

# INTRODUCTION TO SEA-SURFACE TOPOGRAPHY FROM SATELLITE ALTIMETRY

This article examines the reasons for oceanographic interest in the shape of the sea surface, the measurement of that shape from satellite altimetry, and the fundamental fluid dynamics relating sea-surface height to ocean currents. The calculation of sea-surface height using the altimeter measurement system, the satellite orbit, the geoid, and atmospheric variables is reviewed. The combined use of satellite-derived sea-surface heights and a numerical ocean-circulation model for improved ocean forecasting is discussed.

## INTRODUCTION

Sea-surface topography is of interest because many different physical processes cause it to change continually. Besides surface waves and tides, ocean phenomena that influence the shape of the sea surface vary on different scales of length and time. These phenomena include large current systems (e.g., the Gulf Stream, the Antarctic Circumpolar Current, and the Kuroshio), the eddies and rings shed by large currents, El Niño, storm response, and other open-ocean processes. The intimate connection between ocean currents and sea-surface topography is a consequence of the geostrophic force balance (the Coriolis force balanced by the pressure gradient), which is discussed below.

Changes in the sea-surface height produced by currents are too small (characteristic heights less than 1 meter), too spread out (characteristic lengths greater than 50 kilometers), and too slow (characteristic periods greater than 1 week) to be measured directly. Shipboard measurements of the distribution of subsurface water density, however, could be connected to sea-surface topography by making assumptions about geostrophic balance and the existence of uniform currents at some reference level deep in the ocean. Changes in sea-surface topography could then be related to differences in the currents at the surface and at the reference level.

Assumptions about currents are unnecessary when they can be measured by moored current meters or by acoustic tracking of floats. These types of current observations have been crucial to the progress of physical oceanography in recent decades. Another method of measuring currents uses two pressure gauges deployed on the ocean floor. The fluctuations of the pressure difference between the two gauges can be related to the horizontal current between them. However, the use of pressure gauges is hampered by difficulties of leveling and calibration drifts. Besides the expense and difficulty of making accurate current-meter and pressure-gauge

measurements, a major disadvantage is the very limited sampling that such in-situ data yield.

Satellite altimetry bypasses many difficulties of in-situ measurements of the velocity field. Altimetry not only provides rapid global coverage available from satellite-based measurement systems, it also bypasses the problem of knowing currents at a subsurface reference level (since surface topography is related directly to surface currents).

All the applications of altimetry mentioned above are derived from the measurement of the travel time of the radar pulse from the satellite to the ocean surface and back, along with ancillary observations. In addition, changes in the amplitude and shape of the radar pulse occur when it is reflected from the sea surface; these changes allow estimates to be made of the average wave height and the surface wind speed. The results of wave-height and wind-speed studies are not covered here, but examples are presented in the article by Dobson et al. elsewhere in this issue.

In the section below, the fundamental difference between two techniques for measuring sea-surface height—subsurface density measurements from a ship and from satellite altimetry—is described.

## THEORETICAL BASIS

Ocean-current systems that have length scales greater than about 50 kilometers and that vary on time scales longer than about one day have a characteristic force balance dominating their evolution. They are strongly constrained by the relative thinness of the ocean, the spherical shape of the earth, the rotation rate of the earth, and the vertical density changes (mostly due to temperature and salinity) of ocean water. The vertical hydrostatic equation (which holds as well at scales smaller than those mentioned above) states that pressure changes are simply due to the weight of the water above:

$$1/\rho \partial p/\partial z = -g, \quad (1)$$

where  $\rho$  is the water density,  $p$  is the pressure,  $z$  is positive upward, and  $g$  is the gravitational acceleration. Thus,

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pressure changes in the vertical are independent of water motion. The forces are quite different in the horizontal, where the geostrophic balance of pressure gradient and rotational force obtains:

$$1/\rho \partial p/\partial x = fv, \tag{2}$$

where  $v$  is the horizontal velocity at right angles to the pressure gradient and  $f$  (the Coriolis parameter) is related to the rotation rate of the earth by

$$f = 2 \Omega \sin \theta, \tag{3}$$

where  $\Omega$  is the rotation rate in radians per second and  $\theta$  is the latitude. The Coriolis force derives from the principle of conservation of total angular momentum in a rotating frame of reference. A basic result of the conservation law is that a water particle moving in the ocean feels a force to the right of its velocity (in the Northern Hemisphere). The geostrophic balance states that this force is not accompanied by an acceleration in the direction of the force, but rather is balanced by the force of the horizontal pressure gradient. These two force balances are illustrated in Fig. 1.

Since there are no time-dependent terms in Eqs. 1 and 2, one implication is that the vertically displaced water surface shown in Fig. 1 can exist in a steady state in the ocean in the presence of currents. Thus, water flows downhill everywhere except in the ocean, where it goes around! Of course, these force balances are only approximate, so that the steady state posed in Eqs. 1 and 2 is never actually realized. The neglected terms in the fluid dynamical equations of motion (which involve inertial effects, forcing, turbulent dissipation, etc.) cause a complex spatial and temporal evolution of all large-scale ocean phenomena.

The desired connection between currents and surface topography is obtained by eliminating the pressure between Eqs. 1 and 2 and by integrating vertically from an arbitrary reference level,  $z_r$ , to get

$$v - v_r = g/\rho f \int_{z_r}^z \partial \rho/\partial x \partial z', \tag{4}$$

where  $v_r$  is the velocity at the reference level and  $v$  is the velocity at  $z$ . One implication of Eq. 4 is shown in Fig. 2. If the density structure between two levels is known, the integration can be carried out and the velocity difference computed. Regions in Fig. 2 where constant-density surfaces slope are those having associated currents. Horizontal constant-density surfaces (which imply no pressure gradients for Eq. 2) have no currents.

Historically, the density structure needed for Eq. 4 was measured by lowering instrumentation from ships to measure temperature and salinity, from which density could be computed. To compute the velocity,  $v$ , an assumption had to be made about a reference level,  $z_r$ , and the velocity at the reference level,  $v_r$  which appears as an integration constant in Eq. 4. The usual assumption is to choose a reference level so that the velocity

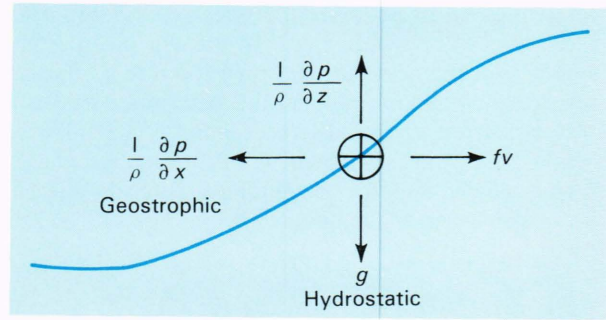


Figure 1—Force balance for ocean currents.

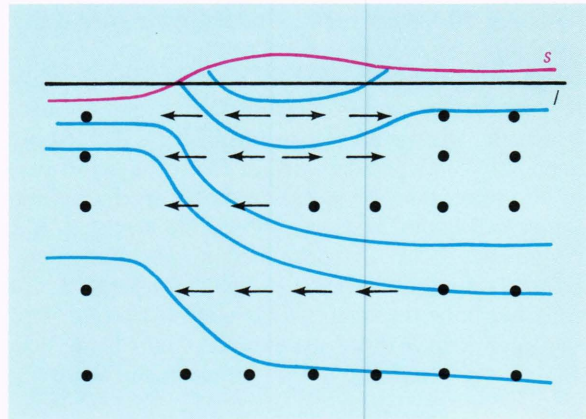


Figure 2—Schematic cross section of the Gulf Stream. The arrows indicate the direction of the horizontal pressure gradient, and the black dots indicate the vanishing horizontal pressure gradient. The current is flowing perpendicular to the plane of the page. The colored lines are contours of equal water density. The line  $l$  is a level surface; the line  $s$  is the actual sea surface (from H. Stommel, *The Gulf Stream*, Cambridge Univ. Press (1966)).

is zero there. The essential assumptions are that nothing much happens deep in the ocean and that the reference velocity is zero at a standard depth (e.g., 1500 or 2000 meters). Recent research<sup>1</sup> has shown, however, that this so-called “level of no motion” is a generally poor assumption; the depth of the contours of zero velocity can vary greatly, and the contours can even intersect the surface and the bottom of the ocean many times on scales of interest.

Satellite altimetry becomes very important at this point. Instead of having to assume something about the velocity at depth or requiring current measurements at depth, the reference level for Eq. 4 can be chosen to be the surface of the ocean. No assumption is necessary about the surface velocity because it is related to the surface topography through the geostrophic and hydrostatic equations by

$$v_s = (g/f) \partial \eta/\partial x, \tag{5}$$

where  $v_s$  is the surface velocity and  $\eta$  is the vertical displacement (i.e., the topography) of the sea surface, which is measured directly by satellite altimetry. Thus, the deep

reference level of no motion has been changed to a surface “level of known motion” by the new measurement system. For many important applications (e.g., initializing ocean circulation models and descriptive studies of surface currents), the observed changes in sea-surface height (Eq. 5) are sufficient. Useful oceanographic information can therefore be obtained merely by measuring relative height changes along the satellite ground track.

## THE ALTIMETER MEASUREMENT SYSTEM

If the ocean were at rest, the height,  $H_s$ , of the sea surface relative to the ellipsoid of revolution would vary by about 50 meters because of bathymetry and the irregular distribution of mass beneath the sea. This irregular shape is called the geoid, i.e., the shape that would be assumed by a motionless uniform ocean on a rotating earth. The dynamic topography,  $\eta$ , is defined as the departure of the ocean surface from the geoid caused by the movement of water as shown in the definition sketch of Fig. 3. The figure shows that height,  $\eta$ , is simply related to (a) the orbit height,  $H_o$ , above an arbitrary reference surface (usually a standard oblate spheroid fit to the earth’s shape), (b) the height,  $H_a$ , of the satellite above the sea surface, and (c) the height,  $H_g$ , of the geoid above the reference surface by

$$\eta(x) = H_s - H_g(x), \quad (6a)$$

where

$$H_s = H_o(x) - H_a(x). \quad (6b)$$

Changes in  $\eta$  are given by

$$\partial\eta/\partial x = \partial H_s/\partial x - \partial H_g/\partial x. \quad (6c)$$

There are several serious complications hidden in these simple equations. The satellite altimeter directly measures the height of the satellite above the sea surface,  $H_a(x)$ . The orbit height,  $H_o(x)$ , is obtained by a worldwide ground-based tracking network and can come either in the form of a best-fit (over a day or two) ellipse or as an ephemeris (a table of position versus time). The geoid height,  $H_g(x)$ , comes from historical ground-based data or high-resolution (less than 100-kilometer) satellite gravity data and is available in the form of global models accurate at large wavelengths or, in certain regions, as more accurate local models. Thus, there is much additional information beyond that measured by the satellite that must be obtained to compute the sea-surface height from Eq. 6.

Another complication is accuracy. The largest values for  $\eta$  in the ocean are slightly greater than 1 meter, and the satellite orbits at a height of about 800 kilometers. The desired signal is obtained from Eq. 6 as a small difference between very large numbers and is therefore sensitive to errors. However, because most of the errors associated with the terms in Eq. 6 occur on different

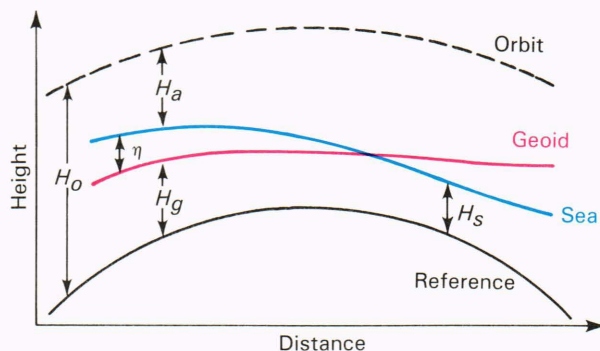


Figure 3—A definition sketch for the measurement system.

length and time scales, a useful estimate of  $\partial\eta/\partial x$  can be obtained by using various smoothing, filtering, or other statistical techniques. Errors associated with the orbit, for example, vary on length scales of the circumference of the earth and on a time scale of one satellite revolution. The mesoscale slope,  $\partial\eta/\partial x$ , however, varies on spatial scales of tens to hundreds of kilometers and on time scales of days to weeks. Therefore, any technique for subtracting long-wavelength errors (e.g., computing the departure from a straight linear or quadratic orbit approximation) from the computed  $\partial\eta/\partial x$  will remove most of the orbit error (along with any long-wavelength ocean phenomena like the general circulation) and leave only the mesoscale variation.

Another source of error is the geoid. Although errors in the geoid can occur on the same length scales as ocean phenomena of interest, both the geoid and its errors are independent of time. Now it is possible to derive time-dependent surface currents without the need for a geoid (or the introduction of geoid errors). To do so, the mean (i.e., time-independent) topography and the fluctuating (i.e., time-dependent) topography can be measured using different approaches. The total height can be written as the sum of mean and fluctuation quantities:

$$\eta(x,t) = \bar{\eta}(x) + \eta'(x,t) \quad (7a)$$

$$H_s(x,t) = \bar{H}_s(x) + H'_s(x,t), \quad (7b)$$

where an overbar indicates mean and a prime indicates fluctuations.

It therefore follows from Eq. 6a that the temporally varying component of topography is given by

$$\eta'(x,t) = H'_s(x,t) - \bar{H}_s(x). \quad (8)$$

The fluctuating part of the dynamic topography can be computed using only the satellite orbit and altimetry information (Eq. 6b). The mean sea surface,  $\bar{H}_s(x)$ , is computed by averaging Eq. 6b, using repeated measurements over the same point on the ocean surface. A geoid is therefore not required to study ocean variability. This is fortunate, since in many areas of the world an accurate geoid is not available. In places where the variability is much stronger than the mean (e.g., in the Gulf

Stream and its vicinity, where rings and meanders dominate the circulation), the restriction to the analysis of time-dependent phenomena may not present a serious problem. However, the price to be paid for this simplification is that knowledge of the mean circulation is lost because the mean dynamic topography has been combined with the geoid to form the mean sea surface, which, by Eq. 6a, is

$$\bar{H}_s(x) = \bar{\eta}(x) + H_g(x). \quad (9)$$

The separation of the problem into mean and fluctuating parts is the basis of the so-called crossover and colinear methods of studying ocean variability, an example of which is given below. (See also a discussion in the article by Cheney et al. elsewhere in this issue.)

Finally, there is a group of errors associated with the effects of the atmosphere on the propagation of the radar pulse. Dry air, water vapor, and ionospheric electrons cause the radar pulse to travel at a variable speed that is somewhat slower than the speed of light in a vacuum. With additional measurements of atmospheric pressure, water vapor content, and solar activity, these effects on radar pulse propagation can be modeled and approximate corrections can be obtained. Again, the propagation errors often have wavelengths longer than the oceanographic phenomena of interest and can therefore be removed, to some extent, by filtering. The most troublesome is the water-vapor correction, because the scales can match those of mesoscale ocean phenomena, in which case the accuracy of the height calculation is degraded.

In summary, the errors mentioned above not only are of different magnitudes (ranging from centimeters to meters), but they also vary on different scales of length and time from each other and from the ocean signal,  $\eta$ , the quantity to be estimated. Differences of scale allow procedures (e.g., curve fitting, smoothing, spectral filtering, separating time-dependent from time-independent components) to be used to separate signal from noise. The entire measurement system must include not only the satellite and its altimeter, but also an accurate global tracking network, measurements of the earth's geoid, and means to measure four atmospheric variables (pressure, temperature, water content, and solar flux). Satellite altimetry involves combining all the corrections and the statistics in such a way that it is possible to compute useful sea-surface topography. Although all the sources of error discussed above must be handled very carefully to get a useful signal, it is possible to do so successfully.

The data have been applied to studies of global mesoscale variability statistics, tides, fluctuations in the Antarctic Circumpolar Current, and equatorial Kelvin waves.<sup>2</sup> Several articles in this issue examined other applications. In the section below, additional uses of the sea-surface topography are illustrated.

## SEA-SURFACE TOPOGRAPHY

Once the sea-surface dynamic topography,  $\eta$ , has been computed as accurately as possible, a proper study of

the implications of topographic variations can begin. Many different physical processes contribute to the displacement of the sea surface, and a study of any particular process involves an attempt to isolate its effect. The sea-surface displacement computed from Eq. 6 can be written as

$$\eta = \bar{\eta} + \eta_{\text{bar}} + \eta_{\text{tide}} + \eta_{\text{meso}} + \eta_{\text{wave}} + \eta', \quad (10)$$

where  $\bar{\eta}$  is due to mean currents (i.e., currents that do not vary with time),  $\eta_{\text{bar}}$  is the inverse barometer response to atmospheric pressure (i.e., high atmospheric pressure pushes the water surface down),  $\eta_{\text{tide}}$  is due to tides,  $\eta_{\text{meso}}$  is due to mesoscale features (i.e., features that vary on time scales of days to weeks and on space scales of 50 to 200 kilometers),  $\eta_{\text{wave}}$  is due to surface waves, and  $\eta'$  is any other time-dependent contribution (e.g., due to wind).

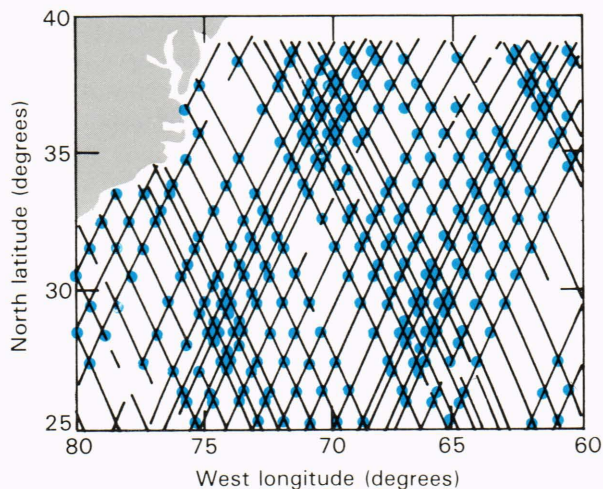
There are many ways to study the individual components of Eq. 10 using altimetry data. The mean displacement is simplest theoretically since all that is involved is averaging data taken over a long period (ideally, years); all the time-dependent components average out to within the noise of the measurements. The inverse barometer effect is slow and large scale. It is usually modeled as an inversely proportional relationship (a 1-millibar increase in atmospheric pressure lowers the ocean surface by 1 centimeter). To study tides by altimetry, a long time-series is obtained and the appropriate tidal frequencies are selected by spectral analysis. If the tidal signal is to be removed to isolate another effect (e.g., mesoscale features), then an analytical tidal model can be used. Long, slow planetary waves such as those associated with El Niño can be studied by following their characteristic propagation across the equatorial Pacific (see the article by Cheney et al. elsewhere in this issue).

To study the evolution of mesoscale features in a particular region, the displacement,  $\eta_{\text{meso}}$ , is computed from Eq. 10 where the barometric, tidal, wave, and mean contributions are subtracted out using various models. A typical pattern of GEOSAT ground tracks covering a region off the east coast of the United States during a 30-day period is shown in Fig. 4. For each of the tracks, displacements due to contributions from the Gulf Stream (1-meter variation) and its associated meanders and rings (10- to 50-centimeter variation) can be monitored. Another way to use the same data is to collect two measurements made at different times at the same location. These crossover data are obtained at points where the ascending and descending ground tracks cross each other (Fig. 4). If the surface displacement,  $\eta$ , is represented at the crossover point,  $x_o$ , at each of the two times,  $t_1$  and  $t_2$ , by a mean and a time-dependent part, then

$$\eta_1 = \eta(x_o, t_1) = \bar{\eta}(x_o) + \eta'(x_o, t_1) \quad (11a)$$

$$\eta_2 = \eta(x_o, t_2) = \bar{\eta}(x_o) + \eta'(x_o, t_2). \quad (11b)$$

By adding the two measurements, an estimate of the mean,



**Figure 4**—Satellite coverage of the northwest Atlantic in 30 days: ground tracks (lines); crossover points (colored circles).

$$\bar{\eta}(x_o) = 0.5 [(\eta_1 + \eta_2) + (\eta'_1 + \eta'_2)], \quad (12)$$

is obtained. To the extent that the time-dependent parts cancel, the estimate improves when more data are averaged. By Eq. 6, this result can be expressed in terms of the mean sea surface, which is computed with orbit and altimetry data (no geoid data):

$$\bar{H}_s(x_o) = 0.5 (H_{s1} + H_{s2}) + 0.5 (H'_{s1} + H'_{s2}). \quad (13)$$

An estimate obtained from Eq. 12, based on 30 days of GEOSAT data, is shown in Fig. 5a. A similar estimate based on 3.5 years of data from GEOS-3 and 90 days of data from Seasat<sup>3</sup> is shown in Fig. 5b. The latter shows a mean Gulf Stream off the coast of Cape Hatteras and strong fluctuations farther to the east. The southeast part of the region was judged to yield an estimate of the spatial scale and amplitude of the error in the calculation. The shorter GEOSAT estimate compares favorably with Seasat results in most of the region. The comparison is good enough to expect substantial improvements by using all available GEOSAT data rather than just 30 days.

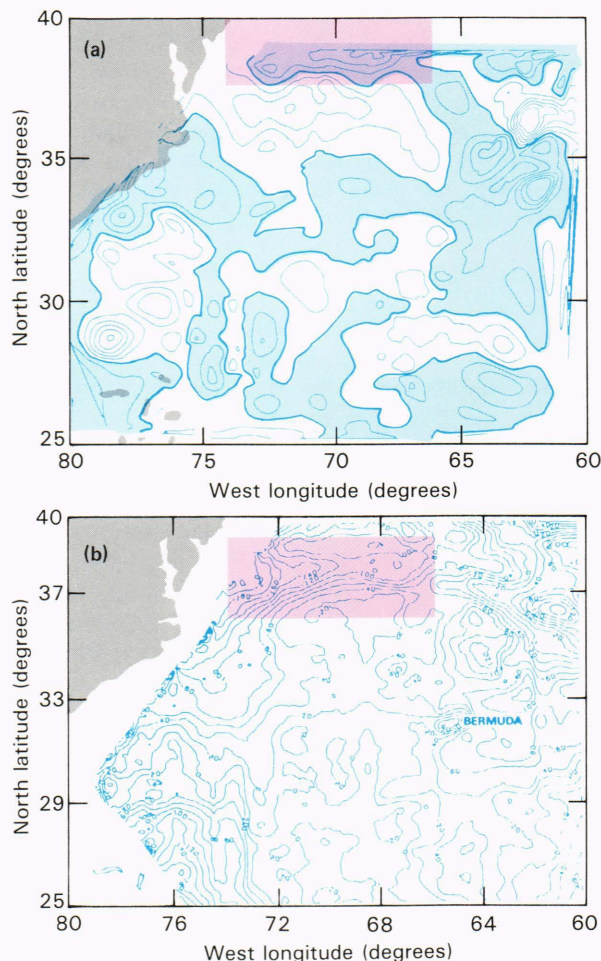
By subtracting the two measurements (Eq. 11), an estimate of the rate of change of sea-surface topography is obtained:

$$\partial\eta/\partial t = \partial\eta'/\partial t = (\eta_2 - \eta_1)/(t_2 - t_1). \quad (14a)$$

An important result is that the fluctuating part of the dynamic topography,  $\eta'$ , can be expressed in terms of the sea-surface height,  $H_s$ , which is computed without knowledge of the geoid by

$$\partial\eta'/\partial t = (H_{s1} - H_{s2})/(t_2 - t_1). \quad (14b)$$

An estimate of the magnitude of the time-dependent topography can be computed from Eq. 14 without divid-

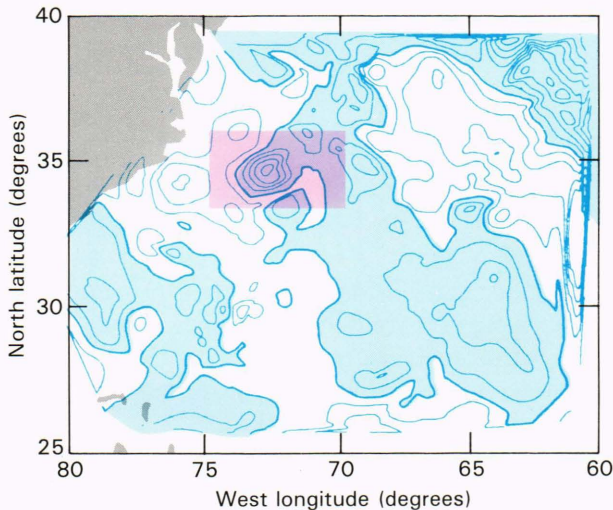


**Figure 5**—Mean sea-surface dynamic topography (a) from 30 days of GEOSAT data. Note the mean Gulf Stream within the highlighted box (contour intervals are 20 centimeters apart; negative values are shaded), and (b) from combining 3.5 years of GEOS-3 and 90 days of Seasat data.

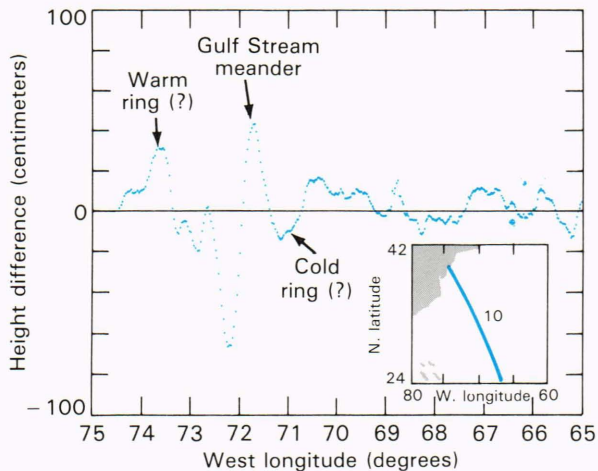
ing by the time differences. This estimate of height difference, based on the same 30 days of GEOSAT data, is shown in Fig. 6. Independent estimates of sea-surface temperature also suggest the presence of a Gulf Stream ring in the neighborhood of the ring-like feature seen in Fig. 6. Estimates of the mean and time-dependent parts of the sea-surface displacement allow descriptive studies of ocean currents to be made through Eq. 5.

In November 1986, the orbit of GEOSAT was changed so that the ground tracks repeated themselves (thus began the Exact Repeat Mission) every 17 days. The simple differencing technique just described for crossover points could now be applied at every point along a ground track. The height differences between two successive passes (one on January 1, 1987, the other 17 days earlier) along that ground track are shown in Fig. 7. This dramatic result shows very clearly the signature of a Gulf Stream meander and possible warm and cold rings on either side.

Another important use of sea-surface topography is in imposing constraints on numerical ocean-circulation



**Figure 6**—Time-dependent sea-surface dynamic topography from 30 days of GEOSAT data. Note the highlighted ring-like feature near 33°N, 73°W. (The contour intervals are 20 centimeters apart; negative values are shaded.)



**Figure 7**—Height difference along an Exact Repeat Mission ground track (shown in insert). The position of a Gulf Stream meander is evident, and possible warm and cold rings on either side of the meander are indicated (E. B. Dobson, unpublished).

models<sup>4</sup> by providing observed surface boundary conditions to the model. In this way, dynamical studies and ocean forecasting can be improved by altimetry. A more extensive discussion on numerical ocean modeling and its relation to satellite altimetry is given in the article by Robinson and Walstad elsewhere in this issue. A collaboration between the APL and Harvard groups is currently under way to optimize the combined use of satellite altimetry and numerical ocean-circulation models to improve ocean forecasting. The collaboration involves

the use of near real-time GEOSAT altimetry in a numerical ocean-circulation model for one- to two-week forecasting of ocean currents, rings, eddies, etc., in the Gulf Stream meander region of the northwest Atlantic Ocean. Subsequent work will seek to improve methods of assimilating altimetry data into the numerical model and to extend the analysis to other regions of the global ocean.

## FUTURE APPLICATIONS

The simple geostrophic relationship expressed in Eq. 4 is the starting point for understanding a wide variety of phenomena in the world's oceans. Because of the close connection between the satellite-measured sea-surface topography and geostrophic surface currents, altimetry can make major contributions to the understanding of those phenomena. Also, because of the unique global coverage, other phenomena (e.g., global tides) can be studied by altimetry. By using the altimeter's wind- and wave-measurement capability, wind and wave phenomena can be examined. When altimetry is combined with in-situ measurements, the subsurface implications of the sea-surface topographic variations can be explored. A new and potentially powerful use of satellite altimetry is in conjunction with numerical ocean-circulation models to improve dynamical studies and ocean forecasting.

As discussed above, a careful treatment of altimetry data allows the separation of oceanographic space and time scales from those of the error sources. A precision of about 3 centimeters in the analyzed field of dynamic topography has been achieved (see the article by Cheney et al. in this issue) with GEOSAT, which is the most accurate altimeter flown to date. Future altimeter missions will undoubtedly include advances in instrumentation and will be even more accurate. In addition to altimeters scheduled for U.S. missions, France, Japan, the Soviet Union, and the European Space Agency are all flying or are planning to fly altimeters in space. No doubt we will learn more about the ocean with these new eyes looking down.

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