DAVID E. IRVINE

# WAVES ACROSS THE OCEAN

Measurements of swell—long ocean waves—made from a satellite have yielded an exceptionally clear field verification of a very simple theory. The clarity is both gratifying in itself and indicative of the fidelity of ocean remote sensors.

## RIPPLES IN A POND

Go to a pond and throw in a rock. For about the first second, the surface within a foot or so of the impact point will look quite ragged. This is certainly no surprise, but what happens next should be. Despite the initially irregular motion, a ring of very regular waves radiates outward almost immediately. Both the remnants of impact and the subsequent ring of waves are evident in Fig. 1. The curious nature of this regularity is generally lost on us because it is so familiar.

You will also notice that the first few waves are a bit longer than those that follow them, and the succeeding ones are still shorter. Thus, not only does the impact create a set of regular, concentric waves, but these waves appear in order of decreasing wavelength.

The explanation for such simple, regular waves as these was basically completed by the middle of the nineteenth century. The elements of that explanation are a pair of fundamental physical laws, an idealized geometry, and a set of approximations. The physical laws are conservation of mass and momentum. The geometry simply envisions an infinitely deep fluid far from any boundary. The approximations can be classified roughly as material or dynamic. One set indicates which of the material properties of water are important and which are not. The other set indicates the parameter range within which the predictions are expected to hold; for example, the usual explanation is applicable only to waves of small slope.

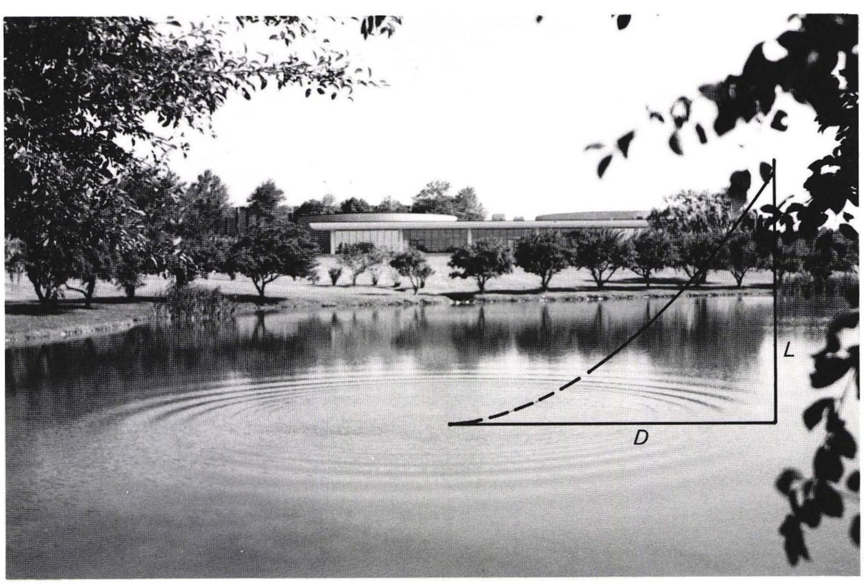
Building on these elements, the mathematical theory predicts that the speed $S$ at which a wave moves out from the impact point can be given in terms of the wavelength $L$ and a proportionality constant $A$:

$$S = \tfrac{1}{2} A\sqrt{L} .$$

This accounts for the observation that the first waves moving out from the disturbance are longer than succeeding ones—they are faster. Therefore, the initially complicated surface, composed of many wavelengths, eventually sorts itself out, with the longest and therefore fastest waves out in front. This is called dispersion and is one of the most distinctive features of water waves.

An additional prediction of the theory relates the speed $S$ to another characteristic of the wave—its period $P$:

$$S = \tfrac{1}{2} A^2 P .$$



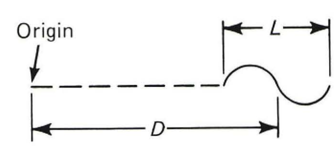




**Figure 1**—Ripples in a pond in front of APL. The approximate variation of wavelength ($L$) with distance ($D$) from the origin is superimposed.

Although the period is not quite as obvious on casual observation as is the wavelength, it is often easier to measure. All you really need is a watch.

From these predictions, one can construct two more that are easily verified. A given "piece" of the wavefield with wavelength $L$ and period $P$ moves at speed $S$. Therefore, in a time $T$ it will travel a distance $D$ equal to $ST$. One could then stand a distance $D$ from the point of origin of the waves, measure the period several times, and compare it to the prediction, $P = (2DA^{-2})T^{-1}$. An alternative would be to wait a time $T$ after the waves are generated, photograph them, and compare the variation in wavelength with the prediction

$$L = (\tfrac{1}{2}\ AT)^{-2}D^2 \ .$$

The latter prediction is verified in Fig. 1. Notice that from the variation of $L$ one can also infer the origin.

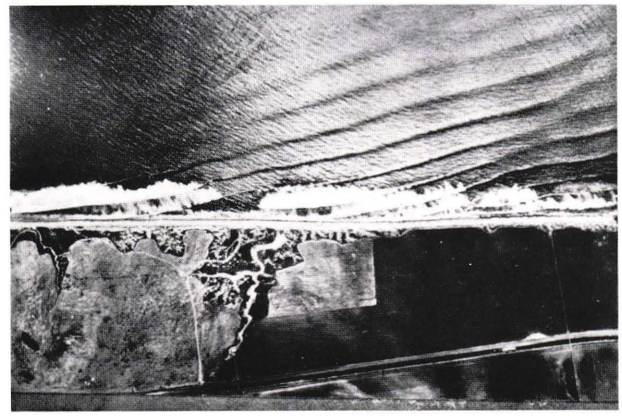


**Figure 2**—Swell in the ocean. This aerial photograph of waves approaching a California shore was taken from the first of the Scripps papers (Munk and Snodgrass) on swell. The coast trends northwest to southeast.

## WAVES ACROSS THE OCEAN

If now you were to go to the shore and watch the waves for several weeks, you would probably notice that only occasionally would they be sufficiently regular to remind you of the ripples in a pond. On the calmer days, you might see a basically regular train of waves coming toward shore. The waves on windy days, however, would be ragged, irregular, and steep, with frequent breakers. With some effort, you would also discover that the choppier waves would be generally much shorter (in wavelength, not height) than the smoother, more regular ones.

The two classes of waves are called wind sea and swell. As you might guess, the term wind sea applies to the rougher, shorter variety. They usually occur when the wind is strong. Progress toward understanding these waves has been slow and arduous. Their connection to ripples on a pond is remote and not very helpful.

On the other hand, you could almost imagine the swell approaching the shore in Fig. 2 to be the ripples from the impact of some tremendous rock thrown into the sea.

Suppose for a moment you were willing to entertain this notion literally and wanted to find out where and when the rock had hit the water. If you tried to duplicate the pond experiment, you would run into an immediate puzzle: the wavelength *decreases* as it approaches shore. The problem is that the formulas were derived assuming the water to be infinitely deep. This clearly will not work too near shore.

Fortunately, an extended version of that theory predicts that, while the wavelength is shortened and the direction altered as the waves approach shore, the period is not. Therefore, you can use the formula from the first section to deduce the *where* and *when* of the imaginary rock from the period of swell as it reaches shore.

So imagine yourself standing on the beach in Fig. 2. You would find the period of the waves to be 14 seconds. Since you want both the where *and* when, you would need another measurement. If you came to the same beach one day later, you would find the period of the waves had decreased to 13 seconds. With this, you can now readily estimate that the rock hit the water two weeks earlier and 7600 miles away!

The swell in Fig. 2 is coming from the southwest. Although the wave direction changes somewhat as it approaches shore, it doesn't change much; you would not do too badly if you assumed that there had been no change of direction at all. Now, get out a globe. Seventy-six hundred miles southwest of a beach in California puts you somewhere just east of Australia.

Finally, a check of the weather records from Sydney would show that there had been a fairly intense storm about two weeks earlier. This storm was the "rock."

The picture that emerges is that of wind sea generated in storms and then radiating away as swell. The simple experiment in this section not only establishes the plausibility of comparing the swell to ripples in a pond, but it clearly points the way to a major field experiment.

## THE SCRIPPS EXPERIMENTS

A group of oceanographers from the Scripps Institution of Oceanography at LaJolla, Calif., performed a set of three experiments to study ocean swell. The measurement device common to all three was a pressure transducer mounted on the sea floor. In moderately shallow water, the bottom pressure is directly proportional to the amplitude of the swell passing overhead; the Scripps instruments could detect amplitudes of 1 millimeter.

The first experiment, conducted in 1956, was a relatively modest attempt to measure the period of the swell at several stations near the California coast. That experiment was much like the imaginary one described above; the period of swell approaching shore was recorded for several days at a time. The data showed not only that the period decreased as predicted, but also that the inferred storm locations were plausible.

In the second experiment, three pressure transducers were arranged in a triangular array 900 feet on a side. With this innovation, the direction of the incoming swell as well as its period and amplitude could be estimated. The observed periods again decreased as predicted and the inferred storm locations, now more accurate, were in reasonable agreement with the available meteorological records, such as they were.

The final experiment was an ambitious attempt to track swell all the way across the Pacific. The amplitude and period were recorded for more than two months in 1964 at six stations deployed along a great circle route from New Zealand to Alaska. Measurements of swell amplitude were taken twice daily. Figure 3 shows data taken during roughly a 30-day period in Honolulu. Again, the predicted decrease in period is evident. The inferred storm locations were again consistent with the existing meteorological records.

By comparing the amplitude of the swell from a given storm at all six stations it was possible to measure attenuation. Waves with a period of 14 seconds and longer experienced only the attenuation that would result from the spreading of a fixed amount of energy over a larger and larger area. The startling conclusion was that swell could propagate halfway around the earth with essentially no loss of energy.

## THE APL EXPERIMENT

The Scripps experiments strengthened beyond anyone's expectation the analogy of ripples in a pond to ocean swell. The picture of storm-generated waves evolving into swell and propagating thousands of miles gains both intelligibility and credibility from this analogy. The technology of the 1960s allowed us to verify part of that picture, and the technology of the 1980s is about to allow us to verify still more.

In June 1978, NASA launched Seasat, a satellite dedicated to probing the ocean. Three months later, Seasat died of a massive power failure. Despite the frustration, oceanographers have salvaged a number of case studies. The principal casualty has been statistical reliability; it is as if the Scripps instruments had failed after measuring one storm. The technology of the 1970s and 1980s has some drawbacks. The bright side is that the research community has focused considerable effort on these case studies.

The eyes of Seasat were a set of microwave radars. The technique basic to several of those instruments is sketched in Fig. 4. When a beam of microwave energy is broadcast toward the ocean surface, a small fraction of that energy is scattered back toward the satellite. The key to the technique rests with the fact that the intensity of the backscattered energy is proportional to the intensity of ocean waves having very short wavelength (on the order of centimeters). The principal mechanism is generally believed to be Bragg scattering.

If you have seen the ocean on a windy day, you may have noticed that the smallest scale waves were very rough; if the wind were to increase, they would become still rougher. One of the instruments on Seasat, called the scatterometer, exploited this effect by simply interpreting the intensity of the backscattered microwave power as a measure of wind speed. Furthermore, since the roughest of the short waves are those moving in the direction of the wind, further interpretation made it possible to estimate wind direction.

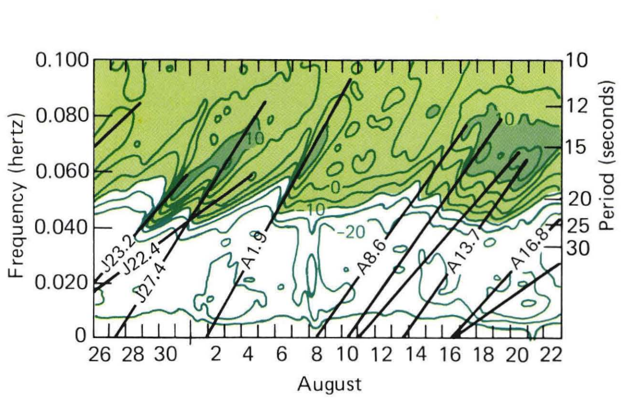



**Figure 3**—Contours of wave energy density constructed from frequency spectra for roughly a 30-day period in Honolulu. The ridges represent the arrival of swell from distant storms and are labeled according to the storm time. For instance, J27.4 means July 27, 9.6 hours GMT. The ticks on the date axis denote midnight GMT. This figure was adapted from the last of the Scripps papers (Snodgrass et al.).

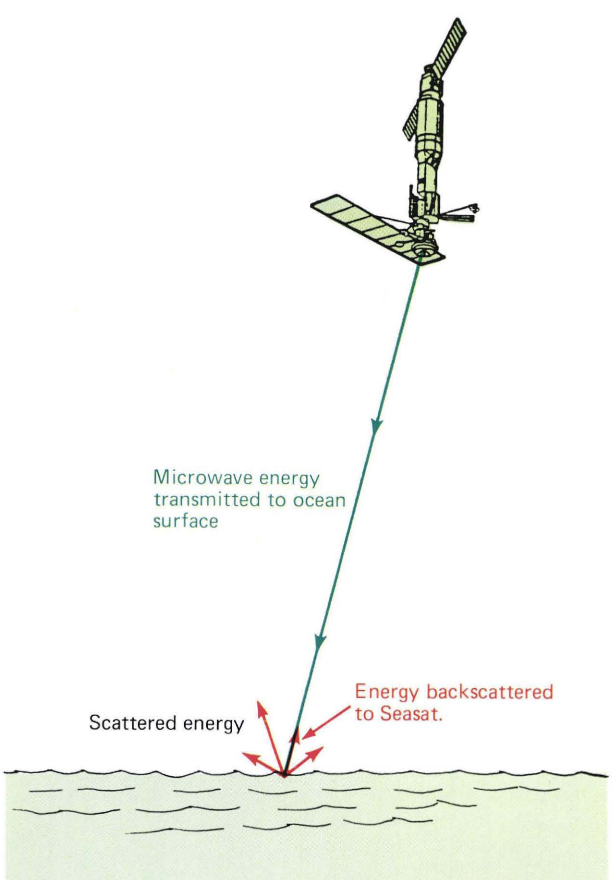



**Figure 4**—The scattering of microwave energy from the sea surface. As indicated, only a small fraction of the energy is backscattered to Seasat.

## LINEAR SURFACE WAVE THEORY

The intent of this section is to outline the derivation of the relations $S = \frac{1}{2}A\sqrt{L}$ and $S = \frac{1}{2}A^2P$, fundamental to the interpretation of both the Scripps and APL experiments. Directed toward readers who have some familiarity with fluid mechanics, the derivation is condensed but essentially complete.

The idealized geometry of this theory is shown in the figure. The ocean occupies the infinite half space bounded above by the surface $z = Z(\mathbf{r}, t)$; this relation defines the surface whose fluctuations constitute waves. The water velocity at any point in the ocean is given by $\mathbf{u} = \mathbf{U}(\mathbf{r}, z, t)$.

The first assumption is that all the motions associated with the wave are small. Quantitatively, this works out to mean that the surface slope, $\nabla Z$, is always much less than one.

The next assumption is that the water is incompressible and inviscid. To neglect viscosity implies that the internal stress field is an isotropic compression whose local intensity is just the pressure, $p = P(\mathbf{r}, z, t)$. Since compressibility is also neglected, the water's density, $\rho$, remains constant, and so compression leads to undulations in the surface, i.e., to waves.

The final assumption is that water waves are irrotational; $\nabla \times \mathbf{U} = 0$. This can be defended on theoretical grounds but ultimately is justified simply by measuring the velocity field of a real wave—it is irrotational.

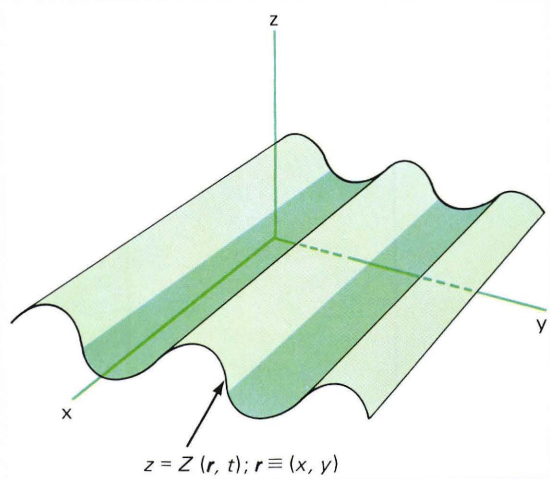
The fundamental physical laws are conservation of mass and momentum:

$$\nabla \cdot \mathbf{U} = 0 \ ,$$

$$\frac{\partial}{\partial t} (\rho \mathbf{U}) = -\nabla P + \rho \mathbf{G} \ .$$

The earth's gravitational field is represented by the vector $\mathbf{G} = (0, 0, -g)$, where $g = 9.81$ meters per second squared.

This mathematical picture is essentially completed with the addition of two surface-boundary conditions and the requirement that the motion far below the surface be small. The first surface condition is that any




The idealized geometry of wave theory.

particle once on the surface remains on the surface. For the small amplitude motions imposed by our initial assumption, this becomes

$$\frac{\partial Z}{\partial t} = W \quad \text{on } z = 0 \ ,$$

where $W$ is the vertical component of $\mathbf{U}$.

The assumption of small amplitude has now allowed two key simplifications. Nonlinear terms have been neglected, and the boundary condition has been evaluated at $z = 0$ rather than at the free surface $z = Z$.

The remaining condition essentially uncouples the atmosphere from the ocean and yields a model of swell rather than wind sea, that is, the surface pressure is held constant.

The program is now straightforward and therefore further abbreviated. The irrotationality of the wave field implies the existence of a velocity potential that allows the problem to be recast in terms of potential theory; the problem is then easily solved. Finally, the general solution for wave height can be built up from elementary solutions by Fourier composition:

Short waves are also sensitive to swell, a feature that was exploited by another Seasat instrument, the synthetic aperture radar (SAR). There is still a lively dispute over exactly how SAR detects ocean waves, but there is little dispute that it can.

Researchers at APL have intensively studied a comparatively small data set collected with both SAR and the scatterometer over the North Atlantic. A SAR image 60 miles wide and 600 miles long was taken off the North Carolina coast. In addition, the scatterometer mapped the surface wind fields of a tropical storm several thousand miles away that had occurred nearly a week earlier. The data taken from these two instruments constitute the foundation for our study. Figure 5 shows the locations of the SAR image and the storm against the backdrop of the North Atlantic.

The initial step was to extract measurements of the swell from the SAR data. Those data, essentially a map of backscatter intensity, showed a distinct system of intensity variation throughout most of the area examined. Since the backscatter intensity is a direct function of swell amplitude, we expected the "waves" on the image to indicate the wavelength and direction of the swell.

Figure 6 shows that the inferred variation of wavelength along the pass (colored line) is plausible. Just

$$Z = \int a(\mathbf{k})\, e^{i\chi(\mathbf{k})}\, d\mathbf{k} \; ,$$

where the phase is represented by $\chi = \mathbf{k} \cdot \mathbf{r} - \sigma t$, the frequency by $\sigma$, and the wavenumber by $\mathbf{k}$. These are related to the period and wavelength in turn by $P = 2\pi/\sigma$ and $L = 2\pi/k$; $k$ is the magnitude of $\mathbf{k}$.

The most significant finding in this analysis, however, is not the final expression for $Z$, but rather a relationship obtained along the way:

$$\sigma = \sqrt{gk} \quad ,$$

which is the dispersion relation. It is at the heart of wave physics and will be discussed again below after its significance is clearly established.

Classically, the expression for $S$ is obtained from the integral expression for $Z$ by a method known as stationary phase. Essentially, the method capitalizes on the fact that the integral of the product

$$a(\mathbf{k}) e^{i\chi(\mathbf{k})}$$

is very nearly zero when $\chi(\mathbf{k})$ is a much more rapidly varying function of $\mathbf{k}$ than $a(\mathbf{k})$. The dominant contribution to the integral will come from those isolated regions where $\chi$ is *not* a rapidly varying function of $\mathbf{k}$—the stationary phase points.

Both in a pond and on the ocean, regular wave patterns only begin to emerge once the waves leave the vicinity of their generation. This translates precisely into the statement that those patterns occur when $\mathbf{r}$ and $t$ are sufficiently large to guarantee that $\chi(\mathbf{k})$ is a rapidly varying function of $\mathbf{k}$ and hence that the stationary phase method is applicable.

Consequently, the stationary phase condition

$$\nabla_{\mathbf{k}} \chi = \nabla_{\mathbf{k}} [\mathbf{k} \cdot \mathbf{r} - \sigma(k) t] = 0$$

is satisfied along the path

$$\mathbf{r} - \mathbf{S} t = 0 \; ,$$

where the magnitude of $S$ is given by

$$S = \frac{d\sigma}{dk} = \frac{1}{2}\sqrt{\frac{g}{k}} = \frac{1}{2} A \sqrt{L} \; ,$$

or, in terms of frequency and period,

$$S = \frac{1}{2}\frac{g}{\sigma} = \frac{1}{2} A^2 P \; .$$

This procedure not only establishes the expressions for $S$, it also yields the vector equivalent to $D = ST$. Since $S$ depends upon $k$ (or $\sigma$), different wavelengths travel at different speeds; in short, they disperse.

While it is difficult to fully "explain" the origin of the dispersion relation without simply reproducing its mathematical derivation, some intuition can be gained by considering a very simple class of wave motion, that is, small-amplitude waves resulting from a balance between momentum variations and a restoring force that depends upon some sort of spatial gradient. For most such waves, the trajectory of a specified particle is a simple closed path with a characteristic length, say $a$. For periodic motion of frequency $\sigma$, the speed is proportional to $\sigma a$, and the momentum, to $\rho\sigma a$; periodic fluctuations of the momentum are thus proportional to $\rho\sigma^2 a$. The restoring force, depending upon a spatial gradient, is some function of $k$—call it $F(k)$. Consequently, to satisfy Newton's law, the momentum fluctuations must be of the same order of magnitude as $F(k)$, i.e.,

$$\rho\sigma^2 a \sim F(k) .$$

The restoring force for surface gravity waves is directly related to the surface slope and is therefore proportional to $k$. Specifically, $F = |\nabla p| = \rho g a k$. Hence, $\sigma^2 \sim gk$, and therefore $d\sigma/dk$ depends upon $k$. Thus, gravity waves are dispersive. Compare this to sound waves, for which the restoring force is proportional to $k^2$. In this case, $d\sigma/dk$ is *independent* of $k$. Therefore, sound waves are nondispersive. The dispersiveness of surface gravity waves is often taken as their most significant feature.

as expected, the wavelength slowly increases as one moves away from the generating storm and toward shore. The inferred swell directions, though generally reasonable, were both noisy and slightly biased.

We next produced maps of the surface winds in the tropical storm based on scatterometer data. A simplified version of those maps is included in Fig. 5. The complete set also included wind direction.

The last step involved the assistance of Oceanweather, Inc., a private company whose business is wave prediction. Over the past 20 years, practical concern for waves coupled with scientific interest has led to a number of numerical models of ocean wave generation and propagation. Given estimates of surface winds, these models first compute the wind sea as it is generated and then follow the waves as they are transformed into swell.

An Oceanweather model put all this together and predicted the swell field radiating from the tropical storm, whose winds were estimated from the scatterometer. Those predictions are shown along with the measurements from SAR in Fig. 6. Given the complexity of both the computer model and the satellite-based measurements, this is a gratifying substantiation of the picture we obtained from the Scripps experiments.

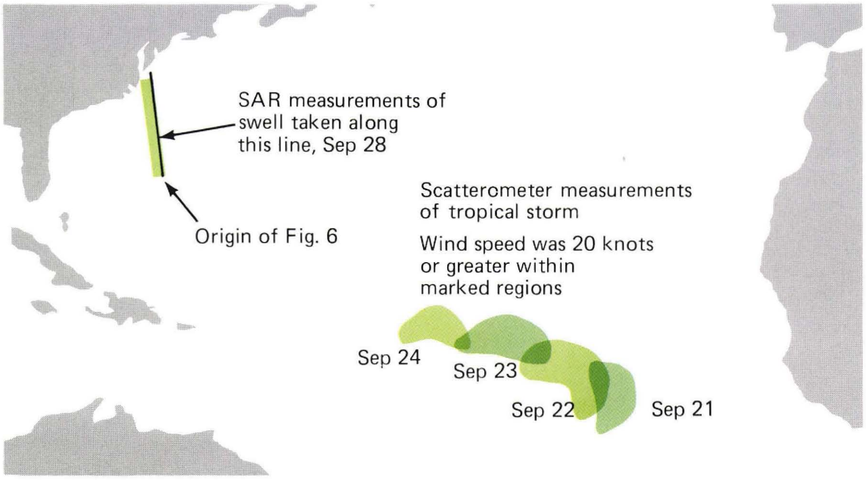



**Figure 5**—The location of the SAR measurements and the tropical storm mapped by the scatterometer.

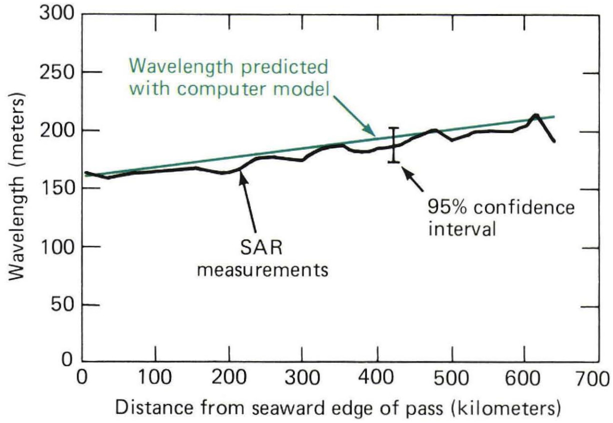



**Figure 6**—SAR measurements of wavelength from the seaward end of the pass to the southern edge of the Gulf Stream. The brackets indicate a 95 percent confidence level. The colored line is based on a computer prediction.

Finally, the analogy of swell in the ocean to ripples in a pond can be confirmed by treating both the SAR measurement and the computer predictions of Fig. 6 just like the wavelength measurement superimposed on Fig. 1, and inferring from them the "point of origin." The only difference is that the pond measurements were taken along a line known to pass through that point and close enough to neglect the earth's curvature; the ocean measurements were not. A suitably generalized version of

$$L = (\tfrac{1}{2}AT)^{-2}D^2$$

was fitted first to the SAR data and then to the Oceanweather predictions. The proximity of the two inferred origins is a striking indication of the agreement in Fig. 6; they were less than 14 kilometers apart and occurred within 8 minutes of one another.

## THE FUTURE

Although the confirmation of theory embodied in Fig. 6 has been treated in this paper as an end, it is really only a beginning. Seen instead as a confirmation of technique, Fig. 6 supports our confidence in the combination of microwave remote sensing and large-scale numerical modeling. We have already begun to apply that technique in situations, such as hurricanes, where older techniques are utterly inadequate. The remaining articles in this issue will describe similar beginnings.

## BIBLIOGRAPHY

Extensive discussions of surface wave physics abound. Four that were consulted are

B. Kinsman, *Wind Waves: Their Generation and Propagation on the Ocean Surface,* Prentice-Hall, Englewood Cliffs, N.J. (1965).
P. LeBlond and L. Mysak, *Waves in the Ocean,* Elsevier, New York (1978).
O. Phillips, *The Dynamics of the Upper Ocean* (2nd edition), Cambridge University Press, New York (1977).
J. Stoker, *Water Waves,* Interscience, New York (1957).

The Scripps experiments are documented in a set of three papers:

W. Munk and F. Snodgrass, "Measurements of Southern Swell at Guadalupe Island," *Deep-Sea Res.* **4**, 272-286 (1957).
W. Munk, G. Miller, F. Snodgrass, and N. Barber, "Directional Recording of Swell from Distant Storms," *Phil. Trans. A,* **255**, 505-584 (1962).
F. Snodgrass, G. Groves, K. Hasselmann, G. Miller, W. Munk, and W. Powers, "Propagation of Ocean Swell Across the Pacific," *Phil. Trans. A,* **259**, 431-497 (1965).

The most recent paper documenting the APL experiment is

R. C. Beal, T. W. Gerling, D. E. Irvine, F. M. Monaldo, and D. G. Tilley, "Spatial Variations of Ocean Wave Directional Spectra from the SEASAT Synthetic Aperture Radar," *J. Geophys. Res.* (in press, 1985).

Two earlier papers in the *Johns Hopkins APL Technical Digest* series that elaborate on the use of SAR for oceanography are:

R. C. Beal, "The Potential of Spaceborne Synthetic Aperture Radar for Oceanography" **1**, 148-156 (1980).
R. C. Beal, "Predicting Dangerous Ocean Waves with Spaceborne Synthetic Aperture Radar" **5**, 346-359 (1984).

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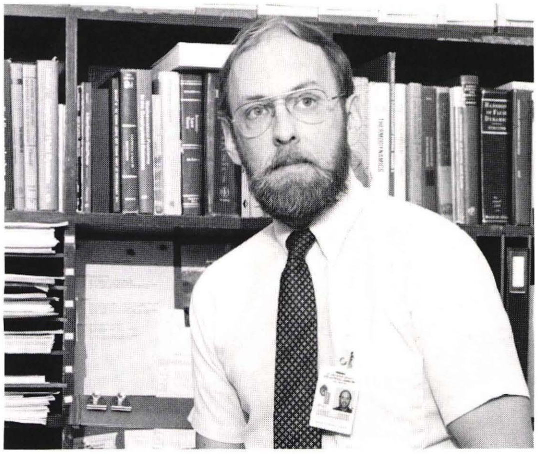

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