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REASONING UNDER UNCERTAINTY FOR A COASTAL OCEAN EXPERT SYSTEM

The Applied Physics Laboratory is conducting research in tactical oceanography whose objective is to develop an Ocean Expert System that (1) captures the physical cause-and-effect relationships of the dominant coastal processes and (2) performs coastal scene reasoning using the available environmental data and products to assess critical environmental factors and their likely effects on mission effectiveness. This article presents the need for evidence-based mechanisms for uncertainty management in such a system and surveys various approaches to uncertainty management that are being investigated for use in the Ocean Expert System.

INTRODUCTION

Recent military operations have been largely maritime with a significant emphasis on joint-service control of the littoral battlespace. Although the particular geographic location may vary, future operations are likely to have a similar nature and emphasis. The coastal environment is extremely complex and can affect littoral military operations both positively and negatively.[1] Research in tactical oceanography focuses on providing naval mission planners and platform commanders with the information required to exploit the oceanographic and lower atmospheric environment across a full range of possible missions.

The tactical use of environmental information will be, at least partially, specific to the particular mission or warfare area (e.g., mine warfare, special warfare, amphibious warfare, strike warfare, antiair warfare, antisurface warfare, and antisubmarine warfare). In particular, which environmental information is even of interest must be determined by its potential effect on the mission. For example, ocean currents would be of great interest in planning an amphibious landing but of much less interest in planning an airborne strike.

Further, the same underlying physical process may have different tactical implications for various missions. An example here might be atmospheric winds. For a strike mission, the winds could influence the orientation of the carrier (to better support takeoff and landing) and route planning (to conserve fuel and extend flight time). The same winds could be of interest when planning an amphibious assault because of their influence on the currents (hence, landing time) and during antisubmarine warfare because of the acoustic effects of the air–sea interaction or the possibility of upwelling deepening the mixed layer. Tactical oceanographic support, therefore, requires insight into the governing oceanographic and meteorological processes as well as their tactical implications for the various warfare areas.

Currently, specially trained naval meteorological and oceanographic (METOC) officers provide the tactical decision maker with the environmental information relevant to the task at hand. The METOC officers use a dedicated environmental workstation called TESS (Tactical Environmental Support System[2]) to acquire and assimilate forecast fields, satellite imagery, and *in situ* data. In addition to determining what environmental information is relevant, they interpret information in the context of the mission and even the individual systems that might be employed. Finally, they are frequently called upon to make tactical recommendations, such as the orientation of the aircraft carrier, on the basis of environmental considerations. Unfortunately, a METOC officer is not available to provide tactical oceanographic support on every naval platform. Currently, METOC officers are limited to the oceanographic forecast centers ashore and the major command ships at sea, such as the aircraft carriers. Recent trends in military force levels suggest the situation will not improve in the near future.

The shift from open-ocean warfighting to littoral operations brings with it shrinkage of the temporal and spatial scales over which environmental changes occur and expansion of the potential effect of the environment on military operations. The inherent complexity of the coastal environment, the small scales of tactical interest, and the likely paucity of coastal data suggest that accurate environmental assessment and tactical interpretation will become more difficult as they become more important.

Dantzler and Scheerer[3] have proposed a knowledge-based or expert systems approach to the problem of coastal scene assessment and mission-specific tactical interpretation. The Ocean Expert System (OES) is intended to capture some of the physical insight and reasoning processes of the METOC officer and make them more widely available. The OES does not negate the need for trained analysts but rather is targeted at extending the availability of analytical skills in an autonomous fashion.

A major functional goal of the OES is to perform the mission-driven coastal scene reasoning required to develop a mission-specific coastal scene description as shown in Figure 1. Each mission area will have associated with

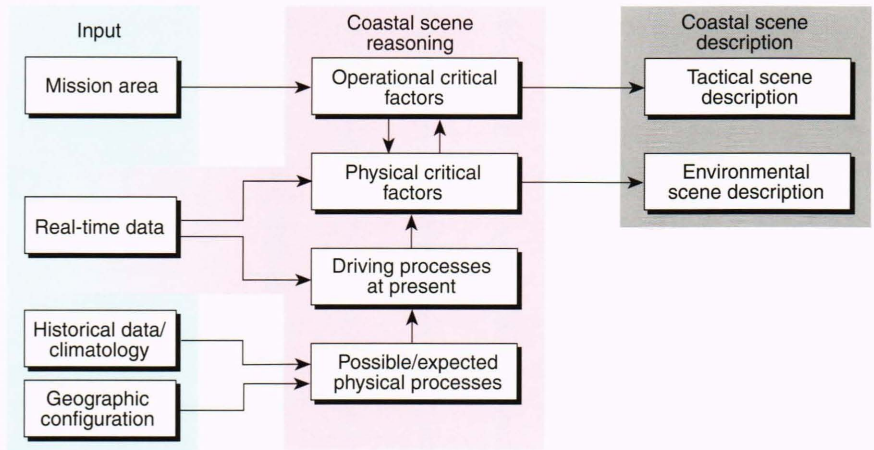


Figure 1. The mission-driven nature of coastal scene reasoning in the Ocean Expert System (OES). Each mission area has one or more operational critical factors (i.e., "time across the beach" for an amphibious landing) that capture the potential effect of the environment on the mission. The values of the operational critical factors are determined by the underlying physical critical factors (i.e., currents [set and drift]). Coastal scene reasoning consists of drawing on the available input and the OES process knowledge base (which captures the cause-and-effect relationships govern-ing the fundamental physical processes) to assess and predict the values of the physical and operational critical factors. The coastal scene description is a geographic presentation of these values.

it one or more operational critical factors, which will directly affect mission effectiveness and are the key parameters for a tactical decision maker/mission planner. Underlying those operational critical factors are physical critical factors that ultimately determine their values. For example, the prevailing currents (physical critical factor) will fundamentally influence the "time across the beach" (operational critical factor) in an amphibious landing.

The coastal scene description that a tactical decision maker desires is mission specific and comprises both the operational and physical critical factors for a given mission. Those factors can be presented geographically and temporally at multiple levels of detail. In the example of the amphibious landing, the coastal scene description might include, at a high level, a display of time across the beach for various landing areas and start-of-mission times. A lower-level display of the coastal scene description might show the actual currents.

Assessing and predicting the values of mission-specific physical and operational critical factors in a coastal region are major goals of the OES. The OES process knowledge base is intended to capture the fundamental coastal, physical cause-and-effect relationships in a manner that supports computer-based coastal scene reasoning. Drawing on the available input (*in situ* and remotely sensed data, historical climatology, numerical model outputs, environmental products, and the rest), OES coastal scene reasoning will identify which physical processes are currently dominating the coastal scene. An understanding of the underlying physical processes will allow the OES to generalize sparse coastal data into regions of similar physics and assess the physical critical factors. From there, the values of the operational critical factors can be determined.

The OES conceptual system architecture (Fig. 2) provides for local and offboard data acquisition and interpretation, guided by a central set of interactive knowledge bases that support the scene description reasoning process. The content of the individual knowledge bases and the manner in which this system architecture reflects the process by which the human analyst develops an environmental scene description have previously been presented.[3] This article examines the need for uncertainty management and evidence-based reasoning in the OES and explores various formalisms that may fulfill this need.

EVIDENCE-BASED REASONING AND UNCERTAINTY MANAGEMENT IN THE OES

The coastal ocean environment is fundamentally determined by the interactions of identifiable physical processes and geographic/oceanographic features including, but not limited to, bathymetry, water masses, long-period waves, coastal upwelling, fronts and eddies, internal motions, kelvin and edge waves, barotropic tides, internal tides, internal waves, and mixing.[4] Extensive literature exists on the significant oceanographic processes and features potentially relevant to the coastal ocean, and it is now being reviewed for knowledge acquisition and computer-based knowledge representation in the process knowledge base (see Fig. 2). One of the functionality goals for the OES is to be able to use this knowledge to identify and predict the critical physical features and processes relevant to a particular region on the basis of available historical, *in situ*, and offboard data.

Evidence-Based Reasoning

In several situations, the OES will need to make inferences and confirm or disconfirm hypotheses on the basis of the available evidence, as in the identification of significant oceanographic features. Consider remote sensing of sea-surface temperature via satellite sensors (Fig. 3). The application of edge-detection algorithms to such satellite images frequently yields many filaments that identify high horizontal temperature gradients in the image similar to those identified by an analyst, as shown in Figure 4. Inferring which of these filaments should be discarded as superficial and which should be grouped as significant water-mass features (e.g., eddies or fronts) could involve multiple hypotheses that would be confirmed or denied by the evidence of historical climatology, previous analyses, and the underlying physical processes. In this example, the remotely sensed data provide diagnostic evidence (high gradients) for the feature, and

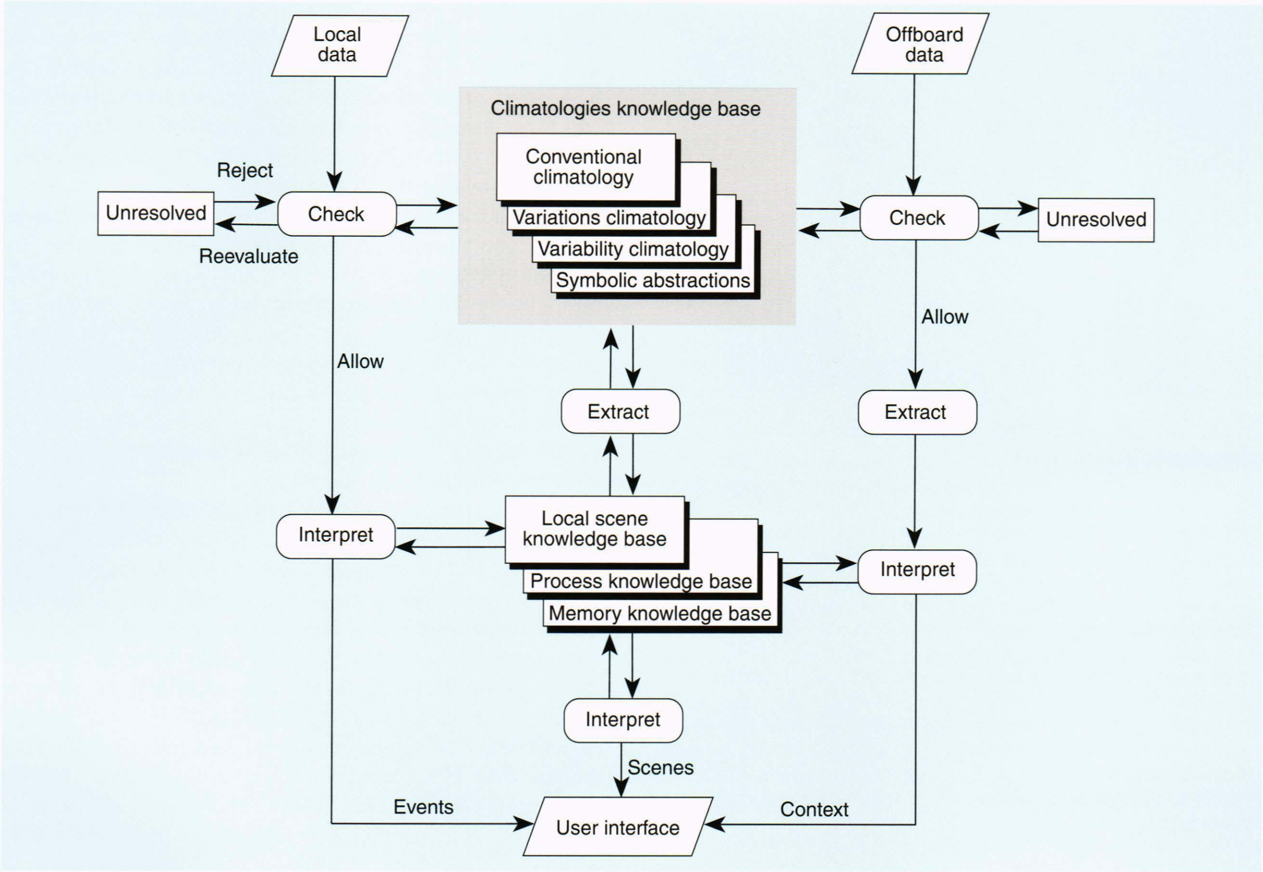


Figure 2. Conceptual design for the Ocean Expert System (OES). The goal of the system is to produce a best estimate of the oceanographic scene description, given the information that might be available at any particular time. Information external to the system is derived from local oceanographic and atmospheric measurements and from information communicated to the system from offboard sources. A key aspect of the OES is the integration of embedded knowledge bases that allow (1) the evaluation of available oceanographic information in a context of conventional algorithmic tests (such as statistical variability) in addition to previous experience in the area and (2) a physics-based representation of coastal dynamics that cannot easily be captured in a conventional computer-based system. The estimated scene description provides information about significant environmental events, the mapping of those events to the local area, and estimates of the larger-scale context within which the scene is to be interpreted.

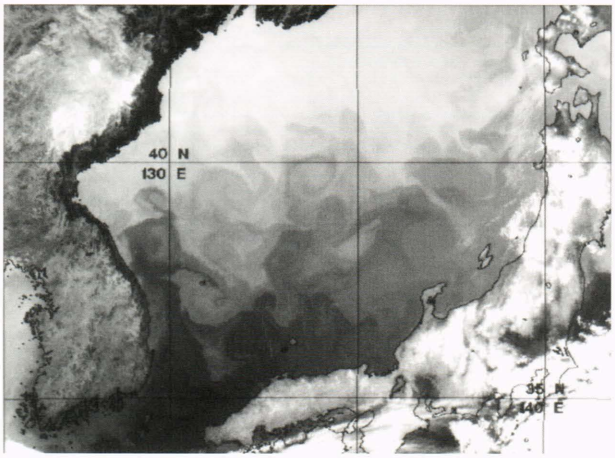


Figure 3. Infrared imagery of the surface of the Sea of Japan in May 1991, obtained by the NOAA-10 satellite. The nearly cloud-free image, taken in late winter, shows the remarkable complexity in the thermal structure within the transition from the warm water temperatures (represented by the dark shades) in the eastern portion of the sea to the colder waters (shown by the light shades) to the west. (Courtesy of the U.S. Naval Oceanographic Office, Operational Oceanography Center, Stennis Space Center, Miss.)

the underlying physical process that could cause the feature (e.g., the pinching off of an eddy from the Gulf Stream) provides causal or predictive evidence.

Evidence-based reasoning is also likely to be required for the identification and prediction of coastal oceanographic processes. Consider coastal upwelling.[5] The Ekman layer is a surface layer in which there is a balance between the wind stress and the Earth's rotational force. In coastal upwelling, persistent along-shore winds produce an Ekman transport (net movement of water in the Ekman layer) away from the coast/continental shelf that, in turn, results in the upwelling of cooler water near the coast and/or shelf break. In this example, a communicated weather forecast from a regional oceanography center provides information on expected wind conditions. A forecast of the appropriate wind conditions would be predictive evidence indicating an increased likelihood of upwelling. In such a situation, the shelf's coastal and bathymetric configuration would be evidence for whether the upwelling would be likely to occur only at the coast or also at the shelf break. A similar approach would be used for other relevant processes.

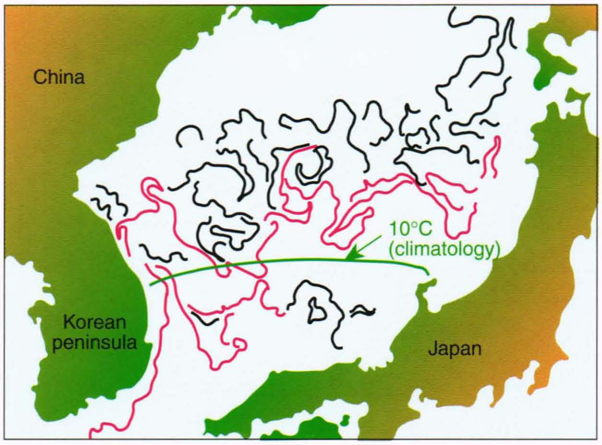


Figure 4. Sea-surface temperature gradient analysis of the infrared image in Figure 3. The original sea-surface imagery is replaced by an oceanographic feature analysis of the locations of the significant temperature gradients in the image. The gradients (red depicting a strong temperature difference and black a weak gradient) often represent the surface expression of the water-mass features (e.g., water-mass boundaries, or fronts, and separated eddies) in the image. The complexity in the imaged water-mass structure is evident by comparison with the mean climatological location of the 10°C sea-surface isotherm shown in green, which is often taken as the indicator of the northern boundary of the Tsushima current. The analyst captures the essential elements of this complexity in an oceanographic feature-oriented analysis, thus reducing the volume of data that must be handled in subsequent analyses and facilitating automated interpretation of the image.

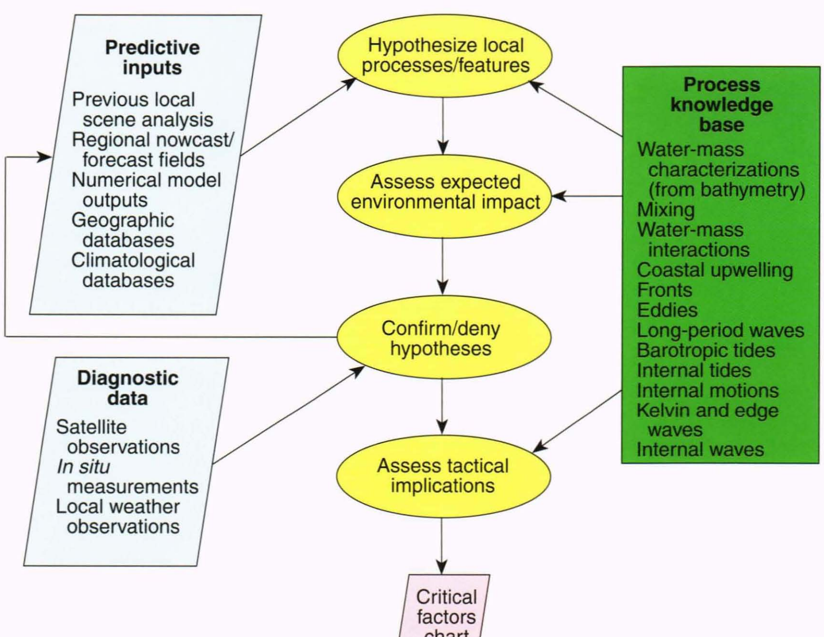
Figure 5 shows the anticipated flow of reasoning. Upon entering a new operating area, the OES would first try to hypothesize which coastal processes would be likely to occur and their significance to the local area. Nowcast/forecast fields from a regional oceanographic center, outputs from numerical models, and geographical and climatological databases would be used to infer which of the potentially relevant processes represented in the process knowledge base are actually important to the specific area at hand. The process knowledge base is intended to be general in the sense that it is not tailored to a particular coastal region. The variations and variability climatology knowledge bases (shown in Fig. 2) are to be instantiated with region-specific information.

Once the oceanographic features and processes that are likely to be significant have been hypothesized, the process knowledge base is consulted to assess their probable effect on the local environment. For example, the process of coastal upwelling could be expected to produce a deeper mixed layer and cooler sea-surface temperatures in the upwelling zone. Similarly, the effect of a local eddy that does not appear in the regional nowcast/forecast can be assessed by "bogusing" it in, that is, by adjusting the adjacent stream lines accordingly.[6]

Next, *in situ* data measured locally or sensed remotely are checked to see if they are consistent with the expectations or if they suggest alternative hypotheses. The complexity of the coastal scene and the interactions between the various processes may make it difficult to assess which processes are causing the various manifestations observed in the local data. Limiting the focus to the dominant processes and their manifestations, however, will help, as will the additional data and understanding gained over time in a particular area. Further, merely hypothesizing the processes in a region that are likely to be significant, even if they cannot be decisively confirmed or denied by local data, has potential utility.

The final step in the reasoning process is to assess the tactical implications (e.g., the implications for acoustic propagation) of the processes/features confirmed or at least believed to be likely. The output of the reasoning process is an on-line geographic critical factors chart showing the dominating conditions for the task at hand (e.g,. regions of bottom-limited acoustic propagation). An

Figure 5. Flow of reasoning for the Ocean Expert System (OES). When a new operating area is entered, the oceanographic environmental scene description would be initialized from historical climatological databases. The OES could then draw on predictive inputs and the process knowledge base to generate hypotheses about the occurrence of various coastal ocean processes and their expected environmental effects. Diagnostic data serve to confirm the hypotheses or suggest alternatives. The conclusions are used as feedback so that the scene description improves with time as more local data become available. Finally, the tactical implications of the oceanographic processes/features are assessed and highlighted to the operator in a geographically based critical factors chart.



explanation facility is envisioned that would explain the process or feature that caused the tactically relevant condition, as well as the predictive and diagnostic evidence that led to the conclusion of that particular process or feature being present or likely.

Refining the Environmental Scene Description with Time

Recent experience during Desert Storm highlighted the benefit of being able to develop improved forecasts over time. Anticipating nontraditional and data-sparse operating areas, the OES must be able to refine its understanding of the coastal ocean environment with time and evidence. In particular, mechanisms must exist to accumulate evidence and refine analyses, estimates, and conclusions. The feedback loop shown in Figure 5 allows the OES to use previous analyses in the ongoing reasoning process for the coastal scene description. The initial field estimate can then be revised as additional evidence is obtained in the form of satellite imagery or other relevant local measurements.

Sources of Uncertainty

Numerous sources of uncertainty exist in both the available data and the reasoning that occurs throughout the process of developing the coastal ocean scene description. Nowcast/forecast fields, for example, may not resolve the physical scales of interest. Similarly, the data may be insufficient to initialize a numerical model such that the scales of interest can be accurately resolved. *In situ* data, such as expendable bathythermograph readings, can be inaccurate. Remotely sensed data may be imprecise if not properly registered to reference points on the coastline. Additionally, satellite image-processing algorithms highlight the significant water-mass features only to some degree of uncertainty. In short, the available data can be incomplete, imprecise, inaccurate, or otherwise uncertain.

When reasoning on the basis of evidence, one frequently makes use of rules of inference where the presence of (causal or diagnostic) evidence implies the truth of a hypothesis. Such a statement can be represented as $E \rightarrow H$, where E represents the evidence and H represents the hypothesis. Inaccurate, imprecise, incomplete, or otherwise uncertain data used as the evidence E can only produce an uncertain conclusion concerning the hypothesis H. Additionally, the rules of inference themselves are subject to uncertainty. In other words, even if the evidence E were known with certainty, the logical implication of the "$\rightarrow$" symbol is often an expression of increased likelihood rather than complete certainty. For example, although one may be able to come up with a reasonable likelihood that an eddy will be pinched off from the meandering Gulf Stream or that coastal upwelling will occur under a particular set of circumstances, complete certainty that these things will occur is unlikely. Finally, even if a precise and certain rule could be developed, the conditions in a given region at a particular time would probably only meet the triggering conditions of that rule to some degree of uncertainty, and, hence, the conclusion would again have an element of uncertainty.

Each of the examples of sources of uncertainty given here has focused on hypothesizing the physical processes that are likely to be significant in a particular region. Similar uncertainties are present when hypothesizing the effects of such processes on the coastal environment.

Although both the data and the feature/process knowledge involved in developing a description of the coastal scene contain many uncertainties, causal links are known between the underlying physical features/processes and the resulting environmental conditions. Operational naval oceanographic and meteorological analysts currently rely on these causal links and the predictive and diagnostic evidence to develop a geographically based critical factors chart. The OES is intended to extend such an approach to surface and subsurface platforms where expert analysts are not available.

In this section, a flow of reasoning for the OES has been defined that requires the ability to refine the description of the coastal scene with time, to handle both uncertain data and inferences, and to distinguish between competing hypotheses on the basis of accumulating evidence. The remainder of this article introduces various approaches to uncertainty management and evidence-based reasoning currently being investigated for incorporation into the OES. This survey will be of interest to anyone developing a knowledge-based system in which uncertainty management is required.

PROBABILISTIC APPROACH USING BAYES'S THEOREM

Uncertainty about the likelihood that a particular event will occur frequently finds expression in probabilistic terms. If S, the sample space, is the set of all the possible elementary outcomes, and H is an event, that is, a subset of S, then the probability of event H is denoted by $p(H)$. The function p is a probability function if and only if it satisfies the following three axioms:

(P1) $p(H) \geq 0$ for all H that are elements of the set S,
(P2) $p(S) = 1$, and
(P3) if $H_i, \ldots, H_n$ are mutually exclusive, that is, they are pairwise disjoint and cannot occur at the same time, then $p\left(\bigcup_{i=1}^{n} H_i\right) = \sum_{i=1}^{n} p(H_i)$, where $\cup$ refers to set union.

An important consequence of these axioms is that

$$p(H) + p(-H) = 1 \ , \tag{1}$$

where $-H$, the complement of H, is the set difference of S and H. This equality follows since $1 = p(S) = p(H \cup -H) = p(H) + p(-H)$. Rewriting Equation 1 as $p(-H) = 1 - p(H)$ allows one to compute $p(-H)$ from $p(H)$.

The probability that an event H occurs, given that one knows with certainty that an event E occurs, is called the conditional probability of H given E. This conditional probability is denoted $p(H \mid E)$ and is defined to be the probability of both H and E occurring (the joint probability of H and E) divided by the probability of E occurring:

$$p(H \mid E) = \frac{p(H \cap E)}{p(E)} \quad \text{if } p(E) > 0 \ , \tag{2}$$

where $\cap$ refers to set intersection.

Two events A and B are independent (meaning that the occurrence of either one of the events does not affect the occurrence of the other) if $p(A \mid B) = p(A)$ and $p(B \mid A) = p(B)$. From the definition of conditional probability, it can be seen that two events A and B are independent if and only if $p(A \cap B) = p(A) \times p(B)$.

The concept of total probability states that if $E_1, \ldots, E_n$ are exhaustive and mutually exclusive (i.e., $\bigcup_{i=1}^{n} E_i = S$ and $E_i \cap E_j = \varnothing$ [the empty set] for $i \neq j$), then for any event H,

$$p(H) = \sum_{i=1}^{n} p(H \mid E_i) p(E_i) . \tag{3}$$

This result can be shown with the following argument. Since $E_1, \ldots, E_n$ are exhaustive and mutually exclusive, H can be expressed as $H = (H \cap E_1) \cup (H \cap E_2) \cup \ldots \cup (H \cap E_n)$. Using the additivity axiom (P3), we can express the probability of H as $p(H) = \sum_{i=1}^{n} p(H \cap E_i)$. Finally, making use of the definition of conditional probability on each term in the summation gives us the result in Equation 3.

With the necessary fundamentals of probability theory in place, Bayes's theorem, the core of uncertainty management in this approach, can be derived. From the definition of conditional probability, $p(H \mid E)\, p(E) = p(H \cap E) = p(E \cap H) = p(E \mid H)\, p(H)$, since joint probability or set intersection is commutative. This expression can be rewritten as

$$p(H \mid E) = \frac{p(E \mid H)\, p(H)}{p(E)} . \tag{4}$$

Recognizing that any set and its complement are exhaustive and mutually exclusive, total probability (Eq. 3) can be used to express $p(E) = p(E \mid H)\, p(H) + p(E \mid -H)\, p(-H)$. Substituting this equation into the denominator of Equation 4 yields Bayes's theorem:

$$p(H \mid E) = \frac{p(E \mid H) p(H)}{p(E \mid H) p(H) + p(E \mid -H) p(-H)} . \tag{5}$$

If H represents some hypothesis one seeks to confirm or deny and E represents a piece of evidence that has been observed, then $p(H)$ represents the prior probability of H occurring before the evidence E was observed, and $p(H \mid E)$ represents the posterior or updated probability of H after factoring in the new evidence E.

The value of Bayes's theorem for evidence-based reasoning can now be seen. Consider a medical diagnosis in which H is a particular illness that has been hypothesized, and E is a symptom that has been observed. If we know the likelihood of a random person in the population having the illness, $p(H)$, and the likelihood of a patient known to have the illness exhibiting the symptom, $p(E \mid H)$, and the likelihood of the symptom being present in a person known not to have the illness, $p(E \mid -H)$, then Bayes's theorem allows us to conclude the likelihood that a person exhibiting the symptom does, in fact, have the illness, that is, $p(H \mid E)$. Similarly, one could determine the likelihood of a particular physical process or feature (e.g., a front) being present, given the prior probability of the process or feature being present (e.g., from historical satellite imagery) and the conditional probability of the observed environmental conditions, given the presence or absence of the process or feature in question.

Naturally, more than one process or, in general, hypothesis may need to be considered, and Bayes's theorem will need to be generalized accordingly. If $H_1, \ldots, H_m$ are exhaustive and mutually exclusive events (hypotheses), then for any (evidence) E, $p(H_i \mid E) = [p(E \mid H_i) \cdot p(H_i)]/p(E)$ by the definition of conditional probability. Applying total probability (Eq. 3) to the denominator gives

$$p(H_i \mid E) = \frac{p(E \mid H_i) p(H_i)}{\sum_{k=1}^{m} p(E \mid H_k) p(H_k)} . \tag{6}$$

Just as multiple (exhaustive and mutually exclusive) hypotheses may need to be considered, so may multiple pieces of evidence. The most general form, then, is

$$p(H_i \mid E_1 E_2 \ldots E_n) = \frac{p(E_1 E_2 \ldots E_n \mid H_i)}{\sum_{k=1}^{m} p(E_1 E_2 \ldots E_n \mid H_k)\, p(H_k)} . \tag{7}$$

To summarize, assuming that one has prior probabilities for the hypotheses in question and conditional probabilities showing the likelihood of observing the various combinations of evidence, given one of the hypotheses, Bayes's theorem provides a method for determining the updated or posterior probability of any hypothesis in view of any observed evidence. Bayes's theorem has two primary advantages. First, it allows us to determine the probabilities of interest from other probabilities that are, presumably, easier to state. Second, the method can be rigorously proved correct, given the axioms of probability.

The Bayes's theorem approach to evidence-based reasoning, however, is not without difficulty. One issue is the prior and conditional probabilities required. The manner in which these numbers may be obtained can be limited by one's interpretation of probability. The relative frequency interpretation of probability, for example, requires that a repeatable experiment can be performed and defines the probability of an event E to be

$$p(E) = \lim_{n \to \infty} \frac{n(E)}{n} , \tag{8}$$

where $n(E)$ is the number of times that E occurs in the first n repetitions of the experiment. The weak law of large numbers states that if p is the probability of an event E, then for any δ greater than 0 and any confidence level less than 1, a sufficiently large number of experiments exists for which one can conclude with that confidence level that the frequency estimate is within δ of $p(E)$. Someone holding to the relative frequency interpretation of probability will be limited to applying Bayes's rule only when many repeatable experiments can be performed to arrive at the requisite initial values. Although this limitation is not a problem for tossing coins, rolling dice, or drawing cards, situations exist in which one would like to assign probabilities but a repeatable experiment may not be possible. For example, the OES may want to use the probabilities of upwelling occurring along a particular continental shelf break under various wind conditions. Conducting a large number of repeated experiments under each of the wind conditions to determine these probabilities is not feasible.

The subjective interpretation of probability, on the other hand, claims that probability represents a person's degree of belief in a proposition. A central idea of this interpretation is that a probability is the rate at which a person is willing to bet for or against a proposition. Thus, two reasonable people may have different degrees of belief in a proposition, even though they have been presented with the same evidence. The subjective interpretation postulates rational behavior in which people hold only coherent sets of beliefs. In this context, a set of beliefs is said to be coherent if it will not support a combination of bets such that the person holding those bets is guaranteed to lose. Shimony[7] has shown that a set of beliefs is coherent if and only if the beliefs satisfy the axioms of probability. If one accepts a subjectivist interpretation of probability, then subjective estimates provided by a domain expert can legitimately be used for assigning the requisite probabilities for the application of Bayes's theorem.

A reasonable question, then, is "How good are people at estimating probabilities?" Tversky and Kahneman[8] provided empirical evidence that people are poor estimators of probability. Compiled data are another potential source for the needed probabilities. Harris[9] surveyed medical literature for results that compiled data on patients for seven commonly used tests and found significant variability in the reported probabilities for five of the tests and good correlation between the reported probabilities for the other two. Although both experts and compiled data seem to produce suboptimal probability values, Ben-Bassat et al.[10] showed that even poor probabilities may be good enough to discern accurately between competing hypotheses, since large deviations in the prior and conditional probabilities result in only small changes in the value produced by the application of Bayes's theorem.

In addition to the question of where the requisite prior and conditional probabilities come from, other potential difficulties are associated with the application of Bayes's theorem. In particular, Bayes's theorem requires all the evidence and hypotheses to be expressed as propositions, and all the evidence that may be received to be anticipated in advance (since one must supply the conditional probabilities of the evidence being present or not, given the truth of a particular hypothesis). Most troubling, however, is the exponential explosion in the number of values required for the application of Bayes's theorem. In particular, Equation 7 requires the conditional probabilities of all the possible combinations of evidence for each of the hypotheses of interest. Further, extending the system to include the possibility of a new piece of evidence could require updating all the conditional probabilities, since they depend on the combination of the presence or absence of each piece of evidence.

One approach to dealing with the large number of values required is to assume conditional independence among pieces of evidence, given a hypothesis. This assumption reduces Equation 7 to

$$p(H_i \mid E_1 E_2 \ldots E_n) = \frac{p(E_1 \mid H_i) \times p(E_2 \mid H_i) \times \ldots \times p(E_n \mid H_i) \times p(H_i)}{\sum_{k=1}^{m} p(E_1 \mid H_k) \times p(E_2 \mid H_k) \times \ldots \times p(E_n \mid H_k) \times p(H_k)} . \quad (9)$$

Although the conditional independence assumption helps to reduce the number of initial values required, it sacrifices the accuracy if the assumption is not valid. In summary, the use of subjective probability theory and Bayes's rule provides a mathematically well-founded and statistically correct method for handling uncertainty, but such an approach is not feasible for most applications, because too many prior and conditional probability values are required.

DEMPSTER-SHAFER THEORY

The theory of belief functions, or Dempster-Shafer theory, was developed by Arthur Dempster[11] in a series of papers in the 1960s and Glen Shafer[12] in his 1976 book, *A Mathematical Theory of Evidence*. The theory is a generalization of probability theory and was motivated by difficulties these researchers had with the additivity axiom (P3), the representation of ignorance in probability theory, and the demands for prior and conditional probabilities, which they perceived frequently to be unavailable.

In particular, Shafer took issue with the property of a probability function p that states that $p(H) + p(-H) = 1$ for any hypothesis H. Shafer claims that evidence partially supporting a hypothesis need not be viewed as also partially supporting the negation of the hypothesis. In regard to the representation of ignorance, Dempster-Shafer theory provides an explicit representation to make up for the perceived weakness of the lack of such a representation in probability theory. Under a probabilistic approach, in the absence of any further information, probability is distributed among competing alternatives according to the principle of indifference or maximum entropy. Thus, if three alternatives exist and no evidence for or against any of them has been obtained, then each is assigned a probability of 1/3. Shafer argues that such an approach blurs this circumstance of ignorance with the situation in which significant background knowledge or evidence exists to show that the three alternatives are, in fact, equally likely. Dempster-Shafer theory provides an explicit representation of ignorance, where assigning a belief of zero to a proposition implies complete ignorance rather than certain falsehood, as in probability theory.

The starting point in Dempster-Shafer theory is the frame of discernment Θ, which is composed of a set of exhaustive and mutually exclusive events. The frame of discernment can be viewed as a set of possible (mutually exclusive) answers to a question. It is similar to the sample space in probability theory. Consider ocean frontal analysis at a particular location where fronts of various strengths have been observed in the past. One question might be "What type of front is present at this location?" The frame of discernment would be a set of possible (mutually exclusive) answers to this question, perhaps {none, weak, strong}. An important concept in Dempster-Shafer theory is that any subset of the frame of discernment is a hypothesis and is equivalent to the disjunction of the individual elements in the subset. Thus, the hypothesis of some type of front being present would be the subset {weak, strong}. The set of all hypotheses represented by a frame of discernment Θ, that is, the set of all subsets of Θ, is denoted 2^{Θ}.

A basic probability assignment, *m*, assigns a number in [0, 1] to every subset *A* of Θ such that the numbers sum to one. The expression *m*(*A*) is the probability mass assigned exactly to *A* that cannot be further subdivided among the subsets of *A*. Thus, in the frontal analysis example, probability mass that simply supports the presence of a front without giving any insight into whether the front is likely to be weak or strong would be assigned to *m*({weak, strong}). More specific information would be assigned to *m*({weak}) or *m*({strong}). Any probability mass not assignable directly to any subset of Θ (because of ignorance) is grouped under $m(\Theta)$. Formally, a function *m* with domain 2^{Θ} is a basic probability assignment if it satisfies

(M1) $0 \leq m(A) \leq 1$ for any *A* that is a subset of Θ,

(M2) $m(\varnothing) = 0$, and

(M3) $\sum_{A \subseteq \Theta} m(A) = 1$.

Gordon and Shortliffe[13] have presented an example of a simplified medical diagnosis on cholestatic jaundice that helps to make these concepts clear. The frame of discernment comprises four competing causes of the illness, namely, hepatitis (*Hep*), cirrhosis (*Cirr*), gallstones (*Gall*), and pancreatic cancer (*Pan*) (Fig. 6A). There are sixteen possible subsets of Θ to which some of the total belief can be assigned (Fig. 6B plus the empty set $\varnothing$), although only seven of these are of clinical interest (Fig. 6C). One possible basic probability assignment is $m(\{Hep\}) = 0.2$, $m(\{Cirr\}) = 0.1$, $m(\{Hep, Cirr\}) = 0.2$, $m(\{Gall, Pan\}) = 0.4$, $m(\Theta) = 0.1$, and $m(A) = 0$ for all other $A \subseteq \Theta$.

The belief in *A*, denoted *Bel*(*A*), measures the total amount of belief in *A*, not the amount assigned precisely to *A* by the basic probability assignment. Thus,

$$Bel(A) = \sum_{X \subseteq A} m(X), \tag{10}$$

where every belief function corresponds to a specific basic probability assignment. Formally, a function *Bel* with domain 2^{Θ} is a belief function if it satisfies

(B1) $0 \leq Bel(A) \leq 1$ for any $A \subseteq \Theta$,

(B2) $Bel(\varnothing) = 0$,

(B3) $Bel(\Theta) = 1$, and

(B4) $Bel(A_1 \cup A_2) \geq Bel(A_1) + Bel(A_2) - Bel(A_1 \cap A_2)$ and similarly for $n > 2$ subsets.

Returning to the cholestatic jaundice example, $Bel(\{Hep, Cirr\}) = m(\{Hep\}) + m(\{Cirr\}) + m(\{Hep, Cirr\}) = 0.2 + 0.1 + 0.2 = 0.5$.

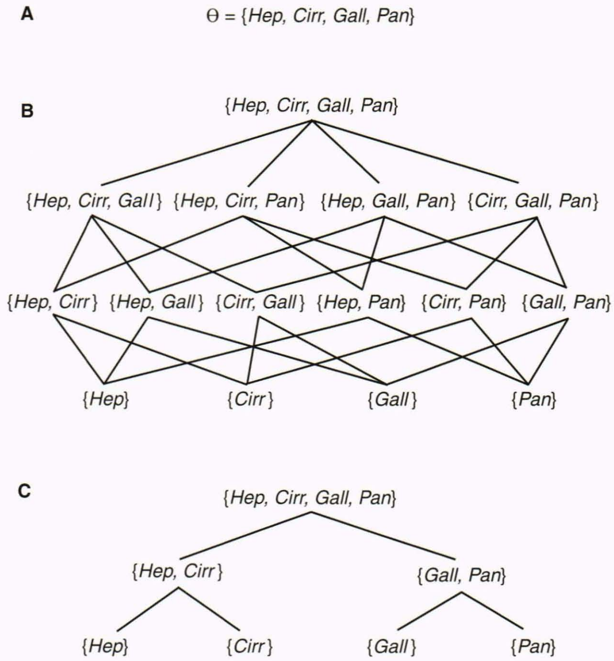


Figure 6. The frame of discernment, Θ, in Dempster-Shafer theory is the set of possible (mutually exclusive) answers to a question. **A.** The frame of discernment for a simplified medical diagnosis of cholestatic jaundice where hepatitis (*Hep*), cirrhosis (*Cirr*), gallstones (*Gall*), and pancreatic cancer (*Pan*) are the possible causes. **B.** 2^{Θ}, the set of all subsets of Θ. **C.** A few subsets of Θ that are of clinical interest. Dempster-Shafer theory supports the assignment of belief and the collection of evidence at any level in the hierarchy of subsets shown in part B. Further, it parallels human reasoning in the narrowing of the hypothesis subset with the accumulation of evidence. Dempster's rule of combination for assessing the combined effect of two pieces of evidence can be computationally demanding, but that problem is mitigated when the evidence is limited to a small subset of 2^{Θ} as shown in part C.

In Dempster-Shafer theory, the observation of evidence against a hypothesis is viewed as evidence supporting its negation. The negation of a hypothesis is defined to be the set difference of the frame of discernment and the hypothesis. Thus, evidence disconfirming the hypothesis {*Hep*} is equivalent to evidence supporting the hypothesis {*Cirr, Gall, Pan*}, that is, *Cirr* or *Gall* or *Pan*. Similarly, $Bel(-\{Hep, Cirr\}) = Bel(\{Gall, Pan\}) = 0.4$. Note that if we call *A* the hypothesis *Hep* or *Cirr*, that is, {*Hep, Cirr*}, we see that $Bel(A) + Bel(-A) = 0.9$. In general, for any hypothesis $A \subseteq \Theta$, $Bel(A) + Bel(-A) \leq 1$. This can be contrasted with probability theory, where $p(A) + p(-A) = 1$.

The plausibility of *A* is defined as

$$Plaus(A) = 1 - Bel(-A), \tag{11}$$

and reflects the extent to which the evidence allows one to fail to doubt the hypothesis *A*. The complete information stored in a basic probability assignment *m* or the corresponding belief function *Bel* can be expressed by the belief interval [*Bel*(*A*), *Plaus*(*A*)], where *Bel*(*A*) represents the amount of belief currently committed to *A*, and

Plaus(A) represents the maximum amount of belief that could be committed to A, since the remaining belief has been committed to $-A$. The width of the interval is a measure of the belief that neither supports nor refutes A, but which may later shift either way on the basis of additional evidence. The width of the interval can be viewed as the amount of uncertainty associated with a particular hypothesis, given the evidence.

In probability theory, the effect of evidence is represented as conditional probabilities, and when new evidence is received, the posterior probability can be calculated from the prior and conditional probabilities using Bayes's rule. The analogous method for updating beliefs in the presence of new evidence under Dempster-Shafer theory is Dempster's rule of combination. If two pieces of independent evidence are represented by two basic probability assignments m_1 and m_2, then Dempster's rule of combination provides a method for calculating a new basic probability assignment denoted $m_1 \oplus m_2$ and called the orthogonal sum of m_1 and m_2. The corresponding belief function $Bel_1 + Bel_2$ can then be computed from $m_1 \oplus m_2$ using Equation 10. Formally, Dempster's rule of combination is as follows. Let m_1, m_2 be basic probability assignments over Θ. If $\sum_{X \cap Y = \varnothing} m_1(X) m_2(Y) < 1$, then the orthogonal sum, $m_1 \oplus m_2$, of m_1 and m_2 is

(i) $m_1 \oplus m_2(\varnothing) = 0$ and

(ii) for any nonempty $A \subseteq \Theta$,

$$m_1 \oplus m_2(A) = \frac{\sum\limits_{X \cap Y = A} m_1(X) m_2(Y)}{1 - \sum\limits_{X \cap Y = \varnothing} m_1(X) m_2(Y)}. \tag{12}$$

An intersection table with values of m_1 and m_2 along the rows and columns, respectively, is a helpful device for computational purposes. Each entry in the table has the intersection of the subsets in the corresponding row and column, as well as the product of the two basic probability assignment values. If we take m_1 to be the sample basic probability assignment defined earlier and $m_2(\{Hep\}) = 0.8$, $m_2(\Theta) = 0.2$, and $m_2(A) = 0$ for all other subsets A of Θ, then the information shown in the following table results:

	m_1				
m_2	$\{Hep\}$ (0.2)	$\{Cirr\}$ (0.1)	$\{Hep, Cirr\}$ (0.2)	$\{Gall, Pan\}$ (0.4)	Θ (0.1)
$\{Hep\}$ (0.8)	$\{Hep\}$ (0.16)	$\varnothing$ (0.08)	$\{Hep\}$ (0.16)	$\varnothing$ (0.32)	$\{Hep\}$ (0.08)
Θ (0.2)	$\{Hep\}$ (0.04)	$\{Cirr\}$ 0.02	$\{Hep, Cirr\}$ (0.04)	$\{Gall, Pan\}$ (0.08)	Θ (0.02)

Using this table, we have $m_1 \oplus m_2(\{Hep\}) = (0.16 + 0.04 + 0.16 + 0.08)/[1 - (0.08 + 0.32)] = 0.44/0.60 = 0.733$.

The primary difference between Dempster-Shafer theory and Bayesian probability theory is that Dempster-Shafer allows one to assign probability mass to subsets of the frame of discernment directly, whereas Bayesian probabilities are only directly assigned to the elementary outcomes or individual elements of the sample space. It is possible to compute the probability of an arbitrary subset of the sample space in Bayesian probability theory, but this value is computed from the probability of the elementary outcomes that compose that subset. For example, the probability of the roll of a single die coming up even is the sum of the probabilities of the die coming up two, four, or six. Dempster-Shafer theory allows one to distribute the probability mass (which must sum to one) across the subsets of the frame of discernment, whereas probability theory requires that the unit probability mass be spread across the elementary outcomes or individual elements of the sample space. Note that a probability function is a special case of a belief function in which belief is only assigned to singleton elements of the frame of discernment. Hence, Dempster-Shafer theory can be viewed as a generalization of probability theory. The explicit representation of ignorance in Dempster-Shafer theory and the fact that *Bel*(A) + *Bel*($-A$) need not sum to one both result from the ability to assign probability mass to subsets of Θ rather than merely to singleton elements.

The difference between these two approaches can also be seen in that probability theory represents the state of knowledge with a single number, whereas Dempster-Shafer theory uses two numbers—the belief and the plausibility (or the lower and upper probability, in Dempster's terms) (see Fig. 7). In probability theory, the upper and lower probabilities are the same, since $p(A) + p(-A) = 1$. In Dempster-Shafer theory, the width of the interval between the belief and the plausibility can be viewed as the

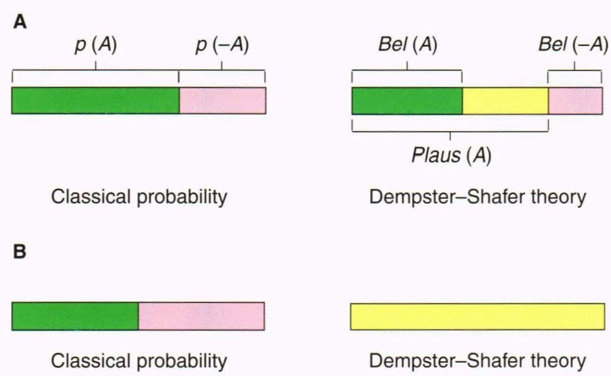


Figure 7. A. Classical probability theory and Dempster-Shafer theory differ in their representations of uncertainty in a proposition. Probability theory uses a single number $p(A)$ to represent the probability that proposition A is true, given the evidence. The remaining probability $p(-A) = 1 - p(A)$ is the probability that the proposition is not true. Dempster-Shafer theory uses two numbers that can be viewed as the lower and upper limits of the probability of A. *Bel* (A) is the belief in A or the lower limit, and *Plaus*(A) = 1 − *Bel*($-A$) is the plausibility of A or the upper limit. The width of the interval between the belief and the plausibility can be viewed as the uncertainty in the probability of A. **B.** Illustration of how complete uncertainty about a proposition would be represented in each of the approaches. Probability theory states that in the absence of further information, it is equally likely that the proposition will be true or false. Proponents of Dempster-Shafer theory point out that this approach does not distinguish the situation of complete ignorance from the situation of significant background knowledge, indicating that truth and falsehood are equally likely. Dempster-Shafer theory explicitly represents ignorance about proposition A by setting *Bel*(A) = 0 and *Plaus*(A) = 1.

uncertainty in the probability, since, in general, the probability cannot be pinned down precisely on the basis of the available evidence. One benefit of this "extra" uncertainty is that not nearly as many values are required to use Dempster-Shafer theory as are required for the Bayesian approach.

Dempster-Shafer theory has many advantages. The most frequently stated advantage is the ability to represent ignorance explicitly. Probability is said to assume more information than is given by using the principle of maximum entropy to assert equal prior probabilities. Cheeseman, however, has countered that "those that make these claims fail to show a single unfortunate consequence that follows from this supposed assumed information."[14]

Dempster-Shafer theory has the advantage of explicitly representing evidence that bears on a subset of the frame of discernment rather than only on singletons within the frame. This explicit representation supports the aggregation of evidence gathered at varying levels of detail. Gordon and Shortliffe[13] have stated that this representation is of great value, since human diagnostic reasoning naturally gathers evidence at multiple levels of detail. For cholestatic jaundice, for example, a test may indicate whether the problem is intrahepatic cholestasis {*Hep*, *Cirr*} or extrahepatic cholestasis {*Gall*, *Pan*}. Dempster-Shafer theory allows one to represent evidence for the single hypothesis, *Hep* or *Cirr*, rather than being forced to divide it between the hypothesis *Hep* and the hypothesis *Cirr*.

Another advantage of Dempster-Shafer theory is that it models the narrowing of the hypothesis set with the accumulation of evidence. That is, as evidence is accumulated, the probability mass tends to move down to subsets representing more specific statements. For example, if m_1 assigns mass to Θ and {*Cirr*, *Gall*, *Pan*}, and m_2 assigns mass to Θ and {*Hep*, *Cirr*}, then $m_1 + m_2$ will assign some mass specifically to {*Cirr*}. Finally, since Dempster-Shafer theory uses probability more loosely than a purely Bayesian approach, it does not require the many conditional and prior probability values that a Bayesian approach would require.

Dempster-Shafer theory is not without disadvantages, however. Chief among them is the fact that although Dempster's rule of combination does seem to reflect the pooling of evidence (it is commutative; multiple positive evidences yield a higher belief than any of the evidences on their own, etc.), Dempster and Shafer have merely stated the combination rule without rigorously establishing it as valid. Further, the requirement that the singletons in the frame of discernment be mutually exclusive and exhaustive, and the assumption in Dempster's rule of combination that the evidence is independent, may not be easily met in some domains. Finally, like Bayes's rule, Dempster's rule of combination suffers from computational complexity, since nearly all the functions require exhaustively enumerating all the 2^{Θ} possible subsets of the frame of discernment. Sometimes, the evidence may actually be limited to a small subset of 2^{Θ}, and Dempster-Shafer theory may be tractable in these instances (see Fig. 6C). Barnett[15] has developed a linear-time algorithm for computing m and Bel when the hypotheses of interest are restricted to mutually exclusive singletons and their negations. Shafer and Logan[16] have developed a linear-time algorithm that computes beliefs and plausibilities for hypotheses in a hierarchical tree. Computing the Dempster-Shafer theory functions for subsets of Θ not in the tree, however, is still exponential in the worst case.

Bayesian probability analysis is generally believed, even by the supporters of Dempster-Shafer theory, to be the best approach to uncertainty management for those situations in which all the inputs required for a Bayesian probability analysis are available. Shafer himself wrote, "I would like to emphasize that nothing in the philosophy of constructive probability or the language of belief functions requires us to deny the fact that Bayesian arguments are often valuable and convincing."[17] Barnett wrote, "This is not to say that one should not use Bayesian statistics. In fact, if one has the necessary information, I know of no other proposed methodology that works as well. Nor are there any serious philosophical arguments against the use of Bayesian statistics. However, when our knowledge is not complete, as is often the case, the theory of Dempster and Shafer is an alternative to be considered."[15] Shafer echoed this idea, stating that "the advantage gained by the belief-function generalization of the Bayesian language is the ability to use certain kinds of incomplete probability models."[17]

BELIEF/CAUSAL NETWORKS

Bayes's theorem provides a statistically correct method for handling uncertainty when the requisite prior and conditional probabilities are available. Dempster-Shafer theory generalizes the Bayesian approach through characterizing each hypothesis by a belief interval rather than a single-point probability, but sacrifices the provable validity of the approach in the process. Assuming conditional independence between all the various pieces of evidence, given a hypothesis, as in Equation 9, similarly greatly reduces the number of values required but also sacrifices accuracy, as this assumption is rarely valid. Bayesian belief or causal networks[18] allow one, working with a domain expert familiar with the cause/effect relationships between the variables, to represent precisely which statements of conditional independence are, in fact, valid. This representation supports efficient computation of accurate probabilities with a greatly reduced number of initial values.

Belief networks are special directed acyclic graphs (DAG's), where a DAG G consists of a finite set of vertices (nodes) V and a finite set of directed edges E connecting pairs of vertices such that G contains no cycles. Each vertex or node in a belief network is a propositional variable, that is, a variable whose value can be one of a set of mutually exclusive and exhaustive alternatives. Edges in a belief network represent a causal influence of one node on another.

An exemplary belief network that depicts some of the cause/effect relationships associated with coastal upwelling is shown in Figure 8. The node labeled upwelling/downwelling represents a propositional variable that might take a value in the set {strong upwelling, weak

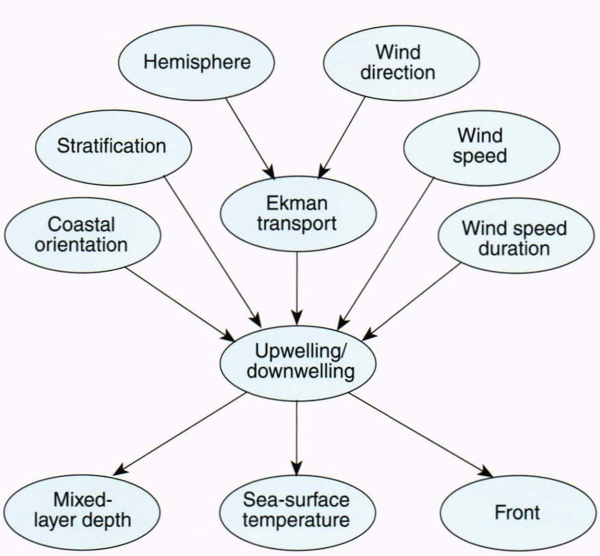


Figure 8. An exemplary belief network that depicts some of the cause/effect relationships associated with coastal upwelling. Each node in a belief network represents a propositional variable that can take on one of a set of possible values. The directed edges between the nodes are drawn from direct cause to effect. Thus, coastal upwelling is caused or influenced by the orientation of the coast, the stratification, the direction of Ekman transport, and the wind speed and wind speed duration. The structure of a belief network captures conditional independencies between the propositional variables. Efficient algorithms have been developed that exploit these independencies to determine and update probabilities as new evidence arrives using a greatly reduced number of initial values.

upwelling, none, weak downwelling, strong downwelling}. The propositional variables for wind direction, coastal orientation, and direction of Ekman transport could take integer values in the range [0, 359]. The hemisphere propositional variable could take the values north or south and so on for the other variables in the belief network.

The five edges directed into the upwelling node represent the causal influence of the coastal orientation, the stratification, the direction of Ekman transport, and the wind speed and wind speed duration on whether or not upwelling takes place. The edges coming out of the upwelling node capture the influence of upwelling on the mixed-layer depth, the sea-surface temperature, and the presence or absence of a front associated with the boundary between the displaced warmer surface water and the cooler upwelled water.

Nodes in a graph can be described using kinship terminology. If two nodes A and B are connected by a directed edge from A to B, then A is said to be a parent of B, and B is called a child of A. Following this concept, the descendents of a node X, $desc(X)$, is the set of all nodes that can be reached by following successive directed edges beginning from node X.

Informally, a DAG is a belief network if the value of every node in the DAG is independent of the values of all nondescendent nodes, given the values of the parent nodes. With reference to Figure 8, discovering the presence of a front or cooler sea-surface temperatures in a coastal region could normally be viewed as evidence for the increased likelihood of a deeper mixed layer, since the front or cool sea surface could have been caused by upwelling, which would also cause a deepening of the mixed layer. If, however, we know for certain whether upwelling is occurring, then discovering the presence of the associated front or cool sea-surface temperatures does not change the probability of a deepening of the mixed layer. Thus, $p(MLD|U, S, F) = p(MLD|U)$, where MLD, U, S, and, F refer to the propositional variables for mixed-layer depth, upwelling, sea-surface temperature, and front, respectively. This equality describes a conditional independence relationship represented in the belief network and ranges over all values of the propositional variables MLD, U, S, and, F. The power of the belief network formalism lies in the ability to represent such conditional independencies in a natural way and to exploit them for the efficient and accurate computation of probability values using a relatively small set of initial values.

To make use of causal networks for uncertainty management, one begins with a domain expert who identifies the perceived direct causes of each variable. The variables and direct causal links are then assembled into a belief network that, by definition, must be acyclic. In Figure 8, the coastal orientation, the stratification, the direction of Ekman transport, and the wind speed and wind speed duration are the direct causes of coastal upwelling.

Next, one must obtain certain probabilities at the various nodes in the network. At each of the root nodes (those without a parent), the prior probability for the possible values of the propositional variable is needed. In Figure 8, historical observations could provide the probability associated with various wind directions at a particular location. Finally, the conditional probabilities for the values of the nonroot node variables, given the values of their parent node variables, are needed. Neapolitan[19] has presented detailed proofs of theorems that show that the joint probability distribution of all the variables can be calculated from these initial values. Using the joint probability distribution, one can obtain the probability of any value of any variable, given the values of any other variables. The reduction in complexity is significant.

For example, suppose one is modeling a domain with ten binary valued propositional values. Then there are 2^{10} values in the joint probability distribution. Now suppose the same domain can be modeled by a causal network where there is one root node, one propositional variable having one parent, and the other eight propositional variables each having two parents. In such a situation, only seventy values are needed to calculate the entire joint probability distribution.

An important aspect of belief networks is the manner in which probabilities are updated when new evidence arrives. Neapolitan[19] has rigorously derived algorithms developed by Pearl[18] for updating the probabilities of all the propositional variables using only local computations. The likelihoods of the various potential values for each of the variables depend, in general, on all the evidence that has been observed (as shown in Eq. 9). The effect of all the evidence on a single node, however, can be broken down into a diagnostic element obtained from the network rooted at the node and a causal element obtained from the portion of the network above the node.

Pearl's algorithm uses a message-passing scheme in which data need only to be communicated between adjacent nodes. The total impact of the causal evidence for a particular node is represented in messages from the node's parents, and the total impact of the diagnostic evidence is represented in messages from the node's children. When new evidence arrives in the form of an observation of particular value for a variable, messages emanate from this node and are propagated throughout the network much as a pebble causes spreading ripples when dropped in a pond. The network is guaranteed to reach equilibrium, at which time message propagation ceases and all the probabilities for all the possible values of all the variables have been updated to reflect the new evidence.

It should be noted that Pearl's algorithm is designed for singly connected DAG's, that is, networks in which there are no cycles in the underlying undirected graph. Pearl[18] has proposed a scheme that would allow the algorithm to be applied to multiply connected networks that are not highly connected. Lauritzen and Speigelhalter[20] have developed a belief propagation algorithm based on graph theory that, although less elegant and intuitive than Pearl's, is directly applicable to multiply connected networks.

Causal networks are attractive because they maintain consistent probabilistic knowledge bases, require far fewer initial values than a naive use of Bayes's rule, and impose no conditional-independence assumptions beyond those provided by the domain expert who enumerates the direct causes of each of the variables. That efficient algorithms exist for updating the probabilities in view of new evidence using only local computations is also attractive. A degree of modularity exists that allows the conditional probability distribution of a variable, given the values of its parents, to be modified without affecting the rest of the network. Portions of the network may be refined, with additional nodes or edges being added to reflect a growing understanding of the domain without invalidating the work done to define the rest of the network and obtain the needed prior and conditional probabilities. Finally, the causal links in the belief network can be used to produce explanations by tracing beliefs back to their sources.

OTHER APPROACHES

Several other approaches to uncertainty management have been proposed or applied to various problems. The certainty factors approach[21,22] was developed for use in the MYCIN medical expert system. This *ad hoc* approach is largely of historical interest, as it has been shown to have inconsistencies, and its developers have gone on to promote Dempster-Shafer theory.[13]

For the PROSPECTOR expert system, rather than providing the conditional probabilities $p(E|H)$ and $p(E|-H)$, the domain expert provided the likelihood ratios

$$LS\,(H,E)=\frac{p(E\mid H)}{p(E\mid -H)}\,,$$

and

$$LN\,(H,E)=\frac{p(-E\mid H)}{p(-E\mid -H)}\,. \tag{13}$$

The odds-likelihood formulations of Bayes's rule[23]

$$O(H\mid E)=LS(H,E)\times O(H)\,,$$

and

$$O(H\mid -E)=LN(H,E)\times O(H)\,, \tag{14}$$

where $O(H)=p(H)/p(-H)$ is the prior odds of H, and the posterior odds $O(H|E)$ and $O(H|-E)$ are similarly defined, were then used to compute the updated odds of the hypothesis H, given the evidence E or $-E$. An advantage of this approach is that people seem to prefer giving likelihood ratios rather than exact probabilities, although the values of LS and LN provided were often inconsistent. Duda et al.[23] outlined methods for dealing with these inconsistencies, and PROSPECTOR was successfully used to predict the location of commercially significant mineral deposits.

Each of the approaches presented thus far has been concerned with uncertainty in the occurrence of an event and has, at least, a loose connection to probability theory. Fuzzy logic,[24,25] by contrast, is altogether different. Fuzzy logic is concerned not with the uncertainty in the occurrence of an event, but rather with the imprecision or vagueness with which a property can be described. Fuzzy logic rejects the law of the excluded middle and thus denies a dichotomy between truth and falseness.

Fuzzy logic is based on fuzzy set theory, which is so named because the boundaries of the sets are imprecisely defined. Figure 9 contrasts conventional set membership with fuzzy set membership. In conventional set theory, a front is either weak, moderate, or strong. Fuzzy set theory, on the other hand, allows a front to belong partially to multiple sets at the same time. Once sets have been redefined in this manner, methods can be developed for specifying the effects of qualifiers such as "very," "not," and "rather" on the possibility distributions of sets. Further, it is possible to define a reasoning system based on methods for combining distributions via conjunction, disjunction, and implication. Fuzzy logic has successfully been applied in control systems for many devices, including trains, washing machines, and videocassette recorders.

Much debate exists as to the adequacy of probability theory for handling reasoning under uncertainty. Cheeseman has strongly defended probability, claiming that Dempster-Shafer theory, certainty factors, fuzzy logic, and other approaches "attempt to circumvent some perceived difficulty of probability theory, but . . . these difficulties exist only in the minds of their inventors."[14] He has further stated that these other approaches are "at best unnecessary and at worst misleading." Horvitz et al.[26] listed seven intuitive properties of measures of belief from which the axioms of probability are a necessary consequence, as shown by Cox.[27] Horvitz et al.[26] also reviewed which of these properties the various nonprobabilistic approaches reject and argued that those proper-

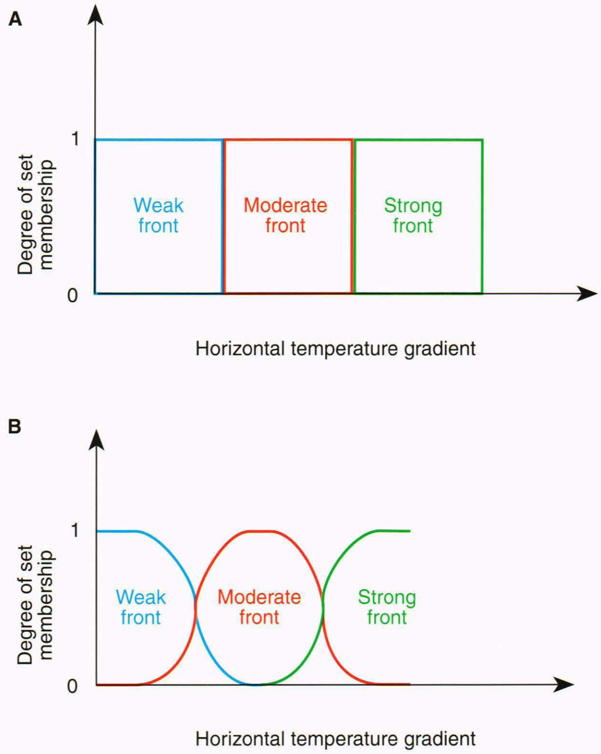


Figure 9. Comparison of two types of set membership. **A.** Conventional (precise). **B.** Fuzzy (imprecise). The vertical axis is the degree of set membership. Fuzzy set theory allows an object to have partial membership in one or more sets at the same time and forms the foundation for fuzzy logic. Fuzzy logic is not concerned with uncertainty about the truth of a proposition, but rather with the linguistic imprecision or vagueness with which a property can be described.

ties should provide a framework for future discussions about alternative mechanisms for uncertainty management.

OES STATUS AND PLANS

The OES project is in its first year. Early efforts have focused on enumerating sources of input, assessing the requirements of the user community, defining a provisional system architecture, conducting preliminary knowledge acquisition, representing the coastal-scene description process, and investigating paradigms for reasoning under uncertainty. Software that implements the algorithms for updating probabilities in Bayesian belief (causal) networks has been acquired, and implementation of the algorithms in Dempster–Shafer theory has begun.

Current efforts are focused on developing the process knowledge base and developing a facility for the automated construction of a static critical factors chart based solely on historical data. Work is progressing toward the demonstration of an initial prototype in 1994. Once this prototype is completed, current data, the process knowledge base, and formalisms for reasoning under uncertainty will be drawn on to support the automated development and continual refinement of multiple, mission-specific, dynamic critical factors charts.

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