system reliability MLE is virtually unchanged. However, the subsystem reliability MLEs increase slightly with information from the 200 successful system-level tests.

To illustrate the properties of the MLE method, consider the following modification to the example. Let the test data on CV 2 and the test data about the LPCI system be modified so that the reliability estimates from system and subsystem data alone disagree significantly (see Table 6). In this case, the system reliability estimate from subsystem testing alone is 0.99985 (or the probability of failure is 1.47×10^{-4}). However, the LPCI system reliability MLE is significantly smaller at 0.99580 (or the probability of failure is more than one order of magnitude greater at 4.19×10^{-3}). Again, the MLEs represent a compromise between the ratio estimates from subsystem and system test data; the degree of change in the estimates depends on the statistical information in the data sample used for estimation. In this case, the CV 2 subsystem MLE differs the most from its ratio estimate because it has the largest statistical variance of any subsystem estimate and, based on the system configuration (see Figure 5), it has the largest potential of any subsystem to affect the system-level reliability estimate.

We now compute the confidence interval for the MLE of the LPCI system in Table 6 using the bootstrap method. The confidence interval for the estimate is computed using 500 Monte Carlo bootstrap samples from the method described in the section on confidence bounds. The 90% bootstrap confidence interval on the LPCI system is (0.99196, 0.99864). [The confidence interval on the probability of failure is (0.00804, 0.00136).] Because the system reliability is so close to

Table 6. Modified subsystem and system test data and MLE reliability estimates for the LPCI system

Subsystem	No. of Tests	No. of Successes	No. of Successes / No. of Tests	MLE Estimate
1. Pump 1	240	236	0.98333	0.98333
2. Pump 2	240	238	0.99167	0.99167
3. Pump 3	190	189	0.99474	0.99474
4. CV 1	14,232	14,231	0.99993	0.99993
5. CV 2	**100**	**99**	0.99000	0.93760
6. MOV	470	469	0.99787	0.99741
LPCI system	**500**	**495**	0.99000	0.99580

Bold denotes the change in the data for the purpose of example. See the associated text for details.

unity, the interval is not symmetric about the MLE estimate. In fact, the normality of the uncertainty about the estimate is questionable given the proximity of the estimate to 1.0. A histogram of the estimates from the bootstrap Monte Carlo procedure is plotted in Figure 6 alongside the Fisher information–derived asymptotic density function about the MLE. For comparison, the bootstrap MLEs in the sample were pooled to estimate the parameters of a normal density function, which is also plotted in Figure 6. The FIM- and bootstrap-derived normal density functions agree well, but these approximations incorrectly distribute a portion of the probability density above 1.0. Note that the bootstrap-derived density is only for purposes of illustrating the closeness of the bootstrap- and FIM-based distributions; actual confidence interval calculations with the bootstrap would only use the data generating the histogram (all ≤ 1.0).

CLOSURE AND FUTURE WORK

This article discusses a general method for reliability analysis that combines information from the testing of subsystems and the full system. It also discusses three examples of the general MLE method for reliability. The examples illustrate a few of the important properties of the method. Namely, the method appropriately combines data from subsystem and full-system reliability tests based on the statistical information in the respective samples. In addition, an extension of the method enables robust test plans to be developed for system reliability estimation involving trade-offs among the MSE (estimation accuracy), the degree of modeling error, and the cost of doing full-system and subsystem tests.

The general maximum likelihood method of reliability estimation combines static success/failure data from subsystem tests and full-system tests. Test data for each subsystem and the full system are assumed to be independent and identically distributed. However, reliability test data can be dependent on dynamic external performance predictors such as age, temperature, manufacturing lot, etc. (for example see Ref. 20). This leads to system and subsystem test data that are independent but not identically distributed. Future work includes extending the general method of Ref. 2 so that the model for the system also reflects dynamic subsystems reliabilities. To this point, a framework for MLE of system reliability from full-system and subsystem tests with dependence on dynamic inputs is established in Ref. 21. However, significant theoretical work remains for the dynamic case to establish the conditions for convergence, convergence rate, and the asymptotic distribution covariance.

The ability to rigorously integrate data from subsystems and full systems has proven to be a flexible and widely applicable capability, including in problems that are not formally presented as reliability. For example, Refs. 22–24 use the formulation to model urban transportation networks, where the full system is the complete network and the subsystems are individual links in the network. References 25 and 26 consider an application in sensor networks, where the full system is a wide-area sensor covering a full region of interest and the subsystems are localized sensors covering only subsets of the region of interest. The method provides for an optimal integration of sensor measurements for area-wide understanding. Reference 9 shows how the method can be extended to estimate structural integrity of shear walls in structures, where the full system is the wall itself and the subsystems are connections between panels forming the wall. The necessary methodological extension for the shear wall application is to the case where the subsystem data are no longer binary, but rather are assumed to be Gaussian distributed. (Another methodological extension is in Ref. 10, where the full-system parameters may include multiple parameters appearing in an exponential family distribution; in particular, the extension goes beyond the scalar mean parameter used above.) Overall, the method provides the analyst a rigorous and practical approach to achieve cost savings and more accurate estimates in test and evaluation through use of all available data.

REFERENCES

[1] C. J. Maranzano and J. C. Spall, "Implementation and application of maximum likelihood reliability estimation from subsystem and full system tests," in *PerMIS'10, Proc. Performance Metrics for Intelligent Systems Workshop*, Baltimore, MD, pp. 146–153, 2010, https://doi.org/10.1145/2377576.2377604.

[2] J. C. Spall, "System reliability estimation and confidence regions from subsystem and full system tests," in *Proc. American Control Conf.*, St. Louis, MO, pp. 5067–5072, 2009, https://doi.org/10.1109/ACC.2009.5160460.

[3] C. J. Maranzano and J. C. Spall, "Robust test design for reliability estimation with modeling error when combining full system and subsystem tests," in *Proc. Amer. Control Conf.*, Baltimore, MD, pp. 3741–3746, 2010, https://doi.org/10.1109/ACC.2010.5531454.

[4] V. Johnson, T. Graves, M. Hamada, and C. Reese, "A hierarchical model for estimating the reliability of complex systems," *Bayesian Statistics*, vol. 7, pp. 199–213, 2003.

[5] S. D. Hill, J. C. Spall, and C. J. Maranzano, "Inequality-based reliability estimates for complex systems," *Nav. Res. Logist.*, vol. 60, no. 5, pp. 367–374, 2013, https://doi.org/10.1002/nav.21539.

[6] H. F. Martz, R. A. Wailer, and E. T. Fickas, "Bayesian reliability analysis of series systems of binomial subsystems and components," *Technometrics*, vol. 30, no. 2, pp. 143–154, 1988, https://doi.org/10.1080/00401706.1988.10488361.

[7] H. F. Martz and R. A. Wailer, "Bayesian reliability analysis of complex series/parallel systems of binomial subsystems and components," *Technometrics*, vol. 32, no. 4, pp. 407–416, 1990, https://doi.org/10.1080/00401706.1990.10484727.

[8] J. C. Spall, "Identification for systems with binary subsystems," *IEEE Trans. Automat. Contr.*, vol. 59, no. 1, pp. 3–17, 2014, https://doi.org/10.1109/TAC.2013.2275664.

[9] L. Wang, G. Bian, J. C. Spall, and B. W. Schafer, "Combining subsystem and full system data with application to cold-formed steel shear wall," in *Proc. American Control Conf.*, Milwaukee, WI, pp. 272–277, 2018, https://doi.org/10.23919/ACC.2018.8431830.

[10] L. Wang and J. C. Spall, "Beyond the identification of reliability for system with binary subsystems," in *Proc. Amer. Control Conf.*, Seattle, WA, pp. 158–163, 2017, https://doi.org/10.23919/ACC.2018.8431830.

[11] J. C. Spall, "Convergence analysis for maximum likelihood-based reliability estimation from subsystem and full system tests," in *Proc. 49th IEEE Conf. on Decision and Control*, Atlanta, GA, pp. 2017–2022, 2010, https://doi.org/10.1109/CDC.2010.5717898.

[12]J. C. Spall, *Introduction to Stochastic Search and Optimization: Estimation, Simulation, and Control*. Hoboken, NJ: Wiley Interscience, 2003, https://doi.org/10.1002/0471722138.

[13]G. Goodwin and R. Payne, *Dynamic System Identification: Experiment Design and Data Analysis*. New York: Academic Press, 1977.

[14]P. Stoica and T. L. Marzetta, "Parameter estimation problems with singular information matrices," *IEEE Trans. Signal Process.*, vol. 49, no. 1, pp. 87–90, 2001, https://doi.org/10.1109/78.890346.

[15]M. A. Gevers, A. S. Bazanella, X. Bombois, and L. Miskovic, "Identification and the information matrix: How to get just sufficiently rich?" *IEEE Trans. Automat. Contr.*, vol. 54, no. 12, pp. 2828–2840, 2009, https://doi.org/10.1109/TAC.2009.2034199.

[16]B. Efron and R. Tibshirani, "Bootstrap methods for standard errors, confidence intervals, and other measures of statistical accuracy," *Stat. Sci.*, vol. 1, no. 1, pp. 54–75, 1986.

[17]L. Ljung, *System Identification Theory for the User*, 2nd ed. Upper Saddle River, NJ: Prentice Hall, 1999.

[18]M. L. Aronsson, L. Arvastson, J. Holst, B. Lindoff, and A. Svensson, "Bootstrap control," *IEEE Trans. Automat. Contr.*, vol. 51, no. 1, pp. 28–37, 2006, https://doi.org/10.1109/TAC.2005.861722.

[19]C. J. Maranzano and J. C. Spall, "Optimum combination of full system and subsystem tests for estimating the reliability of a system," in *PerMIS'09, Proc. Performance Metrics for Intelligent Systems Workshop*, New York, NY, 2009, pp. 73–80, https://doi.org/10.1145/1865909.1865924.

[20]C. J. Maranzano and R. Krzysztofowicz, "Bayesian reanalysis of the Challenger O-ring data," *Risk Anal.*, vol. 28, no. 4, pp. 1053–1067, 2008, https://doi.org/10.1111/j.1539-6924.2008.01081.x.

[21]C. J. Maranzano and J. C. Spall, "Framework for estimating system reliability from full system and subsystem tests with dependence on dynamic inputs," in *Proc. 50th IEEE Conf. on Decision and Control*, Orlando, FL, pp. 6666–6671, 2011, https://doi.org/10.1109/CDC.2011.6161226.

[22]X. Zhao, "Modeling transportation networks and urban traffic dynamics: A Markovian framework," master's thesis, Johns Hopkins University, 2017.

[23]X. Zhao and J. C. Spall, "Estimating travel time in urban traffic by modeling transportation network systems with binary subsystems," in *Proc. Amer. Control Conf.*, Boston, MA, pp. 803–808, 2016, https://doi.org/10.1109/ACC.2016.7525012.

[24]X. Zhao and J. C. Spall, "Modeling traffic networks using integrated route and link data," arXiv, Nov. 4, 2018, http://arxiv.org/abs/1811.01314.

[25]K. Hernández and J. C. Spall, "System identification for multi-sensor data fusion," in *Proc. Amer. Control Conf.*, Chicago, IL, pp. 3931–3936, 2015, https://doi.org/10.1109/ACC.2015.7171943.

[26]L. Wang and J. C. Spall, "Multilevel data integration with application in sensor networks," in *Proc. Amer. Control Conf.*, Jul. 1–3, 2020, pp. 5213–5218, https://doi.org/10.23919/ACC45564.2020.9148012.

Coire J. Maranzano, Force Projection Sector, Johns Hopkins University Applied Physics Laboratory, Laurel, MD

Coire J. Maranzano is a member of APL's Principal Professional Staff. He is a graduate of the University of Virginia, where he earned his doctoral degree from the Department of Systems and Information Engineering. In graduate school, he performed research in the field of Bayesian forecasting and decision science. Since beginning his career at APL in 2006, he has contributed to methodologies for accuracy and reliability estimation of complex systems. He is currently the strike weapons program area manager in the Precision Strike Mission Area. His email address is coire.maranzano@jhuapl.edu.

James C. Spall, Force Projection Sector, Johns Hopkins University Applied Physics Laboratory, Laurel, MD

James C. Spall is a member of the Principal Professional Staff at APL and a research professor in the Johns Hopkins University (JHU) Department of Applied Mathematics and Statistics. He is also chair of the Applied and Computational Mathematics program and co-chair of the Data Science program within JHU's engineering and applied science programs for professionals. Dr. Spall has published extensively in the areas of control and statistics and is the author of several highly cited journal articles, including the most-cited *Johns Hopkins APL Technical Digest* article (vol. 19, no. 4, pp. 482–492, 1998). He holds two US patents for inventions in control systems, both licensed to US companies. He is the editor and coauthor of the book *Bayesian Analysis of Time Series and Dynamic Models* (Marcel Dekker, now CRC Press, 1988) and the author of *Introduction to Stochastic Search and Optimization* (Wiley, 2003). He was one of the inaugural senior editors for *IEEE Transactions on Automatic Control* (2009–2017) and the program chair for the 2007 IEEE Conference on Decision and Control and is a fellow of IEEE. His email address is james.spall@jhuapl.edu.