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LIMITATIONS OF SPECTRAL MEASURES AND OBSERVATIONS OF THE GROUP STRUCTURE OF SURFACE WAVES

Although the group structure of a wave field has a minor effect on its frequency and wavenumber spectra, it is still important when one wants to analyze the corresponding nonlinear response of ships, structures, and surface flux processes. Understanding group structure may also help us understand wave generation, since the modulation of wavelets on the dominant sea may be important in energy transfer to the main waves. It also affects fluxes in other ways: because nonlinear group dynamics depends on higher order interactions as measured in wave steepness, external influences of the same small order can control group formation and propagation. Such nonlinear effects include current variability in space and time, underlying swell, gustiness, and the time history of wind and waves. Observations of group structure are only meaningful in the context of knowledge of such extraneous influences. It may still be possible to understand the group structure of wavelets on longer waves; for the group structure of the dominant wave field, one may have to be content with statistical data.

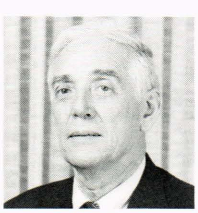
INTRODUCTION

Random wave fields can have very different modulations but nearly identical spectra, as can be seen in Fig. 1. Waveform a is of nearly constant amplitude with suitably inserted phase interruptions, while waveform b is constructed from (a) by multiplying (a) by 2 and replacing the field by zero three-quarters of the time. The energies are alike, and the zero parts are inserted periodically to form a series of wave groups. When encountering these wave fields, the linear response of a floating object to waves will have similar spectra for both wave fields. A ship with a submerged bow or stern, however, will not respond linearly; the buoyancy remains constant with further deflection after submergence. At the same time, the mass to be accelerated increases by the mass of water on deck and the virtual mass when fully submerged. The response then is further slowed. Thus, the response of a ship to waves depends strongly on the likelihood of large wave encounters. A series of large waves will cause a much more dramatic response than a single wave, since recovery will be incomplete before the next wave hits. Wave group structure is important under such circumstances. But most of the time the sea-surface deflection is close to Gaussian in probability, and the wave-amplitude probability density is a Rayleigh distribution. Only for very large deflections, where empirical data necessarily are sparse, will departures from the Rayleigh distribution be crucially important. This will matter in ship failures, fatigue calculations, and the response and survival of marine structures. One needs to know the rare-event part of the probability density to be able to predict the probability of accidents precisely; because one will rarely know enough, one will have to rely on a conservative design of structures and conservative estimates of operational performance.

Sea-surface fluxes also depend on wave breaking and airflow separation, both of which are consequences of large-amplitude events. Clearly, we would like to know more about wave-group structure for scientific purposes of understanding flux processes and the more detailed mechanics of the transfer of wave energy and momentum to the mean current field. The difficulty of the problem becomes clearer after a few reminders about the origins of group structure in a wind-wave field.

ORIGINS OF WAVE GROUPS

Benjamin and Feir[1] discovered the tendency of a constant-amplitude wave train to form groups, and Benjamin[2] did the first analysis of sideband instability, a growing modulation of amplitude and phase propagation at group velocity. Zakharov and Shabat[3] included the linear effect of amplitude variation on wave frequency and found that the complex modulation function, to third order in wave steepness, satisfied the nonlinear cubic Schrödinger equation. They found that the tendency was to form group solitons and that these solitons followed quantum rules. Yuen and Lake[4] confirmed this. Chereskin and Mollo-Christensen[5] showed that the effects of wave damping on an isolated group could be modeled by the inclusion of a linear damping term in the nonlinear cubic Schrödinger equation.



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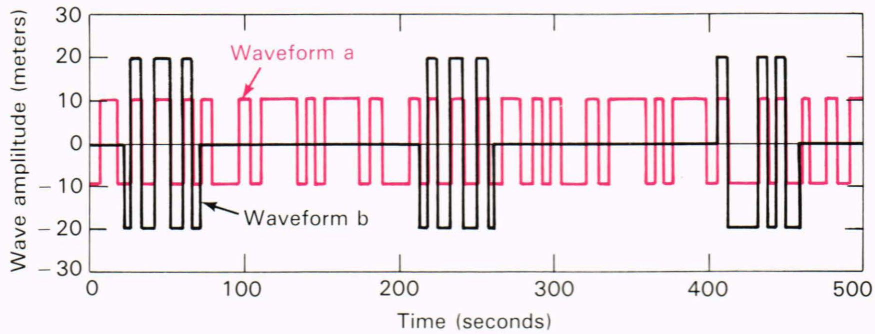

Figure 1—Two different wave fields but with identical (time-averaged) energy and spectra. A ship response to waveform a will be much different than to waveform b.

In his analysis of joint air and water shearflow instability, Blennerhassett[6] found that it followed the same equation with an added linear source term. Melville's experiments[7] showed clear evidence of subharmonic instability in a wind-wave field, confirming the numerical calculations of Longuet-Higgins and Cokelet.[8,9] Bliven et al.[10] showed that for a wavetrain generated by a wavemaker operating at constant amplitude, the Benjamin-Feir sideband instability was inhibited by wind blowing over the waves. This may not be the case for isolated wave groups.

Because wave groups are results of weak (on the order of the cube of the slope) wave-wave interactions that act persistently over many wavelengths and wave periods, other weak effects can affect and possibly dominate the process of group formation.

Flow separation over waves in a group is necessarily a fluctuating phenomenon since waves enter the group and grow in steepness as they advance. At some stage of the wave's progress through the group, flow separation may occur and flow reattachment will follow further on as the wave decreases in amplitude while it decays toward its exit from the group. Each flow reattachment implies the shedding of a vortex disturbance into the air boundary layer. The frequency of shedding will be half the frequency of the underlying waves. The vortices, propagating in the wind wake of the group, will generate waves by a mechanism similar to that proposed by Phillips[11] and investigated by Giovanangeli and Memoponteil[12] in a laboratory experiment. This suggests that wave groups can engender other wave groups of half the frequency. The importance of this mechanism may be negligible; it is probably just one of many candidate mechanisms for group formation in the developing wind-wave field.

Gusts can also generate wave groups, and the locally higher drag due to wave generation can cause disturbances in the air boundary layer; such disturbances will then tend to propagate at a speed higher than the group speed and disturb the water surface further on in a different manner.

Most of the mechanisms that lead to group formation are weak compared to the dynamics that causes wave propagation. Since they have to act over distances of many wavelengths and times of many wave periods to influence the group structure of the wave field, the mechanism that dominates in a given situation will depend on circumstances. Therefore, it is unlikely that in any one situation a current shear or a current divergence will dominate. In an offshore wind situation, the turbulence structure of the air boundary layer may be the dominant influence on surface-wave modulations. Bottom topography, usually a weak effect, may be the dominant persistent weak effect, and the amplitude structure of the wave field and the modulation may be dominated by topographical effects. Inertial oscillations in the surface layer of the ocean engendered by the sudden onset of wind or by abrupt wind shifts can also provide sufficiently organized surface-current shears and oscillations to influence group structure, although, to my knowledge, there have been no observations of such effects.

BIFURCATIONS AND ASYMPTOTIC GROUP STRUCTURE

Because of the many parameters that enter into wave group dynamics, one cannot assert that there exists any single "equilibrium" group statistical structure for all possible combinations of time history of the wind field, surface current, and influence of preexisting wind fields. There seem to be too many possibilities for bifurcations and shifts in wave modulations and dominant wave frequency for one to expect that all possible paths that can be followed in sea-state development would end up at a single asymptotic state as measured in terms of wave-modulational structure. But we may discover that there are classes of external conditions that lead to classes of modulational structure. This is suggested by local folklore among sailors and fishermen. Although most of the observed differences are due to fetch and possibly also to air-sea stability, there seem to be opinions about "lumpiness" of the waves that may warrant investigation.

In order to study group structure, one also must observe the external parameters that can affect it. This makes it exceedingly difficult to study wave groups in the environment because one would also have to observe a large number of external parameters, such as weak persistent currents, wind fluctuations, and so on. But one may hope that in certain "standard" situations, such as near the eye of a hurricane moving at a "normal" speed, the local sea state and group structure will be found to have a kind of universal structure, similar for all comparable hurricanes. Similarly, for certain locally typical disturbances, such as the low pressure areas that originate in the Labrador Sea and move east to the south of Iceland, there may be typical features of the wave field that are worth noting. But we also have to be skeptical about the interpretation of observations that seem to suggest universal structure in the wave-field modulations.

Of course, the central limit theorem applies to many features of random wave fields, but for the joint probability of occurrence of wave groups of certain characteristics, the requirements of independence may not be satisfied well enough to make the central limit theorem apply. For less esoteric measures, such as wave-amplitude probability, the resulting wave field may have a Rayleigh distribution in wave height. When delving further into details about probability for the observation of successions of waves of related amplitudes, we may find departures from independence. If we should isolate "big" wave groups, we may find them to have distinct characteristics of their own.

In laboratory experiments, one needs to show a similar vigilance about the documentation of parameters if one wants to do research on group structure in the wave field. The presence of sidewall boundary layers in a wind-wave tunnel, the steadiness of the wind, and the dependence of secondary flow structures on boundary-layer transition all may affect wave modulations. Therefore, one needs to be quite careful if one wants to study "natural" group structure. However, it may be possible to study the dynamics of isolated groups and simple group interactions without excessive trouble or need for care.

It may well be possible to study the group structure of small wavelets on longer waves, since the dominant external disturbances that affect the wavelets will be the long waves and the wind field over the longer waves. These effects may dominate external and less systematically organized effects.

CONCLUSIONS

There is still systematic work to be done in the study of wave group dynamics, but, because of the sensitivity to rather small external influences, experiments need to be chosen carefully and done well. Observational programs also have to be carried out with due regard for the possibility that there may not exist a unique asymptotic statistical state for the wave-modulation field, but rather that there may be several. One will still be able to find practical estimates of the likelihood of the occurrence of groups with certain specified properties and gather other information useful in ship design and for actuarial purposes.

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