Integrating Cost and Performance Models to Determine Requirements Allocation for Complex Systems

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The engineering of complex systems or a "system of systems" has become increasingly problematic in recent years, yet effective "architecting" approaches that enable cost/performance trades are still immature. This article describes a systematic approach to allocating top-level system-of-systems requirements to component systems, which has been demonstrated on a naval mine countermeasures system-of-systems representation. This integrated analysis produces system effectiveness as a function of cost, corresponding subsystem requirements allocations, and a corresponding force structure or inputs to an overarching force-level cost/performance analysis. Variants of this approach are now being applied to support cost/performance analyses for the Navy Theater Wide Program and to focus future science and technology investments for mine countermeasures. (Keywords: Cost-effectiveness analysis, Nonlinear programming, Requirements allocation, Stochastic optimization, Systems engineering.)

BACKGROUND

The engineering of complex "systems of systems" has been receiving increased attention recently. System-of-systems terminology is now widely used to describe how the successful, combined operation of many platforms, weapon systems, and communication systems is necessary to achieve an overall warfare objective, especially in Joint operations. This increased level of complexity has become a concern at the highest levels of command, as General John Sheehan, former Commander in Chief of U.S. Atlantic Forces, has observed: "Victory will depend on the ability to master the 'system of systems' composed of multiservice hard- and soft-kill capabilities linked by advanced information technologies."[1]

These systems of systems have arisen not by design, but in response to the vision of users who recognize the tremendous potential of systems working together toward broad, common objectives, as expressed by Admiral William Owens,[2] Vice Chairman of the Joint Chiefs of Staff:

> We have cultivated a planning programming and budgeting system that tends to handle programs as discrete entities. . . .Yet, the interactions and synergisms of these systems constitute something new and very important. What is happening is driven in part by broad conceptual architectures—and in part by serendipity: It is the creation of a new system of systems.

Although the characteristics and systems engineering challenges associated with systems of systems are becoming well understood, effective "architecting" approaches are still immature.[3,4] Until successful methodologies have been demonstrated, there will be little justification for the services to move away from the current acquisition focus on single systems procurements.

This article addresses how best to upgrade a complex system of systems. A quantitative methodology for requirements allocation to formulate an optimal upgrade suite under cost and technology constraints is demonstrated. The methodology uses a multidisciplinary approach including operations analysis, cost modeling, nonlinear optimization, and stochastic modeling and simulation (M&S). Appropriate sensitivity analyses on technology constraints can help guide an effective technology investment strategy.

SYSTEMS-OF-SYSTEMS DEFINITIONS AND CONCEPTS

A complex system of systems generally has the following characteristics[5]:

- It comprises several independently acquired systems, each under a nominal systems engineering process.
- Time phasing between each system's development is arbitrary and not contractually related.
- System couplings are interdependent.
- Individual systems are generally unifunctional.
- Optimization of each system does not guarantee overall system-of-systems optimization.
- Combined operation of the systems represents satisfaction of an overall mission or objective.

Although the definition of system of systems is somewhat arbitrary, it is generally viewed as a coherent entity, considering that overall management control over the autonomously managed systems has become mandatory. Unfortunately, large, complex systems of systems are not developed under a single architecture resulting from a strategic development decision. Component systems are developed individually, and the full system of systems can evolve over decades as various leaders develop enhanced visions of how systems can be used together to achieve larger objectives. Although each system may have been justified and designed on the basis of sound systems engineering principles, its requirements and design most likely did not develop in response to concerns over the complete system-of-systems objectives.

System-of-Systems Engineering

A framework for conducting systems engineering at the system-of-systems level has been developed[6] but has not been widely accepted. The elements of system-of-systems engineering are listed below. Those aspects that require a quantitative analysis of alternatives when a system of systems is upgraded are in bold. Table 1 presents the quantitative analysis tasks required for each of these aspects. The methodology discussed in this article has been developed to support those analyses.

- Integration engineering
  - Requirements
  - Interfaces
  - Interoperability
  - **Impacts**
  - Testing
  - Software verification and validation
  - **Architecture development**
- Integration management
  - Scheduling
  - Budgeting/costing
  - Configuration management
  - Documentation
- Transition engineering
  - **Transition planning**
  - Operations assurance
  - Logistics planning
  - **Preplanned product improvement**

Table 1. System-of-systems elements requiring quantitative analysis of alternatives.

Element	Quantitative analysis task
Impacts	Compare system performance vs. requirements
	Assess effects of proposed upgrades
	Utilize M&S to predict performance
Architecture development	Define top-level functional capability
	Assure intersystem performance
	Verify system of systems is truly an integrated architecture vs. random collection of systems
	Attempt to optimize overall system performance
Transition planning	Develop transition alternatives and strategy
	Assess and select
	Document
Preplanned product improvement (P^3I)	Review all component system P^3I plans
	Identify key areas from system-of-systems perspective
	Feed results and priorities back to system activities

Recurrent Management Issues

Often a program executive officer will be responsible for a collection of system acquisition programs, each of which belongs to a larger system of systems. However, this collection may not necessarily fully constitute that system of systems. Rather than architecting an entirely new system of systems, the program executive must often decide how best to upgrade within an existing system of systems. This generally means either beginning a new acquisition program to add a new system to the overall system of systems (additional functionality) or inserting advanced technology into an existing system via an upgrade or modification process.[7]

Significant constraints are placed on these executives, including budgets, politics, ill-defined and competing mission objectives, and the technology itself. Many new initiatives have begun under the umbrella of acquisition reform to encourage acceleration of systems development time, delivery of affordable systems, and risk mitigation through the adoption of commercial off-the-shelf (COTS) components or technologies and industrial best practices. These attempts at reducing the usual acquisition cycle include such innovative and complementary measures as Advanced Technology Demonstrations and Advanced Capability Technology Demonstrations, often described, respectively, as "technology pushes" and "military need pulls."[8]

Although these initiatives promote the quick fielding of new, militarily useful technologies, they do not represent a disciplined approach to considering how best to upgrade specific, complex systems of systems under the constraints already noted. The development of such an approach is the objective of this research effort.

In summary, management issues are focused on upgrading versus systems-of-systems design because:

- All proposed systems and upgrades must fit into an existing system of systems.
- Opportunities rarely exist to architect a major system of systems from scratch.
- Requirements usually evolve in relation to legacy systems' capabilities and management.
- We can often take advantage of available models and simulations that can be adapted for decision support.

OBJECTIVES AND APPROACH

Upgrade Decisions

The decision maker is generally trying to solve one of two problems: (1) maximize the system-of-systems' performance subject to a cost constraint or (2) minimize additional cost under performance constraints. Although the former is clearly applicable to upgrading or architecting a system of systems, the latter arises in the operations and maintenance phase of a system life cycle. That is, we may wish to maintain a proven capability while reducing legacy infrastructure activities.

Although cost-reduction approaches have included "design to cost," recent DoD acquisition reform initiatives have softened hard budget allocations in favor of an approach known as cost as the independent variable (CAIV). The application of the CAIV approach requires a quantitative understanding of the relationship between cost and performance for major system elements. The representation of a system element's performance as a function of cost is referred to as a performance-based cost model (PBCM). Whereas the CAIV terminology has come to represent a specific government approach to acquisition at the individual system level, it is used here simply to indicate that system-of-systems performance will be displayed and understood as a function of the independent variable, cost.

System-of-systems upgrade decisions are reviewed annually for all warfare or program areas as part of DoD strategic planning and budgeting processes. There are four forms of upgrade options, depending on which conditions are most pressing:

1. Adding a new type of system (i.e., additional functionality) to the system of systems
2. Procuring additional numbers of existing component systems (enlarging the scope and capability of the system of systems and offering an opportunity to insert advanced technology)
3. Replacing aging or obsolescent component systems (also offering an opportunity to enhance the system-of-systems' performance and functionality through advanced technology insertion)
4. Upgrading existing component systems because of requirements pressure or availability of advanced technology

Legacy Decision Support

In assessing whether to proceed with the development of a new system or a major upgrade, DoD usually conducts an analysis of alternatives to determine whether the proposed system is the most cost-effective alternative to meeting a certified military need.[9] A typical analysis approach is to use M&S to estimate the marginal utility of proposed system point designs to a larger warfare or campaign mission objective. The system performance is represented by a set of measures of performance (MOPs), and its contribution to the mission is referred to as a measure of effectiveness (MOE).

The simulation is run on a carefully selected set of applicable scenarios, with and without the system alternatives, to characterize the hypothesized system alternatives' value-added. A multi-objective metric that combines costs and multiple MOEs into a single scalar metric may be used to compare alternatives. This metric may also attempt to reflect expert opinion as to the military value of the alternatives that are not captured by quantitative analyses owing to limitations of fidelity, scope, or tractability. A primary shortcoming of the analysis-of-alternatives process from a system-of-systems perspective is that just one component system is considered at a time, in a "stovepipe" fashion. In a cost-constrained environment, this approach will not normally generate the best alternative from the system-of-systems perspective.

The DoD acquisition community strongly prefers quantitative engineering analysis over qualitative decision support methods such as the Analytic Hierarchy Process. This is true perhaps because the community is dominated by engineers and scientists who recognize the difficulties in converting opinion and judgments into meaningful metrics, hence, the heavy emphasis on M&S as the basis for decision making. The reliance on M&S seems to be a widespread preference throughout the technical and scientific community.[10] This article attempts to provide objective, quantitative information to decision makers at the system-of-systems level, thereby minimizing the introduction of subjective judgments at the single-system level.

Proposed Approach

The challenge is to develop a quantitative process or methodology to support system-of-systems upgrade decisions to determine where the limited upgrade resources should be applied. The methodology should also help determine the optimal requirements allocation as a function of overall cost given a system-of-systems architecture. "Architecture" here implies that the system-of-systems functional requirements are well understood and are embodied in the definition of the system-of-systems scope. Whereas the architecture will specify what functions must be accomplished, the CAIV requirements allocation process must address how well each function must be performed by which component system and how many of each system are required.

The process should enable a domain-expert systems architect or engineering team to generate an optimal allocation of design requirements in accordance with a specified MOE for a particular system of systems. Here we formulate the general problem and apply it to a real-world, contemporary system of systems in sufficient detail to demonstrate the feasibility of the approach—a practical proof-of-principle demonstration. The demonstration goes beyond applying closed-form representations of system performance by using simulation to represent system effectiveness. Substantial investments have been made in system-of-systems simulations, and their use avoids the unnecessary simplification of system abstraction resulting from closed-form expressions of typical, complex system-of-systems behavior. Model fidelity and execution time must be balanced because of the intense computational burden of many contemporary warfare simulations. These considerations will drive the selection of the system-of-systems' MOE/MOP and PBCM structure.

There are seven key steps to the proposed system-of-systems CAIV optimization process (Fig. 1):

1. Define the overall system of systems, its components and functionality, and its missions or scenarios of interest.
2. Define critical MOPs and MOEs.
3. Specify initial boundary conditions for the system of systems, as necessary.
4. Formulate PBCMs for each component system by parameterizing each subsystem's cost as a function of one key MOP.
5. If possible, formulate an appropriate closed-form model that will capture the mapping from component system MOPs to system MOEs and eventually the overarching MOE. Alternatively, select an appropriate M&S implementation that evaluates the desired objective function and MOE constraints as a function of component systems' MOPs. (Constructing closed-form expressions that model the system-of-systems' top-level performance is important for initial problem understanding, but will probably not be sufficient to adequately capture system interactions and performance drivers. It will be necessary to use high-fidelity M&S to represent the complexity needed to provide credible analyses to support decisions regarding complex, high-value systems.)
6. Solve the resulting constrained nonlinear (stochastic) performance optimization problem repeatedly, gradually relaxing the overall cost constraint. A solution to a specific constrained problem formulation yields an optimal set of MOP values that represents one system-of-systems requirements allocation corresponding to the most effective system-of-systems design and force structure. The set of solutions will provide insight as to performance and design as a function of CAIV.
7. Effectively communicate results of the process to the decision maker(s). The solution will still require further evaluation to determine design implications for each system. Sensitivity studies should be conducted on secondary MOE constraints and MOP technology constraints to generate operational and technology investment strategy insights, respectively. In this way, the process supports the decision process rather than makes it.

COST/PERFORMANCE MODEL

Consider n types of systems S_i that comprise a system of systems S with the following characteristics and

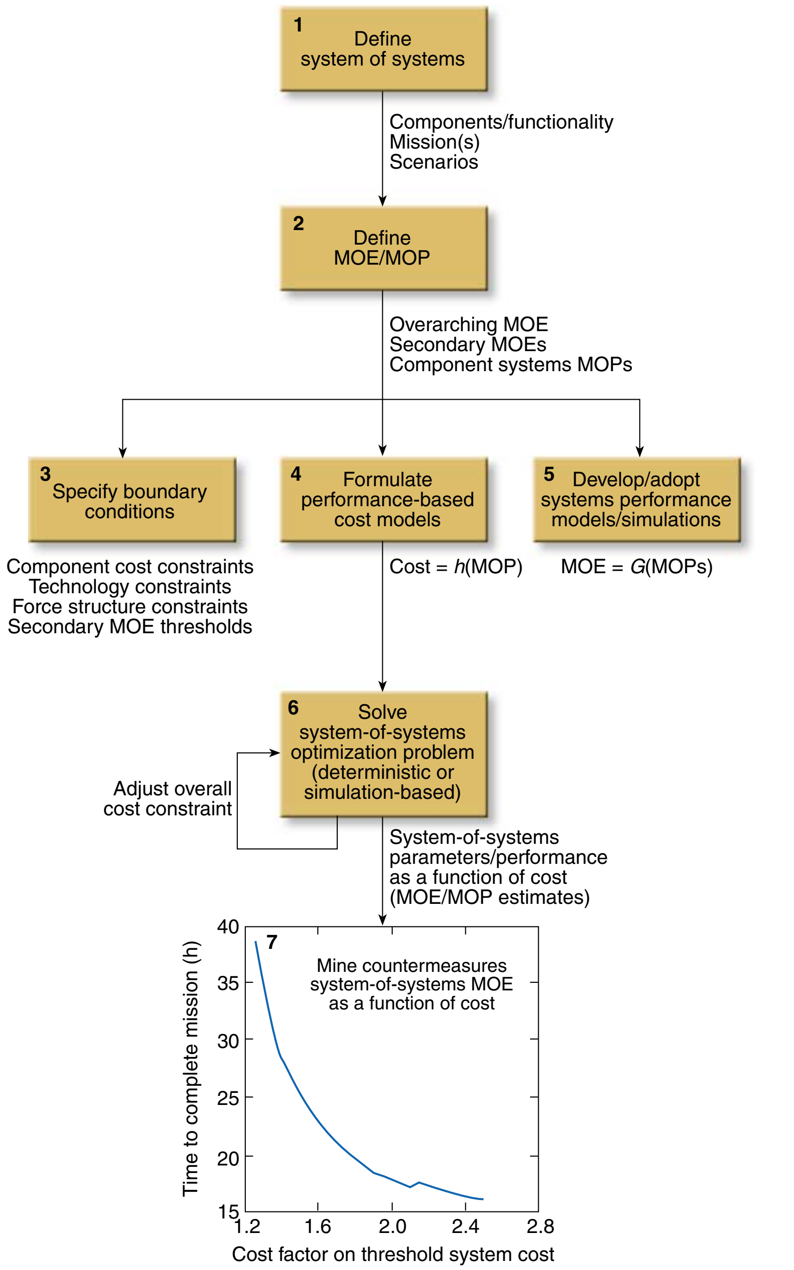

Figure 1. System-of-systems CAIV optimization process (h = hours; G is defined in the insert labeled Nomenclature).

constraints (see the Nomenclature insert for a complete list of terms and their definitions):

- $S = \{S_1, \ldots, S_n\}$.
- There are m_i systems of type i, and the total number of systems is m: $\mathbf{m} = \{m_1, \ldots, m_n\}$ and $m = \sum_{i=1}^{n} m_i$. The minimum number of each system type required for the system of systems is designated $\mathbf{m}^{\mathrm{L}}$.
- Each system type has a set of r_i MOPs: $\mathbf{p}_i = \{p_{i,1}, \ldots, p_{i,r_i}\}$. Thus, each $\mathbf{p}_i$ has dimension r_i and $r = \sum_{i=1}^{n} r_i$.
- Each system's MOPs are constrained by low-performance threshold specification values, $\mathbf{p}_i^*$, and realistic technology limitations at the high-performance end, resulting in the following upper- and lower-bound constraints: $\mathbf{p}_i^{\mathrm{L}} \le \mathbf{p}_i \le \mathbf{p}_i^{\mathrm{U}}$ or $p_{i,j}^{\mathrm{L}} \le p_{i,j} \le p_{i,j}^{\mathrm{U}}$, for all j. Note that for some parameters, such as navigation accuracy, small values are better than large values, hence $\mathbf{p}_i^*$ is not simply the lower bound, $\mathbf{p}_i^{\mathrm{L}}$. In the most general case, these MOP constraints could be functions of program schedule as well, in anticipation of requirements creep and advancing technology.
- Each system's unit cost is a nonlinear function of performance, expressed in terms of its critical MOPs: $c_i(\mathbf{p}_i) = h_i(\mathbf{p}_i)$, $\mathbf{c} = \{c_i, \ldots, c_n\}$. We denote $c_i^* = h_i(\mathbf{p}_i^*)$ as the cost associated with the threshold system. This PBCM is generated by considering each critical MOP as a cost driver of a particular subsystem, whose cost can be parameterized on that MOP. The total system-of-systems cost is then $C(\mathbf{p}) = \mathbf{m}\mathbf{c}^{\mathrm{T}}(\mathbf{p})$.
- The system of systems has one overarching MOE, E, a function of each system's set of MOPs and the number of systems: $E = G(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n)$.

It is clear from the last assumption that each system type has its own overall MOE, say E_i. From the single-system perspective, each system's overarching MOE, E_i, would only be considered as a function of its own MOPs, $\mathbf{p}_i$. But if any E_i depends on not just $\mathbf{p}_i$ but some elements of $\mathbf{p}_j$, where $i \neq j$, then we say that the system of systems is interdependent, and we would have to express the individual systems' MOEs as $E_i = f_i(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n)$. Therefore, in general, E will be a complicated function of the full set of component systems' MOPs, $E = G(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n)$, and the single-system MOEs become uninteresting from the system-of-systems perspective.

When describing a system of systems comprising relatively simple component systems or using simplified models of complex systems, we could express E as a

NOMENCLATURE

A	Reconnaissance system area coverage rate during detection pass (nmi^2/day)
$C(\mathbf{p})$	Total cost for S: $C = \mathbf{mc}^T(\mathbf{p})$
$C^*(\mathbf{p}^*)$	Cost to produce the threshold system
C_f	Number of nonmines falsely classified as minelike
C_k^U	Upper bound for cost constraint: $C_k^U = \text{costfactor}_k C^*(\mathbf{p}^*)$
C_m	Number of mines correctly classified as minelike
$c_i(\mathbf{p}_i)$	$h_i(\mathbf{p}) = \sum_{j=1}^{r_i} h_{ij}(\mathbf{p}_i)$, cost for system S_i
c_i^*	Cost for system S_i with threshold MOPs, $\mathbf{p}_i^*$
$\mathbf{c}^L$, $\mathbf{c}^U$	Lower and upper bound for $\mathbf{c}(\mathbf{p})$
$c(\mathbf{p})$	$\{c_1(\mathbf{p}_1), \ldots, c_n(\mathbf{p}_n)\}$
D_{fa}	Number of mine false alarms
D_{ft}	Number of false targets detected
D_m	Number of detected mines
d_{mine}	Average distance between mines (yd)
E	Overarching MOE for S
E_i	MOE for system S_i: $E_i = f_i(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n)$
F_0	Number of false targets contained in the MCM area $S_{minefield}$
G	Overarching MOE objective function $E = G(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n)$
$h_{i,j}(p_{i,j})$	Performance-based cost model for MOP $p_{i,j}$
M_0	Number of mines originally laid in the MCM area $S_{minefield}$
m	Total number of systems, $m = \sum_{i=1}^{n} m_i$
$\mathbf{m}$	$\{m_1, \ldots, m_n\}$
$\mathbf{m}^L$, $\mathbf{m}^U$	Lower and upper bound for $\mathbf{m}$
P_c	Probability of correctly classifying a detection as minelike or nonminelike at range R_c
P_d	Detection probability at range R_d
P_{fa}	Detection false alarm rate (false alarms/nmi^2)
P_{ID}	Probability of correct mine identification following detection and classification
P_L	Localization (or reacquisition) probability
P_α	Probability that the MCM area will be cleared to the desired minefield clearance rate α
p	Mine clearance probability, i.e., the probability that a mine in the MCM area will be cleared
$\mathbf{p}_i$	MOPs vector for system S_i: $\mathbf{p}_i = \{p_{i,1}, \ldots, p_{i,r_i}\}$
$\mathbf{p}_i^*$	Low-performance threshold specification values for $\mathbf{p}_i$
$p_{i,j}$	jth MOP for system S_i
$\mathbf{p}_i^L$, $\mathbf{p}_i^U$	Lower and upper bound for $\mathbf{p}_i$
q_T	Threshold value for mine clearance rate (quality threshold)
$q(\mathbf{x})$	Mine clearance rate, $q(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n)$
R_c	Minelike object classification range (yd)
R_d	Target detection range (yd)
R_r	Range at which S_2 has an 80% chance of reacquiring S_1's detections
r	Total number of MOPs for S: $r = \sum_{i=1}^{n} r_i$
r_i	Dimension of $\mathbf{p}_i$; number of MOPs for S_i
S	System of systems, comprising n types of systems: $S = \{S_1, \ldots, S_n\}$
$S_{minefield}$	Area to be searched (nmi^2), referred to as the MCM area
T_c	Time required to classify a mine (min)
T_{cf}	Time required to classify a nonmine (min)
T_{class}	Time required to classify all detections within the search area $S_{minefield}$ (h)
T_{detect}	Time required to complete a detection pass through the search area $S_{minefield}$ (h)
T_n	Time spent neutralizing (prosecuting) a classified mine (min)
T_{pf}	Time spent unsuccessfully attempting to reaquire a detection (min)
$T_{transit}$	Reconnaissance system transit time
V_{class}	Reconnaissance system speed during classification operations (kt)
$V_{transit}$	Reconnaissance system speed during detection and transit (kt)
$\mathbf{x}$	r-dimensional MOP vector for S: $\mathbf{x} = \{\mathbf{p}_1, \ldots, \mathbf{p}_n\}$
y	Noise-corrupted objective function measurement
α	Desired MCM area clearance rate
β	Confidence level associated with MCM area clearance rate α
λ	Minefield density (mines/nmi^2)
λ_{ft}	False target (nonmine minelike object) density (objects/nmi^2)
σ	Standard deviation of minelike object localization error (yd)
ω	Simulation-induced noise on objective function G

Note: All vectors are row vectors, hence $\mathbf{c}^T$ denotes transpose.

closed-form function of the MOPs. The simplified (but realistic) naval mine countermeasures (MCM) example developed here has a closed-form, nonlinear expression for E, which is intuitive and quite useful. However, MOPs are themselves typically sensitive to scenarios, concepts of operations (CONOPS), and environments. So to obtain representative, robust, full-fidelity results, it will generally be necessary to use a simulation to evaluate G.

In addition to the constraints on MOPs given in the preceding list, several other constraints can occur and should be considered:

- *Force structure constraints*. There is generally a practical operational or programmatic limitation as to how many systems of each type can comprise the system of systems, known as force structure constraints: $\mathbf{m}^L \le \mathbf{m} \le \mathbf{m}^U$ or $m_i^L \le m_i \le m_i^L$, for all *i*.
- *System effectiveness constraints*. Similar to the MOP constraints, a minimum threshold could exist for each system's MOE. Although such a constraint may have been generated by a technical performance analysis, this threshold may simply be associated with the existing component system whose current performance must be met or exceeded. Therefore, the threshold MOE for each system, S_i, is $E_i^* = f_i(\mathbf{m}^L, \mathbf{p}_i^*, \ldots, \mathbf{p}_n^*) \le E_i$ for all *i*. When trying to minimize cost subject to performance constraints, there should be a minimum overall system-of-systems MOE constraint as well: $E^L \le E$. Without loss of generality, the single-system effectiveness constraint will not be addressed further because it would only be applied in practice to ensure that minimal system performance is achieved separately from the system of systems under consideration.
- *Cost constraints*. When applicable, cost constraints can apply to both individual systems and the full system of systems: $\mathbf{c} \le \mathbf{c}^U$ and $C(\mathbf{p}) \le C(\mathbf{p})^U$, respectively. Implicitly, $\mathbf{c}$ is also bounded below because of the presence of minimum performance thresholds. Hence, we have $\mathbf{c}^* \le \mathbf{c} \le \mathbf{c}^U$. Without loss of generality, we will take the system-of-systems viewpoint and consider only the cost constraint at the macro level, $C(\mathbf{p}) \le C(\mathbf{p})^U$.
- *Secondary MOE constraints*. As will be illustrated by the MCM example, there may be one or more secondary MOEs that must be achieved to some minimum level to achieve mission objectives. This can also be necessary in the case where the system-of-systems effectiveness is not fully expressed by one MOE. Without loss of generality, we will consider just one secondary MOE as a quality constraint: $q(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n) \ge q_T$.

When addressing the system-of-systems upgrade from the CAIV perspective, we would optimize a sequence of nonlinear programs formed by discretely parameterizing the system-of-systems cost constraint. This is accomplished by defining a sequence of upper cost bounds, $C_k^U = \text{costfactor}_k C^*(\mathbf{p}^*)$, where $C^*(\mathbf{p}^*)$ is the cost to produce the threshold system of systems defined by the parameter set $\{\mathbf{m}^L, \mathbf{p}_i^*, \ldots, \mathbf{p}_n^*\}$. The resulting nonlinear programming problem (with only one MOE constraint) is then to maximize $S = \{S_1, \ldots, S_n\}$ system-of-systems performance subject to force level, technology, cost, and performance threshold constraints as shown below.

$$\text{Max } E = G(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n)$$

subject to

$$\mathbf{m}^L \le \mathbf{m} \le \mathbf{m}^U$$
$$\mathbf{p}_i^L \le \mathbf{p}_i \le \mathbf{p}_i^U$$
$$C(\mathbf{p}) \le C_k^U = \text{costfactor}_k C^*(\mathbf{p}^*)$$
$$q(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n) \ge q_T \,.$$

MCM SYSTEM-OF-SYSTEMS COST/ PERFORMANCE MODEL

A simplified but realistic model of naval MCM operations and systems has been developed as a proof-of-principle demonstration. This limited system of systems consists of a minefield reconnaissance system and a mine neutralization system. The reconnaissance system first surveys the entire suspected minefield area, attempting to detect, classify, and localize minelike objects. These contacts are then passed to the neutralization system, which must reacquire the contacts and neutralize each minelike object, if necessary (that is, if it is identified as an actual mine). System descriptions, functionality, MOEs, MOPs, and PBCM are provided in sufficient detail to support system-of-system upgrade decisions and trade-off analyses (see MCM analysis terminology in Nomenclature insert).

The overarching MOE, *E*, for this MCM system of systems *S* is the time required to achieve a specified MCM area clearance rate α with specified confidence level β. Knowing the form of *E* guides our performance model formulation for the component systems S_1 and S_2. For this analysis, we assume there is only one system of each type, therefore $n = 2$ and $\mathbf{m} = \{1,1\}$.

Following the process described earlier, the mission scenario and minefield to be cleared must be specified. The mission is to search a mine danger area of 20 nmi^2, seeded with 100 mines, corresponding to $S_{\text{minefield}} = 20$ and $M_0 = 100$. The mines are laid out in four rows of 25 each, with a 400-yd separation between mines within each row, and 800 yd between the rows. Hence, $d_{\text{mines}} = 600$ yd. Figure 2 illustrates a minefield layout with these characteristics, although this "ground truth" information is unknown to the system of systems.

S_1: MCM Reconnaissance System

This system is used to survey a suspected minefield area, performing the typical MCM minehunting functions of detection, classification, and localization. The CONOPS is that the area is completely covered with a detection pass followed by a second pass for classification. Detection and classification must be done at a reduced standoff range from each detected object, necessitated by the much higher frequency sensor generally required for this more precise function. Localization

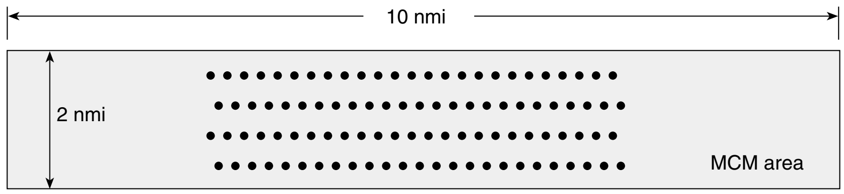

Figure 2. Minefield layout and area to be searched and cleared (not to scale).

is done concurrently with detection and classification, and therefore takes no additional time. In consideration of the overarching system-of-systems MOE, the MOE for S_1 is E_i = time (h) to complete reconnaissance of area $S_{\text{minefield}}$ given λ, λ_{ft}, and M_0, where $M_0 = \lambda S_{\text{minefield}}$ and $F_0 = \lambda_{\text{ft}} S_{\text{minefield}}$. The time to complete the detection pass over the area in hours is simply

$$T_{\text{detect}} = \frac{24 S_{\text{minefield}}}{A} = \frac{24 M_0}{\lambda A}.$$

Following the detection pass over the MCM area, the reconnaissance system will revisit its localized contacts and attempt to classify each one as either minelike or nonminelike. (Later, the neutralization system will attempt to reacquire and neutralize all declared minelike objects.) To calculate the time to complete classification, we must know the number and type of detections expected to be made:

$D_{\text{m}} = P_{\text{d}} M_0$ = number of detected mines

$D_{\text{fa}} = P_{\text{fa}} S_{\text{minefield}}$ = number of mine false alarms

$D_{\text{ft}} = P_{\text{d}} F_0$ = number of false targets detected

To generate expressions for time to classify a real mine as well as false alarms and targets, we must assume a specific classification CONOPS. If we assume that S_1 takes the shortest route between contact locations and then executes a semicircle of radius R_{c} about the contact location, then an approximate expression for the time to classify (in minutes, assuming 2000 yards per nautical mile) is

$$T_{\text{c}} = \frac{60 d_{\text{mine}}}{2000 V_{\text{transit}}} + \frac{60(\pi R_{\text{c}})}{2000 V_{\text{class}}}.$$

What about time spent attempting to classify a target that is a false alarm? Let's assume that the CONOPS would be to execute a full circle about the contact location in the event that the first classification pass was unsuccessful during the first half-circle maneuver. The time required to travel to the contact and execute the full circle in minutes is then

$$\begin{aligned} T_{\text{cf}} &= \frac{60 d_{\text{mine}}}{2000 V_{\text{transit}}} + \frac{60(2\pi R_{\text{c}})}{2000 V_{\text{class}}} \\ &= 2T_{\text{c}} - \frac{60 d_{\text{mine}}}{2000 V_{\text{transit}}}. \end{aligned}$$

This formulation for T_{cf} keeps it independent of cost drivers for the classification sonar performance, which reduces the number of MOPs necessary in the optimization problem, since the terms d_{mine} and V_{transit} will be considered as fixed for the scenario. The time (h) required to classify all detections is then

$$\begin{aligned} T_{\text{class}} &= \frac{1}{60}\left[P_{\text{c}} T_{\text{c}} D_{\text{m}} + (1 - P_{\text{c}}) T_{\text{cf}} D_{\text{m}} + T_{\text{cf}}(D_{\text{fa}} + D_{\text{ft}})\right] \\ &= \frac{1}{60}\left[\begin{array}{l} P_{\text{c}} T_{\text{c}} P_{\text{d}} M_0 \\ + (1 - P_{\text{c}})\left(2T_{\text{c}} - \dfrac{d_{\text{mine}}}{2000 V_{\text{transit}}}\right) P_{\text{d}} M_0 \\ + \left(2T_{\text{c}} - \dfrac{d_{\text{mine}}}{2000 V_{\text{transit}}}\right)(P_{\text{fa}} S_{\text{minefield}} + P_{\text{d}} F_0) \end{array}\right], \end{aligned}$$

and we can now formulate the system MOE as the sum of T_{detect} and T_{class}:

$$E_1 = \frac{24 M_0}{\lambda A} + \frac{1}{60}\left[\begin{array}{l} P_{\text{c}} T_{\text{c}} P_{\text{d}} M_0 \\ + (1 - P_{\text{c}}) T_{\text{cf}} P_{\text{d}} M_0 \\ + T_{\text{cf}}(P_{\text{fa}} S_{\text{minefield}} + P_{\text{d}} F_0) \end{array}\right].$$

Under the assumptions stated above, we can now list the minimum set of MOPs that are necessary to formulate an expression for E_1 as well as describe performance parameters that will affect the performance of the second system, S_2. There will be five MOPs, hence $\mathbf{p}_1 = \{p_{1,1}, \ldots, p_{1,5}\}$.

1. *Area coverage rate*: $p_{1,1} = A = 2R_{\text{d}} V_{\text{transit}}/2000$. This expression represents a two-sided detection sonar. A typical approximation is that for a particular sonar/

target/environment set, R_d is determined by fixing P_d and $V_{transit}$.
2. *Probability of classification*: $p_{1,2} = P_c$. For this analysis, the sidescan sonar's P_c is determined at fixed classification range.
3. *False alarm rate*: $p_{1,3} = P_{fa}$.
4. *Time required to classify a mine*: $p_{1,4} = T_c$.
5. *Minelike object localization error standard deviation*: $p_{1,5} = \sigma$. The localization accuracy is a critical parameter for reacquisition, a major function of S_2. As a simplification, we have chosen to neglect its effect on S_1's reacquisition during the classification pass, because the reacquisition would be done with the identical sensor suite that performed the initial detections.

After some manipulation,[11] the final form of the MOE for S_1 as a function of the MOP vector is

$$E_1(\mathbf{p}_1) = S_{\text{minefield}}\left\{\frac{24}{p_{1,1}} + \frac{1}{60}\left[\begin{array}{l}\lambda(p_{1,2}p_{1,4}P_d) \\ +(2p_{1,4} - T_{\text{transit}}) \\ \times\left(\begin{array}{l}(1-p_{1,2})P_d\lambda \\ +p_{1,3} + P_d\lambda_{ft}\end{array}\right)\end{array}\right]\right\},$$

where

$$T_{\text{transit}} = \left(\frac{d_{\text{mine}}}{2000V_{\text{transit}}}\right).$$

Note that this MOE does not reflect the quality of the reconnaissance, only its duration. If we were considering the effectiveness of the stand-alone reconnaissance system, then we would want E_1 to reflect other mission MOEs as well to effect a measure of minefield characterization. Reconnaissance survey quality will be automatically reflected in E_2 via expressions that utilize all the elements of $\mathbf{p}_1$ that affect initialization of the neutralization function provided by S_2. Additionally, a minimum threshold-quality MOE constraint at the system-of-systems level will also be imposed.

S_2: MCM Neutralization System

The MCM neutralization system attempts to relocate, identify, and neutralize all minelike objects detected and classified as such by the reconnaissance system. For this analysis, the probabilities of identification and subsequent neutralization are assumed to be one, and we will focus on uncertainty related to the reacquisition of all minelike objects passed to S_2 from S_1 as contacts. In consideration of the overarching system-of-systems MOE, the MOE for S_2 is E_2 = time (h) to complete neutralization and neutralization attempts on all contacts and objects classified as minelike by the reconnaissance system S_1.

Clearly, E_2 will depend on the number and types of objects detected and subsequently classified as minelike by S_1. Since the neutralization system will attempt to neutralize all declared minelike objects, it is important to know how many such objects are expected. Expressions for the number of mines correctly classified as minelike, C_m, and the number of nonmines incorrectly classified as minelike, C_f, are as follows:

$$C_m = D_m P_c = P_d P_c M_0$$

and

$$\begin{aligned}C_f &= (D_{fa} + D_{ft})(1 - P_c) \\ &= (P_{fa}S_{\text{minefield}} + P_d F_0)(1 - P_c),\end{aligned}$$

respectively. E_2 can now be formulated using three MOPs: $\mathbf{p}_2 = \{p_{2,1}, p_{2,2}, p_{2,3}\}$.

1. *Contact reacquisition range*: $p_{2,1} = R_r$. This is the standoff distance from the localized target, which yields an 80% probability of reacquisition.
2. *Failed reacquisition time*: $p_{2,2} = T_{pf}$. The average time (min) spent in a failed attempt to reacquire a target handed off from S_1.
3. *Neutralization time*: $p_{2,3} = T_n$. The average time (min) required to neutralize a correctly classified mine.

The contact reacquisition range is used to calculate the probability of reacquisition or localization as

$$P_L = e^{\frac{-\sigma}{4.481R_r}} = e^{\frac{-p_{1,5}}{4.481p_{2,1}}}.$$

This yields P_L = 0.80 when $R_r = \sigma$. This model assumes an exponential decay depending on the localization accuracy and reacquisition capability of the neutralization system, S_1 and S_2 MOPs, respectively. The dependence of P_L on R_r and σ is illustrated in Fig. 3. It is this localization error quantity from S_1 that has the most direct effect on the performance of S_2.

Therefore, the MOE for S_2 can be expressed as the sum of (1) time to successfully reacquire and neutralize minelike objects, (2) time spent in unsuccessful attempts to reacquire minelike objects, and (3) time spent prosecuting nonminelike objects classified incorrectly. After some manipulation,[11] the final form of the MOE for S_2 as a function of the MOP vector is

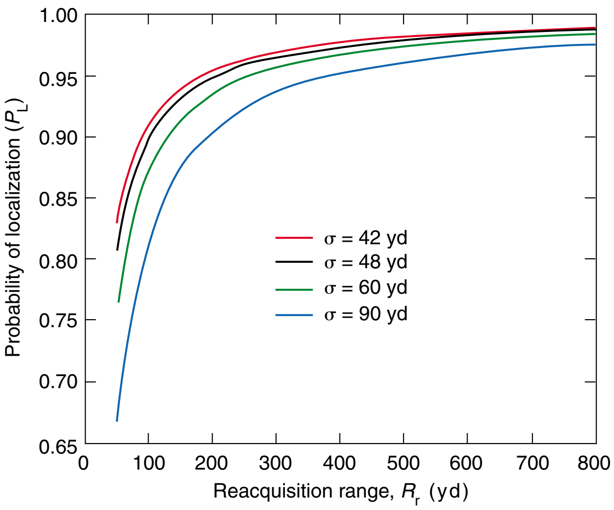

Figure 3. Probability of localization as a function of reacquisition range.

$$E_2 = \frac{1}{60}[P_L C_m T_n + (1-P_L)(C_m T_{pf} + C_f T_{pf})]$$

$$= \frac{S_{\text{minefield}}}{60}\left\{\begin{array}{l} P_d p_{1,2} p_{2,3} \lambda e^{\frac{-p_{1,5}}{4.481 p_{2,1}}} \\ + \left(1 - e^{\frac{-p_{1,5}}{4.481 p_{2,1}}}\right)\left(P_d p_{1,2} p_{2,2} \lambda\right) \\ + \left[(1-p_{1,2})(p_{1,3} + P_d \lambda_{ft}) p_{2,2}\right] \end{array}\right\}.$$

S: MCM Clearance System of Systems

For the full system of systems, the overarching MOE is then simply the total time to complete clearance operations: $E = G(\mathbf{m}, \mathbf{p}_1, \mathbf{p}_2) = E_1 + E_2$. However, an overall performance or quality constraint must be imposed on the clearance operations, otherwise the optimization will result in a very fast yet ineffective system of systems. Specifically, this constraint specifies an MCM area clearance rate α with an associated confidence level β. This should actually be considered as a secondary-quality MOE that has a threshold requirement. Recall that p is the probability that a particular mine will be cleared, which is the product of the sequential operations' probability of success:

$$p = P_d P_c P_L = P_d p_{1,2} e^{\frac{-p_{1,5}}{4.481 p_{2,1}}}.$$

The expected number of mines successfully cleared is then pM_0.

The selection of α = 0.80, M_0 = 100, and β = 0.90 will yield a constraint that $p \geq 0.846$.[11] Therefore, with 100 mines present, we will be at least 90% confident that at least 80 mines will be cleared. In summary, the performance quality constraint is then

$$q(\mathbf{p}_1, \mathbf{p}_2) = p = P_d P_c P_L$$
$$= P_d p_{1,2} e^{\frac{-p_{1,5}}{4.481 p_{2,1}}} \geq 0.846.$$

PBCMs and Parameter Bounds

The reconnaissance system performance ranges and cost modeling are derived from design considerations for an unmanned undersea vehicle. The neutralization system performance ranges and cost models are based on a combination of factors, certain operational MCM systems, and COTS information regarding marine navigation systems.

MOPs developed earlier in this article are grouped by the major subsystem for which they act as major cost drivers. The PBCM provides an approximation of subsystem cost as a function of those same primary subsystem MOPs. This synchronization of cost and performance model parameters is crucial and should become a fundamental feature of the systems engineering process.

Since this type of MCM system would be produced in very small numbers, only developmental costs are considered, neglecting the full system life-cycle costs. COTS or nondevelopmental item technologies are also assumed so that developmental costs approximate research and development and production costs combined. Note that since the PBCMs can include nonlinear expressions, a full life-cycle model for each PBCM can be accommodated with no change in the approach. The subsystem and associated MOPs are illustrated in Fig. 4.

To illustrate the concept of PBCMs, only area coverage rate will be discussed here; the other seven required models are developed in Luman.[11] There are two sonars in the sensor subsystem: detection and classification. Critical performance parameters affecting area coverage rate are probability of detection, range, and maximum vehicle speed at which the sonars can remain effective in the presence of flow noise. They are, of course, sensitive to many environmental parameters as well as assumed target characteristics. The approach here is to assume one environment, a fixed P_d, and a fixed vehicle speed, and then utilize modeled results to derive the PBCM for the search sonar MOP, area coverage rate, A. Table 2 represents the data used to generate the PBCM. (Note that sensitivity studies are advised to understand dependence upon these assumptions.)

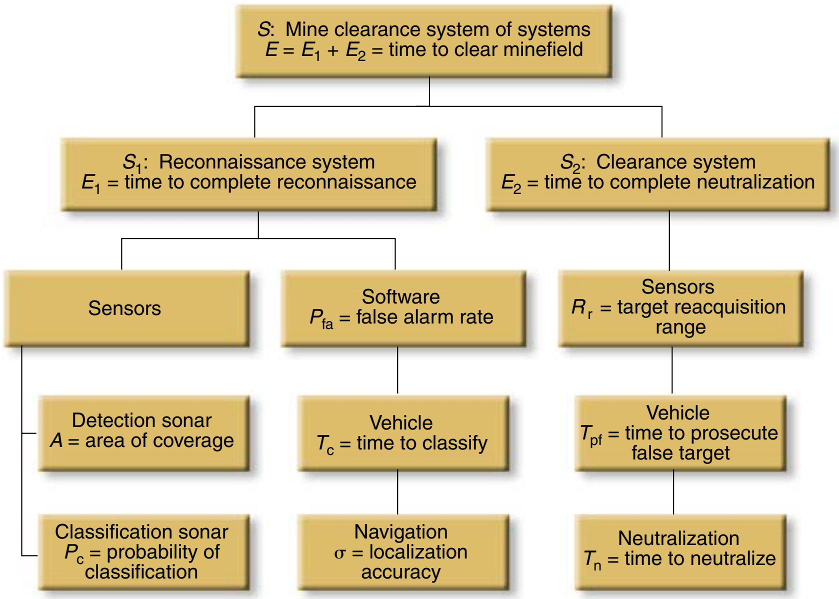

Figure 4. Mine clearance system-of-systems MOE/MOP structure.

which is a multiplier on the threshold system costs indicating the maximum amount the decision maker is willing to spend. In this way, we will consider a series of optimization problems that will provide insight from the CAIV perspective.

RESULTS

Phase I: Closed-Form Objective Function

This section presents the results of optimizing the closed-form representation of the MCM system-of-systems model. For ease of reference, the MCM system-of-systems MOP definitions are repeated with the correspondence to the optimization vector **x**.

A third-order polynomial was fit to the data in Table 2. Together with the upper and lower bounds for A, this constitutes the PBCM. The resulting model is therefore

$$h_{1,1}(p_{1,1}) = (4.5034\times10^{-5})p_{1,1}^3 - 0.0053861p_{1,1}^2 + 0.21593p_{1,1} + 1.3342\,,$$

with constraints as $p_{1,1}^{L} = 10, p_{1,1}^{U} = 100,$ and $p_{1,1}^{*} = 10.$ Figure 5 illustrates the cost/performance relationship for A.

The complete closed-form MCM system-of-systems model is shown in the boxed insert, which incorporates PBCMs for all eight system-of-systems MOPs. The cost constraint indicated is parameterized by costfactor$_k$,

$\mathbf{x}(1) = p_{1,1} = A =$ system S_1 area coverage rate during detection pass (nmi^2/day)

$\mathbf{x}(2) = p_{1,2} = P_c =$ probability of correctly classifying a detection as minelike or nonminelike at range R_c

$\mathbf{x}(3) = p_{1,3} = P_{fa} =$ detection false alarm rate (false alarms/nmi^2)

$\mathbf{x}(4) = p_{1,4} = T_c =$ time required to classify a mine (min)

$\mathbf{x}(5) = p_{1,5} = \sigma =$ standard deviation of minelike object localization error (yd)

Table 2. Data used to generate the PBCM for the search sonar MOP for area coverage rate.

Area coverage rate, A (nmi^2/day)	Cost ($ millions)
10	3.000
57	4.483
82	7.655
94	11.445

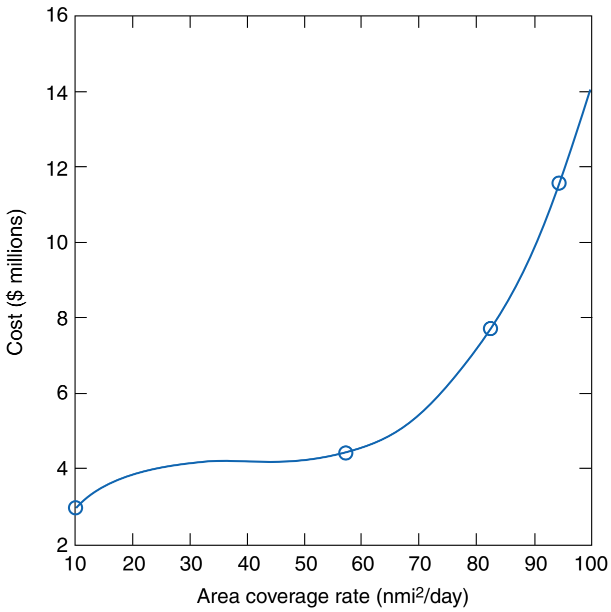

Figure 5. PBCM for area coverage rates. Circled points are rates from Table 2.

SUMMARY OF MCM SYSTEM-OF-SYSTEMS OPTIMIZATION PROBLEM

Maximize $S = \{S_1, S_2\}$ system-of-systems performance (minimize time) subject to technology, cost, and performance threshold constraints:

Minimize

$$\begin{aligned} E(\mathbf{p}_1, \mathbf{p}_2) &= E_1(\mathbf{p}_1) + E_2(\mathbf{p}_1, \mathbf{p}_2) \\ &= \frac{S_{\text{minefield}}}{60}\left\{\frac{24 \cdot 60}{p_{1,1}} + \lambda(p_{1,2}p_{1,4}P_{\text{d}}) + (2p_{1,4} - T_{\text{transit}})[(1 - p_{1,2})(P_{\text{d}}\lambda + p_{1,3} + P_{\text{d}}\lambda_{\text{ft}})]\right\} \\ &= \frac{S_{\text{minefield}}}{60}\left[P_{\text{d}}p_{1,2}p_{2,3}\lambda e^{\frac{-p_{1,5}}{4.481p_{2,1}}} + \left(1 - e^{\frac{-p_{1,5}}{4.481p_{2,1}}}\right)P_{\text{d}}p_{1,2}p_{2,2}\lambda + (1 - p_{1,2})(p_{1,3} + P_{\text{d}}\lambda_{\text{ft}})p_{2,2}\right] \end{aligned}$$

subject to

$$(3.0,\ 0.9,\ 0.25,\ 3.0,\ 42.0)^{\text{T}} \le \mathbf{p}_1 \le (100.0,\ 0.98,\ 2.0,\ 9.17,\ 90.0)^{\text{T}},$$
$$(75.0,\ 1.0,\ 3.0)^{\text{T}} \le \mathbf{p}_2 \le (700.0\ 7.0,\ 10.0)^{\text{T}},$$
$$C(\mathbf{p}_1,\ \mathbf{p}_2) \le C_k^{\text{U}} = \text{costfactor}_k C^*(\mathbf{p}^*),$$

and

$$q(\mathbf{p}_1,\ \mathbf{p}_2) \ge q_{\text{T}} = 0.846,$$

where

$$\begin{aligned} C(\mathbf{p}_1, \mathbf{p}_2) = {} & (4.5034 \times 10^{-5})p_{1,1}^3 - 0.0053861p_{1,1}^2 + 0.21593p_{1,1} + 1.3342 \\ & + 283.4646p_{1,2}^2 - 507.63784p_{1,2} + 227.4598 \\ & - 2.0484p_{1,3}^3 + 9.9873p_{1,3}^2 - 17.9420p_{1,3} + 20.3220 \\ & + 0.11597p_{1,4}^2 - 2.1757p_{1,4} + 15.2040 \\ & + (2.0618 \times 10^{-4})p_{1,5}^2 - 0.0376p_{1,5} + 1.7778 \\ & + (1.5049 \times 10^{-7})p_{2,1}^3 - (1.5782 \times 10^{-4})p_{2,1}^2 + 0.055167p_{2,1} - 1.8133 \\ & - 0.28504p_{2,2}^3 + 3.8462p_{2,2}^2 - 17.2640p_{2,2} + 33.3440 \\ & + 0.21024p_{2,3}^2 - 4.1096p_{2,3} + 25.3970, \end{aligned}$$

$$C^*(\mathbf{p}^*) = \$28.066 \text{ million},$$

and

$$q(\mathbf{p}_1, \mathbf{p}_1) = p = P_{\text{d}}P_{\text{c}}P_{\text{L}} + P_{\text{d}}p_{1,2}e^{\frac{-p_{1,5}}{4.481p_{2,1}}}.$$

$\mathbf{x}(6) = p_{2,1} = R_{\text{r}} =$ contract localization error standoff which yields an 80% probability of reacquisition

$\mathbf{x}(7) = p_{2,2} = T_{\text{pf}} =$ time spent prosecuting a nonmine classified as a mine or unsuccessfully attempting to reacquire a correctly classified mine (min)

$\mathbf{x}(8) = p_{2,3} = T_{\text{n}} =$ time spent neutralizing (prosecuting) a classified mine (min)

The system-of-systems constrained MOE optimization has been solved for an increasing sequence of multipliers (costfactor_k) on the cost of the threshold system, denoted by $C^*(\mathbf{p}^*)$, which happens to be \$28.066 million (see the boxed insert). This provides the decision maker with information to apply the CAIV approach to system upgrade or initial design. Plots are provided so that one can visualize the top-level MOE improvement and corresponding MOP requirements as the system-of-systems cost upper bound is allowed to increase.

The baseline results are obtained by using MATLAB's constrained sequential quadratic programming (SQP) algorithm to solve the fully general nonlinear programming problem in which both objective and constraint functions can be nonlinear.[12] The particular routine is called CONSTR and is contained in the optimization toolbox. Basically, the method formulates a sequence of quadratic programming subproblems based on a quadratic approximation of the Lagrangian function and linearizing the nonlinear constraints about the current iterate. The simpler quadratic programming subproblem (quadratic function with linear constraints) is solved by using an active set projection method. The original nonlinear function and constraint sets are then approximated about the new iterate, and the sequence is repeated until convergence criteria are satisfied.

Figure 6 summarizes these results, which constitute an allocation of MOE requirements to the lower-level MOPs as a function of overall cost. Figures 6a and 6b present the top-level MOE (E = time to complete minefield clearance) as a function of increasing cost factor and dollar cost upper bounds. Figures 6c and 6d present the corresponding optimal MOPs as a function of increasing cost factor. The MOPs are normalized to their upper and lower bounds, with 0 corresponding to their threshold system values and 1 corresponding to their technology limitations. Several significant insights can be obtained from examination of Fig. 6:

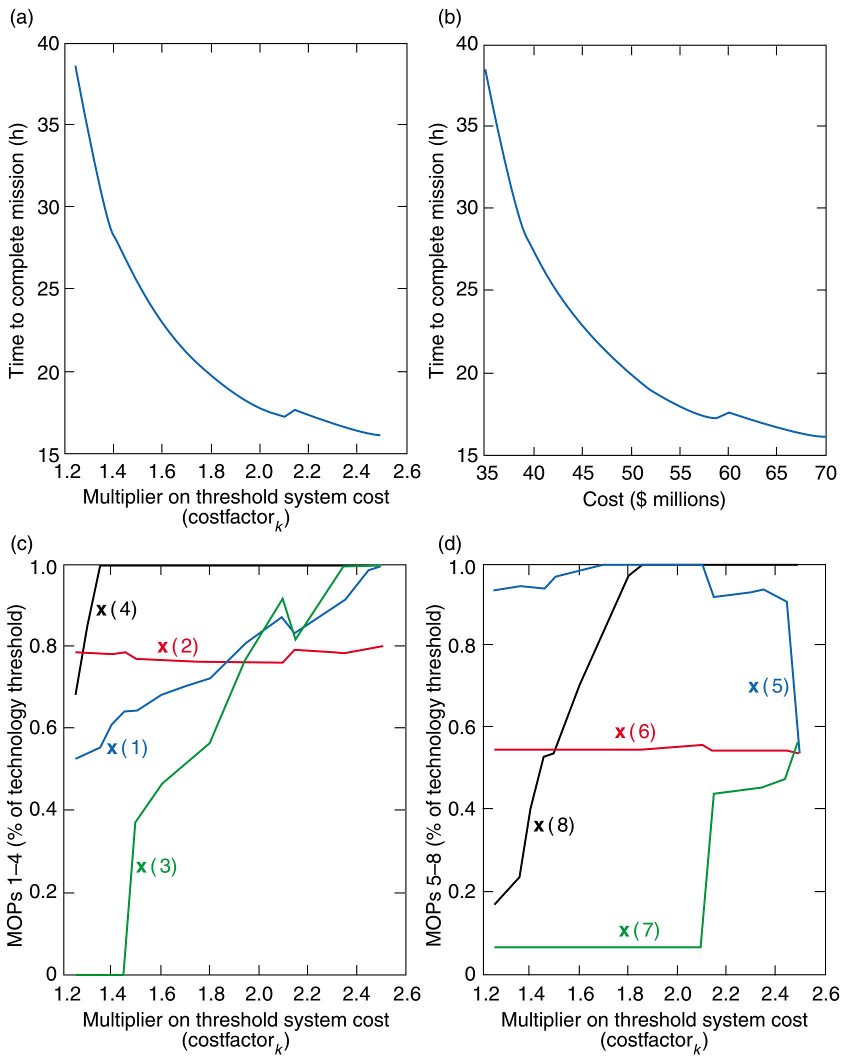

Figure 6. Constrained sequential quadratic programming optimization results for a system of systems. The top-level MOE as a function of increasing cost multiplier is shown in (a) and MOE as a function of dollar cost upper bounds is shown in (b). Corresponding optimal MOPs **x**(1)–**x**(4) and **x**(5)–**x**(8), (c) and (d), respectively, are presented as a function of increasing cost multiplier.

- The system-of-systems MOE improves steadily to an asymptotic lower bound as the cost limit increases. Because of the imposed technology constraints, after a certain point, no amount of money will enhance system performance.
- At the other extreme, if at least 1.25 $C^*(\mathbf{p}^*)$ is not spent, a feasible solution that satisfies the quality constraint cannot be found. That is, even a very slow system cannot achieve the clearance rate constraint.
- A subjective "knee of the curve" can be observed to occur somewhere around 1.8–2.0 times the threshold system cost (about $50–56 million), after which the rate of MOE improvement significantly decreases.
- The component systems' MOP requirements can be determined from these plots as a function of cost factor. One can see which MOPs become stressed (i.e., move away from their threshold system values) and approach their technology constraint limits as the cost constraint is relaxed. Of course, this behavior depends on the PBCM function developed for each MOP, as well as its significance relative to the objective function and quality constraint. Specifically, initial performance is gained by improving **x**(4) (speed) and **x**(5) (location accuracy). Additional

performance gains are most effectively achieved by improving $\mathbf{x}(1)$ (coverage rate), $\mathbf{x}(3)$ (false alarm rate), and $\mathbf{x}(8)$ (neutralization time).

- The hump in the MOE curve near $\text{costfactor}_k = 2.15$ corresponds to a local objective function minimum caused by a prolonged flat area in the PBCM for $\mathbf{x}(7)$.[9] This effect is common in generating realistic PBCMs wherein there may be only a few discrete technology solutions widely separated in performance and cost. Depending on the situation, a discrete optimization method may be more appropriate.

These results can be used to design a specific cost-constrained upgrade to the threshold system of systems. For example, if the allowable cost constraint is twice that of the threshold system, then selecting $\mathbf{p}_1 = \{85.3, 0.961, 0.555, 3.0, 42.0\}$ and $\mathbf{p}_2 = \{423.2, 7.0, 3.0\}$ yields $E = 17.768$ h, with clearance rate $q = 0.846$ at cost $C^*(\mathbf{p}^*) = \$56.132$ million. This is a substantial enhancement to the threshold system represented by $\mathbf{p}_1 = \{10.0, 0.9, 2.0, 9.17, 90.0\}$ and $\mathbf{p}_2 = \{75.0, 6.6, 10.0\}$ that results in an overarching MOE of $E = 93.33$ h with clearance rate of only $q = 0.620$ at cost $C^*(\mathbf{p}^*) = \$28.066$ million. Since CONSTR would not converge for cost factors less than 1.25, the analysis indicates that a system that satisfies the stringent requirement for 84.6% clearance will cost at least 25% more than a system of systems composed of the threshold component systems, but would take 38.38 h to complete the clearance mission with a single pass from each system.

Critical to any cost/performance trade analysis is the concept of sensitivity analyses. Three specific types of sensitivity analyses are especially appropriate for this class of problems:

1. *Sensitivity to mission or scenario*. This is achieved by varying the parameters that define the threat, environment, mission objective, and systems CONOPS.
2. *Sensitivity to secondary MOE constraints*. Since these constraints were arbitrarily set, the optimization should be parameterized for excursions about the nominal value to produce families of CAIV curves.
3. *Sensitivity to PBCMs*. Examination of sensitivities to subsystem cost models and especially technology-driven limitations on MOPs can yield significant insights needed to focus a supporting warfare area technology investment strategy.

Moreover, it can be shown that optimizing each system separately is suboptimal to optimizing the system of systems as a whole.[11] In quantifying the suboptimality of single-system optimization relative to simultaneously optimizing the entire system of systems, several significant insights were obtained and verified by examining some reasonable assumptions that might be held by component systems' management concerning the concurrent systems engineering processes of other systems. For example, if one systems team assumes that the other component systems are being developed for high performance, they will "under-engineer" their own system with respect to interfacing parameters and will tend to allocate resources to enhance the single-system MOE. Conversely, if they assume that the other systems are not performance driven, the result is to over-engineer their own system at the interface. In this case, since resources are constrained, this over-engineering forces degradation in the single-system MOE. In both cases, the overall system of systems is suboptimal, because all systems engineers are making the same erroneous assumptions.

These effects are accentuated with restrictive cost constraints but become insignificant as overall cost constraints are relaxed to the point where the most advanced technology is affordable for all system components—an intuitive result. In other words, if we are not resource constrained, then the correct course of action is simply to optimize each component system for performance without regard to cost. But as the cost constraint is tightened, it becomes increasingly important to consider the full impact of design decisions on the whole to get the most performance per unit dollar. The results in this regard vividly illustrate the maxim, "We are short of money, therefore we must think."

Phase II: Simulation Objective Function

As mentioned previously, obtaining a closed-form, deterministic expression for the system-of-systems' MOE objective function is not always feasible or would introduce unacceptable simplifying assumptions. A growing number of application areas rely on stochastic M&S to predict system-of-systems performance under certain conditions of interest. Therefore, future practical implementations of this approach for warfare area systems of systems will include the use of simulation to evaluate the objective function—an extension that will put a premium on minimizing the search algorithm's function evaluations.

Simulation will generally be of the Monte Carlo type, hence, there will be process noise associated with the function evaluation. The simulation produces a stochastic realization of the objective function of the form

$$y(\mathbf{p}_1, \ldots, \mathbf{p}_n) = G(\mathbf{p}_1, \ldots, \mathbf{p}_n)\boldsymbol{\omega} ,$$

where $\boldsymbol{\omega}$ = simulation noise.

This stochastic nature of the objective function and quality constraint functions means that we have a stochastic optimization problem to which classical optimization methods are not directly applicable. Since G will be evaluated by the simulation, the gradient of y will not be available explicitly. Many stochastic optimization methods, like classical methods, require approximations of gradients, but they become extremely costly to compute in this domain since each function evaluation represents a simulation run. Until recently, finite-difference–based gradient-search stochastic approximation procedures that are adaptations of deterministic algorithms have been most widely used for this type of optimization. A major drawback of these methods is that the number of function evaluations required at each step is linear in the dimension of the search parameter vector for first-order methods and quadratic for second-order methods.[13] Since we envision eventually using large-scale system-of-systems simulations with tens of parameters, a much more efficient method is desirable.

The Simultaneous Perturbation Stochastic Approximation (SPSA) method[14,15] is the most efficient estimator in this domain with respect to function evaluations per iteration, and its first- and second-order versions have been adapted here to solve the MCM system-of-systems problem. The first-order SPSA method is a type of gradient search that requires only two-function evaluations per iteration, independent of the number of parameters to be estimated. The current solution estimate is perturbed in all elements simultaneously in a sort of central difference fashion rather than one component at a time, which is generally done to estimate the partial derivatives that comprise the gradient vector.

Although SPSA per se is an unconstrained (global) optimization algorithm, a penalty function approach was developed[11] to adapt it to the class of nonlinearly constrained problems represented by this system-of-systems optimization process. However, owing to the large range of numerical values of the MCM system parameters, the first-order SPSA (1SPSA) algorithm exhibited poor convergence. Therefore, a second-order version, or 2SPSA,[16,17] which emulates the convergence acceleration and scaling invariant properties of deterministic Newton–Raphson algorithms, was also adapted to the constrained nature of this class of problems. Because of the need to estimate the Hessian matrix, 2SPSA requires five-function evaluations per iteration, but produced much better results than 1SPSA. It is beyond the scope of this article to discuss the details of the penalty function approach, but they are covered in Luman[11] and generalized and extended by Wang and Spall.[18]

Simulation Description

To examine stochastic optimization feasibility, the MCM system-of-systems model was implemented as a simulation, patterned directly after the parameter dependency diagram shown in Fig. 7. The simulation was implemented as a MATLAB function that produces one Monte Carlo realization of E and q with each function call. It randomly generates the specified events in accordance with the MOPs. For example, looking at block 4 in Fig. 7, if there are 100 mines in the minefield (i.e., M_0 = 100) and P_d = 0.90, then the number of detected mines (D_m) is generated simply as 100 Bernoulli success/failure trials with probability of success equal to 0.90. The randomly generated D_m is then passed to block 7, which in turn similarly generates the number of correctly classified mines, and so on. Eventually the MOEs for that realization are produced and the resulting penalty function evaluation is returned by the simulation function MCMSIM after calculating the resultant system cost.

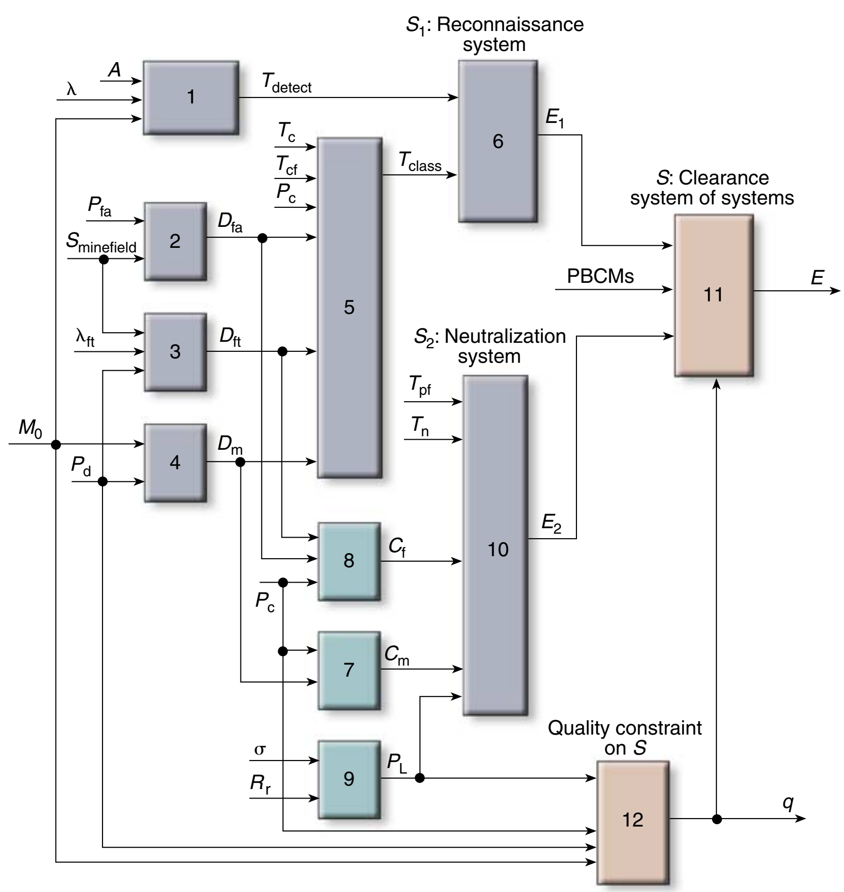

Figure 7. MCM simulation block diagram (refer to Nomenclature).

Second-Order Constrained SPSA Optimization

Figure 8 compares several 2SPSA simulation MOE results to the closed-form analytic model results, which represent the best possible cost/performance for this application. Note that 2000 iterations are required to approach the analytic results. Certain postprocessing methods, common in stochastic optimization practical applications, were applied to achieve the final results, generated by interpolating MOP estimates across the CAIV continuum.[11] These interpolated simulation results are very smooth, approximating the baseline results curve.

However, the overall MOE domain does not reflect the entire situation, and we must examine the degree to which the secondary MOE and cost constraints are satisfied. These comparisons are displayed in Figs. 9 and 10, respectively, and show acceptable levels, considering the variability induced by the simulation. Actually, the interpolated MOP values result in underspending the cost constraint by as much as $4 million (about 6%) at the higher levels of the cost constraint, implying that a bit more performance could be extracted.

An interesting aspect of stochastic optimization is that the optimization process is itself a stochastic process in addition to the system under analysis. Therefore, the issue arises as to how to express the "final" answer in both the MOP and MOE domains. For example, what are the values for the MOEs E and q that are associated with the solution vector $\mathbf{x}$ (MOPs)? Since MCMSIM produces a random realization of the objective function, it must be called many times and results averaged to generate expected values for E and q. The results in Fig. 8 display the standard deviation bars of 100 such function evaluations about the mean simulation results.

In the baseline analytic results at a representative costfactor$_k$ constraint value of 2.0, CONSTR produced $\mathbf{p}_1$ = {85.3, 0.961, 0.55, 3.0, 42.0} and $\mathbf{p}_2$ = {423.2, 7.0, 3.0}, yielding E = 17.8 h, with clearance rate q = 0.846 at cost $C^*(\mathbf{p}^*)$ = $56.1 million. The final results of the nonlinear, constrained, stochastic optimization implementation produced $\mathbf{p}_1$ = {74.3, 0.965, 1.1, 3.2, 55.8} and $\mathbf{p}_2$ = {495.6, 4.4, 3.3}, yielding E = 18.7 h, with clearance rate q = 0.841 at cost $C^*(\mathbf{p}^*)$ = $54.0 million. The overall MOE is about 5% worse for $2 million less cost and a very slight decrease in clearance rate of

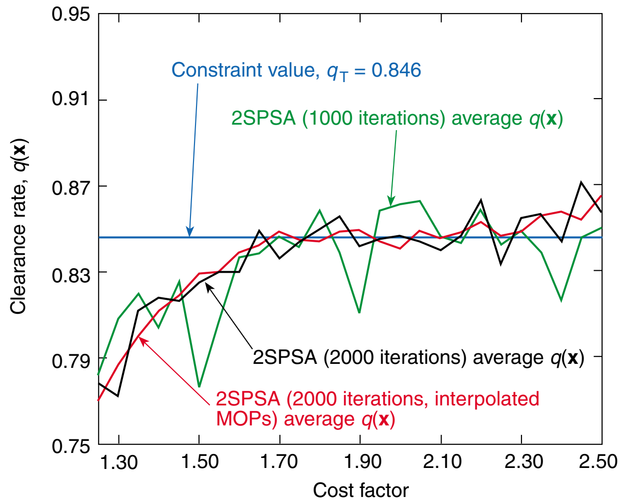

Figure 9. 2SPSA simulation clearance rate results.

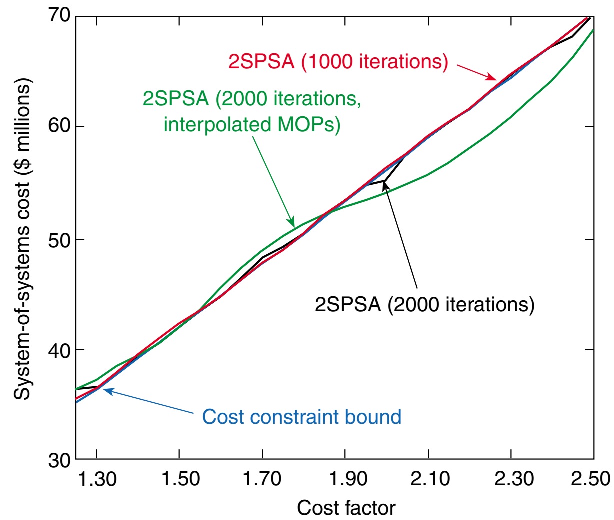

Figure 10. 2SPSA simulation cost results.

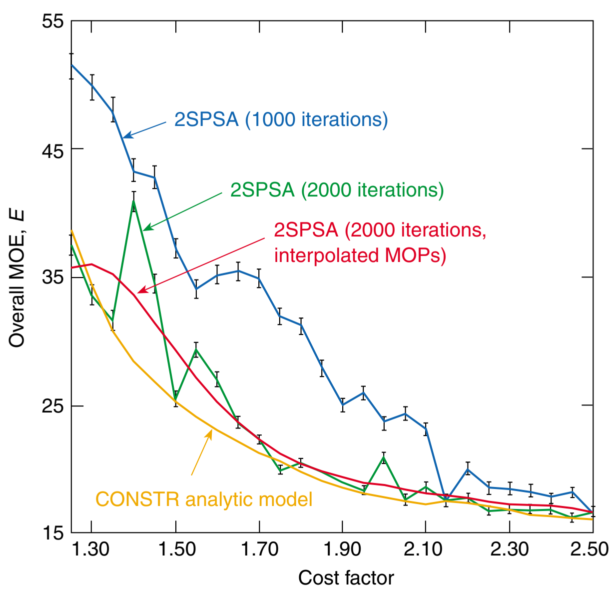

Figure 8. 2SPSA simulation versus analytic model results. (Bars show standard deviations about mean simulation results.)

0.005. As expected, this is suboptimal to the analytic formulation owing to the complexity introduced by simulation variability or "noise."

SUMMARY

A systematic, disciplined, quantitative approach to developing system-of-systems requirements allocations has been demonstrated for upgrading complex systems of systems. The process treats cost as the independent variable and seeks to find the "best" point design for upgrading a particular system of systems, subject to cost, operational, and technology constraints, relative to an overarching MOE. The design requirements generated represent an improved system of systems that may involve upgrading all component systems simultaneously, not just one at a time. Although final systems requirements decisions must subjectively balance multiple factors, this method objectively integrates cost and performance factors at the initial stage of analysis.

The process has been demonstrated on a naval MCM system-of-systems representation of sufficient complexity and detail to demonstrate its feasibility. This proof-of-principle demonstration features a constrained, nonlinear optimization algorithm adapted to both closed-form representation of the objective function (i.e., MOEs) and simulation-based objective function. Owing to the nature of complex system-of-systems interactions, the latter approach will be necessary to address full warfare areas or problems of national interest. Their complexity requires the simulation to represent the mapping of system MOPs to single-system MOEs and on to the overarching system-of-systems MOE. Various optimization methods have been demonstrated and differences quantified, including the suboptimality of considering just one system at a time.[11] The application of the system-of-systems approach can result in more effective and comprehensive systems acquisition and technology investment strategies, with the secondary benefit that the process can be used as a framework to determine how to utilize campaign-level simulation to support acquisition decisions.

Variants of the process are now being applied to support CAIV analyses for the Navy Theater-Wide Program and to focus future science and technology investments for MCM. These applications at the *warfare area* system-of-systems level will enable acquisition executives to move from our legacy single-system acquisition approach[7] to a comprehensive warfare area architecting process (Fig. 11) with a scope spanning new acquisition starts, technology insertion upgrades, force structure, and technology investment strategy.[19]

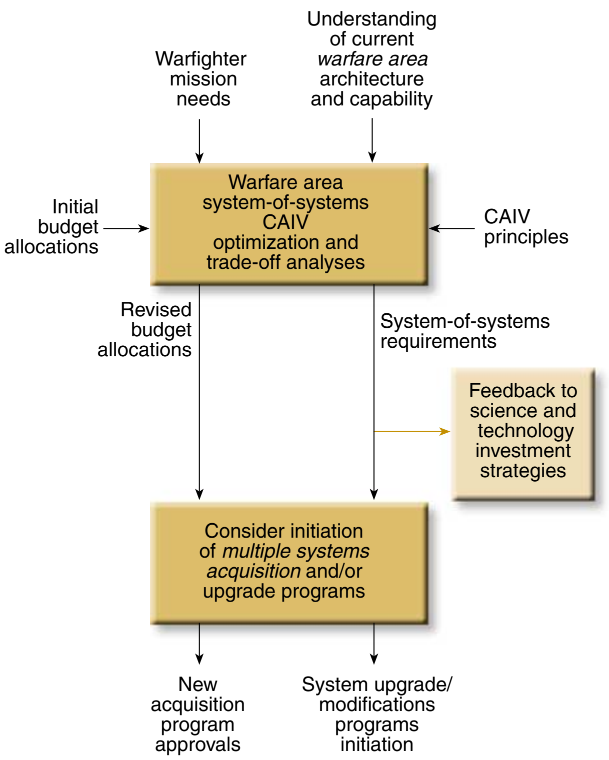

Figure 11. System-of-systems architecting can support an acquisition paradigm shift.

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