C. ALLAN BOYLES, LEWIS B. DOZIER, and GERALDINE W. JOICE

APPLICATION OF COUPLED MODE THEORY TO ACOUSTIC SCATTERING FROM A ROUGH SEA SURFACE OVERLYING A SURFACE DUCT

The technique of local normal modes can be used to obtain an exact numerical solution of the wave equation for the case of an oceanic waveguide that can possess both vertical and horizontal sound-speed gradients as well as a time-varying, randomly rough sea surface. The solution is then applied to long-range propagation in a surface duct that is formed in the upper layer of the ocean when certain oceanographic conditions occur. When a duct forms, significant acoustic energy that can become trapped in it propagates along it by repeated refractions and surface reflections and can be carried long distances.

INTRODUCTION

Issues in passive ocean acoustics[1] motivated the development of the coupled mode model that we will discuss. Passive acoustic systems detect sound energy radiated by a target. In the passive case, we must be able to describe long-range propagation in a surface duct over hundreds of miles. A description of the formation and characteristics of surface ducts will be discussed later. Suffice it to say now that a significant quantity of acoustic energy can become trapped in an upper layer of the ocean. The energy propagates along a surface duct by repeated refractions and surface reflections. Since the wave traveling along a duct interacts many times with the surface, it is necessary to have a model that includes rough-surface scattering effects. Also, since variations in duct thickness can have a profound effect on the transmission of energy along the duct, we must be able to take into account the variation of the vertical sound-speed profile with range. The variation of oceanographic properties with range can occur for many reasons.[2] Since we are talking about acoustic propagation in surface ducts 500 to 800 nautical miles long, the acoustic track could span water in different latitudes or with different current circulations. Also, large-scale eddies could disrupt the duct.

Rough-surface scattering models can be divided into two groups. The first group consists of those with a rough surface overlying a homogeneous medium that extends to infinity. A point source is located either a finite distance from the surface or at infinity, in which case the incident waves are planar. Common to all these models is a single act of scattering from the rough surface. Once the incident wave interacts with the rough surface, it then propagates to infinity, never again interacting with the surface. Slight perturbations in the boundary can produce only slight distortions in the scattered field.

The second group of models can handle multiple scattering from a domain bounded either by two surfaces or by a scattering surface overlying an upward-refracting medium. The source is still modeled as a point source, but now the wave propagating along the waveguide interacts repeatedly with one or both rough surfaces. Hence the field at the receiver is the phased sum of the waves multiply scattered by the irregularities that are distributed along the entire waveguide. Even very slight boundary perturbations can give rise to considerable distortions in the scattered field by virtue of the accumulated effects of repeated scatterings. This is the situation in naturally occurring waveguides such as the ocean, particularly when a surface duct is present.

The problem of treating wave propagation in an inhomogeneous, oceanic waveguide with a rough surface has been addressed by many investigators, notably Harper and Labianca,[3] Bass et al.,[4,5] Bass and Fuks,[6] Kryazhev et al.,[7] Kuperman and Ingenito,[8] McDaniel,[9] and Dozier.[10] The common factor in all these works, except that of Dozier, is the use of perturbation theory in various forms to solve the wave equation. Dozier obtained a numerical solution to the parabolic approximation of the wave equation; however, the parabolic approximation neglects backscattering. There are no such restrictions on the coupled-mode model presented in this article.

There are many approaches to applying the concept of coupled modes to waveguide problems. The term "coupled modes" arises from the following considerations. We are seeking a solution of the wave equation for a wave that is confined to a waveguide. The waveguide is bounded in one space coordinate only (the z coordinate) by two surfaces. It is open to infinity in the x and y directions. Because the problem is bounded in the z direction, the wave functions characterizing that direction are a discrete set of complete, orthonormal eigenfunctions. Each eigenfunction, called a mode, describes the characteristic vibrations of the waveguide in the z direction. If the two surfaces bounding the waveguide are smooth parallel planes, then the energy that starts out initially in a mode remains unchanged in that mode as the wave propagates along the waveguide. When the surfaces are rough, energy can be transferred from mode to mode as the wave propagates. Hence the modes are "coupled" to each other.

The particular approach used for the model described in this article was developed by Boyles.[11] Dozier[12] devised a numerical algorithm and wrote the computer code to integrate numerically the equations obtained by Boyles. However, Dozier's code treated the case of single-frequency continuous-wave propagation. This computer code was extended by Joice to treat the propagation and scattering of an arbitrary pulse waveform.

Two models have been developed. The first solves the problem of a cylindrically symmetric oceanic waveguide with both horizontal and vertical sound-speed gradients and a single realization of a time-varying, randomly rough sea surface; the model's geometry is shown in Fig. 1a. Statistics of the scattered field could be obtained via a Monte Carlo simulation. The main limitation of this work is the assumption of cylindrical symmetry, which precludes any scattering out of the vertical plane. In the second model, the limitation of cylindrical symmetry was removed, and a sea running in one direction was treated; the geometry of this model is shown in Fig. 1b. Both models assume a point source and include full mode coupling and forward- and backscattered waves. Thus both yield an exact solution of the wave equation for a randomly rough surface. Only the cylindrically symmetric model will be discussed in this article.

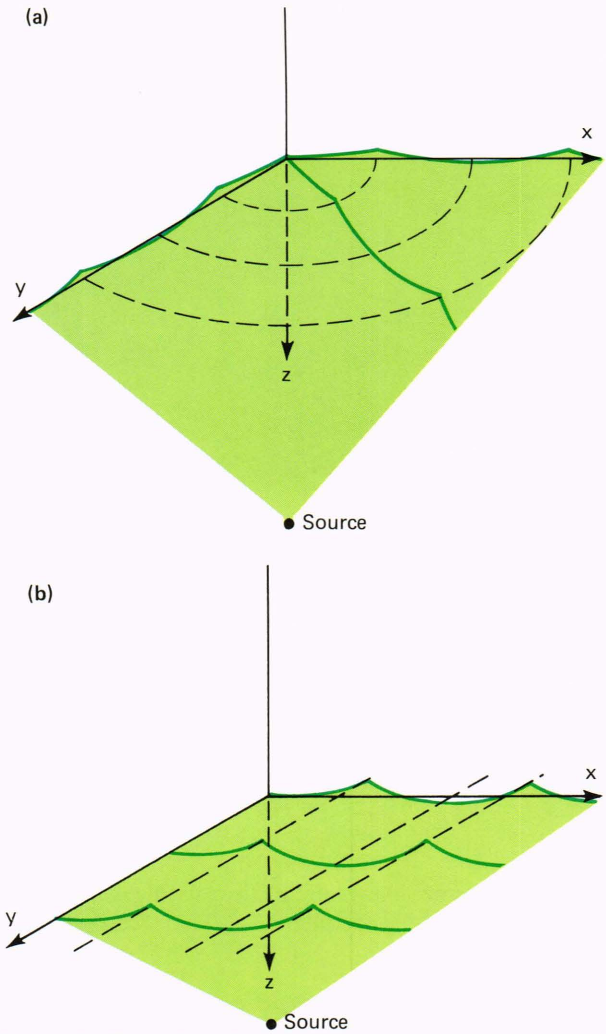


Figure 1—Examples of realization of the sea surface in (a) cylindrical geometry and (b) Cartesian geometry.

COUPLED MODE THEORY

Separation of variables is a powerful technique for solving partial differential equations. However, there are only 11 different three-dimensional coordinate systems in which the wave equation is separable.[13] Only if, in one of these systems, each boundary of the medium coincides with a coordinate surface, and further, if the refractive index is additively separable in the coordinates, can the technique of separation of variables be used.

One method of solving nonseparable problems is the technique of coupled modes, which replaces the wave equation (a partial differential equation) by a system of coupled, ordinary differential equations. These coupled differential equations give an exact and rigorous description of wave propagation for nonseparable problems, a method that has existed at least since 1927 when Born and Oppenheimer[14] applied it to the Schrödinger wave equation. The theory of coupled modes was introduced to the field of underwater acoustics in 1965 by Pierce.[15]

The method of mode coupling is not unique. There are many ways to obtain a coupled system of equations, but we shall discuss only one. A very detailed treatment of the subject can be found in Ref. 11.

We consider the problem of a point source located at a range, r, of $r = 0$ and a depth, z, of $z = z_s$ in a cylindrically symmetric oceanic waveguide. The coordinate system in relation to the waveguide is shown in Fig. 2. At the bottom of the sediment layers, the final boundary (representing a rock basement) is assumed to be rigid. The surface is given by a function of range and time, t, as

$$z = s(r,t) . \quad (1)$$

The speed of sound, $c(r,z)$, in the waveguide is a function of both r and z. The wave equation that governs the acoustic field away from the source is

$$\left(\nabla^2 - \frac{1}{\rho_0} \nabla \rho_0 \cdot \nabla - n^2 \frac{\partial^2}{\partial t^2}\right) P(r,z,t) = 0 , \quad (2)$$

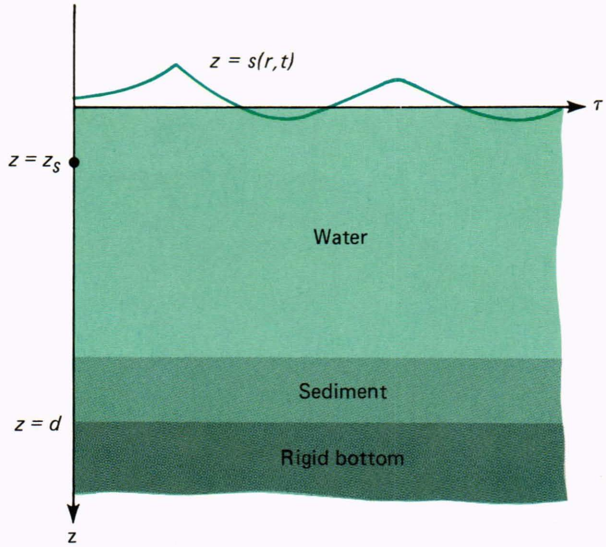


Figure 2—Coordinate geometry for the cylindrical waveguide.

where P is the acoustic pressure, ρ_0 is the density of the medium (water and sediment layers), and n is the refractive index defined by

$$n(r,z) = 1/c(r,z). \tag{3}$$

Let us assume that we are dealing with a sound wave of a single angular frequency ω. Then we can write

$$P(r,z,t) = p(r,z)e^{-i\omega t}, \tag{4}$$

where $p(r,z)$ is that portion of the total acoustic pressure that is independent of time. If we further assume that ρ_0 is a function only of depth, Eq. 2 can be written in cylindrical coordinates

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} - \frac{1}{\rho_0(z)}\frac{d\rho_0}{dz}\frac{\partial p}{\partial z} + k^2(r,z)p = 0 , \tag{5}$$

where we have set

$$k(r,z) = \omega n(r,z) . \tag{6}$$

This is called the Helmholtz equation or the space part of the wave equation. The equation is exact when the boundaries of the medium in which the wave is propagating are independent of time. However, we want to consider a time-varying sea surface. This would normally mean that instead of solving the Helmholtz equation, which is an elliptic partial differential equation, we would have to solve the full wave equation (Eq. 2), which is a hyperbolic partial differential equation. However, the numerical solution of the full hyperbolic problem with a rough, time-varying boundary is impractical; even if a numerically satisfactory algorithm could be devised, it would no doubt require an unreasonable amount of computer time. Fortunately, since the frequencies of the moving ocean surface that control the scattering process are much lower than the acoustic frequency in all cases of interest to us, we can invoke the narrowband approximation to the wave equation, expressed mathematically by

$$\frac{\partial^2 P}{\partial t^2}(r,t) \approx -\omega^2 P(r,t). \tag{7}$$

This approximation is discussed in detail in the papers by Fortuin[16] and Labianca and Harper.[17] It is equivalent to solving the Helmholtz equation for a time-dependent boundary. However, instead of Eq. 4, we now have

$$P(r,z,t) = p(r,z,t)e^{-i\omega t}, \tag{8}$$

where $p(r,z,t)$ becomes a slowly varying function of time through the application of the boundary conditions associated with the Helmholtz equation and $i = \sqrt{-1}$.

Fourier transforming the solutions $P(r,z,t_i)$ obtained at a set of discrete times, t_i, then yields the Doppler frequency spectrum at the point (r,z).

We postulate a solution of the form

$$p = \sum_{n=1}^{\infty} \phi_n(r)\, \psi_n(z,r), \tag{9}$$

where ϕ is an arbitrary function of range and ψ is an arbitrary function of range and depth.

Note that in contrast to the usual normal mode solution, using separation of variables, the depth wave functions, ψ_n, depend on the range as well as the depth. Consequently these are called "local" normal modes. For the range-independent problem, the depth modes are independent of range and hence form a "global" solution, i.e., a given mode is supported unchanged along the entire length of the waveguide.

We also postulate that the local depth modes satisfy the partial differential equation

$$\frac{\partial}{\partial z}\left[\frac{1}{\rho_0(z)}\frac{\partial\psi_n}{\partial z}\right] + \left[\frac{k^2(r,z)}{\rho_0(z)} - \frac{\kappa_n^2(r)}{\rho_0(z)}\right]\psi_n = 0 , \tag{10}$$

(where κ_n is the eigenvalue) and the following boundary conditions: (a) on the sea surface,

$$p[r,s(r,t),t] = 0 ;$$

and (b) on the rigid bottom,

$$\left[\frac{\partial p}{\partial z}(r,z,t)\right]_{z=d} = 0 ,$$

where d is the thickness of the waveguide.

The validity of the postulates given by Eqs. 9 and 10 must be confirmed by experimental measurement.

The variable $\kappa_n(r)$ is the "local" eigenvalue of the problem. It is determined at each point of the waveguide by the sound-speed profile, the density profile, and the boundary conditions. In the usual range-independent normal mode solutions, the eigenvalues are constants independent of the horizontal range.

The only difference in mathematical structure between Eq. 10 for the local modes and the corresponding equation for the range-independent case is that the equation for the local modes is a partial differential equation and the equation for the global modes is an ordinary differential equation. A partial differential equation is necessary here because we are postulating that the depth modes can now be a function of the range as well as the depth.

Now it can be shown that the eigenfunction solutions, ψ_n, of Eq. 10 that are subject to the boundary conditions form a complete, orthonormal system relative to the weight function $(1/\rho_0)$ at each range point r. The orthonormality condition is

$$\int_s^d \rho_0{}^{-1}(z)\psi_n(z,r)\psi_m(z,r)dz = \delta_{nm} , \qquad (11)$$

where δ is the Kronecker delta given by

$$\delta_{mn} = 0, \text{ if } m \neq n$$
$$= 1, \text{ if } m = n ,$$

where $m,n = 1, 2, 3, \ldots$

Thus the modal structure of a range-dependent waveguide varies from point to point along the waveguide. At each point, the modal structure depends on the acoustic frequency, the sound-speed profile, the density profile, the height of the water column, and, of course, the boundary conditions (which are the same along the waveguide). The modal structure can be different at each point because the sound-speed profile, the density profile, and the height of the water column can vary from point to point. Equation 11 implies that the depth modes must be renormalized at each point of the waveguide.

The fact that the modes are different at each point along the waveguide implies the existence of a mode-coupling process. This process is best expressed in terms of waves propagating in the forward and backward radial direction rather than in terms of the total field, $\phi_m(r)$. Consequently, we let

$$\phi_m(r) = a_m^+(r) + a_m^-(r) , \qquad (12)$$

where $a_m^+(r)$ is the forward-propagating wave and $a_m^-(r)$ is the backward-propagating (or backscattered) wave. It is the application of the boundary condition that determines whether a function represents the forward- or backscattered wave. These boundary conditions will be discussed later in this section.

The coupled equations that determine the radial propagating waves are

$$\frac{da_m^+}{dr} - \left(i\kappa_m - \frac{1}{2r}\right)a_m^+ = \sum_{n=1}^{\infty} B_{mn}^{++}a_n^+ + \sum_{n=1}^{\infty} B_{mn}^{+-}a_n^- ,$$

and

$$\frac{da_m^-}{dr} + \left(i\kappa_m + \frac{1}{2r}\right)a_m^- = \sum_{n=1}^{\infty} B_{mn}^{-+}a_n^+ + \sum_{n=1}^{\infty} B_{mn}^{--}a_n^- , \qquad (13)$$

where the coupling coefficients are given by

$$B_{mn}^{++} = -\frac{1}{2\kappa_m}\left[(\kappa_n + \kappa_m)B_{mn} + \delta_{mn}\frac{d\kappa_n}{dr}\right] ,$$

$$B_{mn}^{+-} = \frac{1}{2\kappa_m}\left[(\kappa_n - \kappa_m)B_{mn} + \delta_{mn}\frac{d\kappa_n}{dr}\right] ,$$

$$B_{mn}^{-+} = B_{mn}^{+-} ,$$

and

$$B_{mn}^{--} = B_{mn}^{++} , \qquad (14)$$

and where, for $m \neq n$:

$$B_{mn} = S_{mn} + N_{mn},$$

and

$$B_{nn} = 0 ,$$

where, for $m \neq n$:

$$S_{mn} = \frac{1}{\rho_0(\kappa_m^2 - \kappa_n^2)} \left(\frac{\partial \psi_m}{\partial z} \frac{\partial \psi_n}{\partial z} \frac{ds}{dr} \right)_{z=s} ,$$

and

$$N_{mn} = \frac{-2\omega^2}{(\kappa_m^2 - \kappa_n^2)} \int_s^d \rho_0^{-1} \, n \, \frac{\partial n}{\partial r} \, \psi_m \psi_n \, dz. \quad (15)$$

Consider the coupling coefficient, S_{mn}, that controls the exchange of energy due to the rough sea surface. We see that it is inversely proportional to the difference of the squares of the modal wavenumbers, κ_n, so that coupling is strongest for two neighboring modes. Since the squares of the modal wavenumbers appear, S_{mn} feeds the same power into both forward- and backward-traveling waves at a given range point. Whether or not this power builds up in either the forward- or backscattered wave depends on the phase relationship of the incremental amounts of power fed to the wave at other points. A wave builds up only if the incremental contribution at all points along the waveguide adds up in phase. We note that S_{mn} is proportional to the slope, $\partial \psi_n / \partial z$, of the wavefunctions at the sea surface; this is a very reasonable result. One way to look at this is to consider the ray equivalent[18] for a mode. As the order of the mode increases, the slope of the wave function at the surface increases, and, consequently, the strength of the coupling increases. But as the order of the mode increases, the ray equivalent strikes the surface at larger and larger grazing angles; consequently, we would expect more pronounced scattering. Finally, we see that S_{mn} is proportional to the slope of the sea surface.

The second term, N_{mn}, in the expression for the coupling coefficient, B_{mn}, is associated with mode coupling due to horizontal gradients, $\partial n / \partial r$, in the index of refraction, $n(r,z)$.

We confine the range-dependent properties of the waveguide to an interval $R_0 < r < R_1$; that is, the surface is assumed flat, and outside this region the sound speed depends only on depth. The only requirement we put on R_0 is that $\kappa_m R_0 \gg 1$. This is not a limitation of the theory, but it does simplify the numerical code. The value for R_1 can be hundreds of nautical miles. The boundary conditions on the variables a_n^+ and a_n^- are specified by their values at R_0 and R_1, respectively. To satisfy the radiation condition at infinity, we assume that

$$a_n^-(R_1) = 0.$$

Since the waveguide has no range-dependent properties from R_1 to infinity, an outgoing wave at infinity is guaranteed. Since the initial section of the waveguide ($0 < r < R_0$) is range-independent, a normal mode expansion of a point source is used to calculate $a_n^+(R_0)$.

A very common approximation applied to the coupled set of differential equations to make them tractable is the adiabatic approximation, in which all coupling coefficients are neglected, thus uncoupling Eq. 13. However, the equations are still range-dependent through the eigenvalue $\kappa_n(r)$. With this technique, a weakly range-dependent environment can be treated quite simply. Here, the adiabatic approximation is not used, and full mode coupling is always taken into account.

A MODEL FOR A TIME-VARYING, RANDOMLY ROUGH SEA SURFACE

It should be borne in mind that the propagation model is independent of any surface model and that any other model simulating a random sea surface could be used instead of the one described in this section. The model we will use is that described by Harper and Labianca,[19] in which the random ocean surface is simulated by

$$z = s(r,t) = \sum_{j=1}^{M} h_j \cos(\Gamma_j r - \Omega_j' t + \gamma_j) , \quad (16)$$

where Ω_j' is the angular frequency, $\Omega_j' > 0$, Γ_j is the wavenumber, and the dispersion relationship is

$$\Gamma_j = \Omega_j'^2 / g , \quad (17)$$

(g is the acceleration of gravity). The quantity h_j is the wave amplitude at the frequency Ω_j'. In Eq. 16, the phases γ_j are statistically independent random variables uniformly distributed between 0 and 2π. The frequencies Ω_j' are also taken to be statistically independent random variables.

Assume that the power spectrum, $P(\Omega')$, for the sea surface is band limited: $\Omega_{min} < |\Omega'| < \Omega_{max}$. Define

$$\Delta\Omega = (1/M)(\Omega_{max} - \Omega_{min}) .$$

Then the wave amplitudes, h_j, are defined as

$$h_j = \sqrt{2P(\Omega_j')\Delta\Omega} . \quad (18)$$

The Pierson-Moskowitz spectrum was used to represent the sea surface and is given by

$$P(\Omega') = (0.0081 g^2 / \Omega'^5) \exp[-0.74(\Omega_0/\Omega')^4] , \quad (19)$$

where $\Omega_0 = g/u$ and u is the wind speed in meters per second.

SURFACE DUCT PROPAGATION

There are just two parameters in the wave equation that describe the physical characteristics of the ocean and that affect the propagation of acoustic waves. These are the density, ρ_0, and the sound speed, $c(r,z)$.

The variation of density from the surface to the bottom of the ocean is so slight that it is usually neglected. Typically, a single value of the density is used throughout the water column. It is only in the sediment layers of the bottom that density variations are usually taken into account.

However, the variation of the sound speed within the water column cannot be neglected. It has been found that the speed of sound in the ocean is a function of temperature, salinity, and pressure. The nature of the variation of these three quantities with depth depends crucially on the geographic location.

There have been many equations proposed for determining the speed of sound as a function of temperature, salinity, and pressure. We do not intend to discuss the relative merits of those equations but will simply present one of the most recent determinations given by Mackenzie:[20]

$$c = 1448.96 + 4.591\,T - 5.304 \times 10^{-2}\,T^2 + 2.374 \times 10^{-4}\,T^3 + 1.340\,(S - 35) + 1.630 \times 10^{-2}\,D + 1.675 \times 10^{-7}\,D^2 - 1.025 \times 10^{-2}\,T\,(S - 35) - 7.139 \times 10^{-13}\,TD^3 \text{ (meters per second)},$$

where D is the depth in meters, T is the temperature in degrees Celsius at depth D, and S is the salinity in parts per thousand at depth D. One of the conveniences of this equation is that the more easily calculated quantity of depth is used instead of pressure. The range of validity of this equation encompasses temperature from -2 to 30°C, salinity from 25 to 40 parts per thousand, and depth from 0 to 8000 meters. The variation of the sound speed with depth is called the sound velocity profile.

In certain areas of the world's oceans, the temperature profile shows the presence of an isothermal layer beneath the sea surface. The layer of isothermal water is created and maintained by wind mixing, and being isothermal, it is called a mixed layer. The sound velocity in the layer increases with depth because of the pressure effect. In some areas, this positive sound-velocity gradient can extend to many hundreds of meters because of the existence of other oceanographic conditions below the wind-mixed layer that create a positive sound-speed gradient. With exceptions in some northern waters, the positive sound-speed gradient layer is followed by a layer with a negative sound-speed gradient. Such a sound-speed profile is illustrated in Fig. 3.

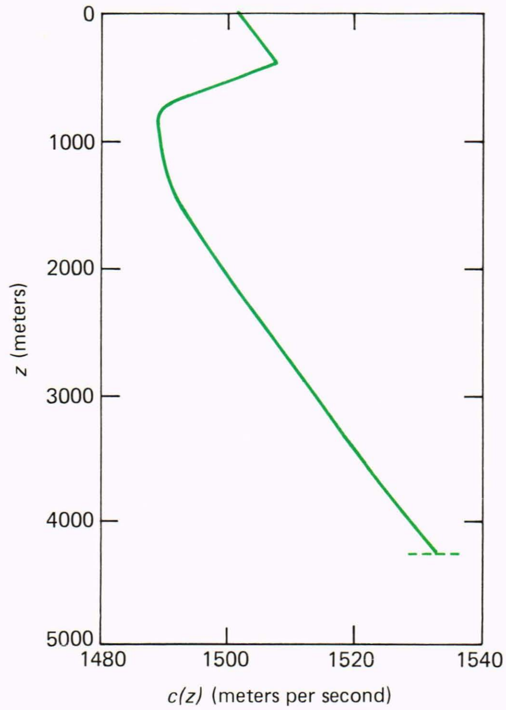


Figure 3—A North Atlantic sound-speed profile in March with a surface duct.

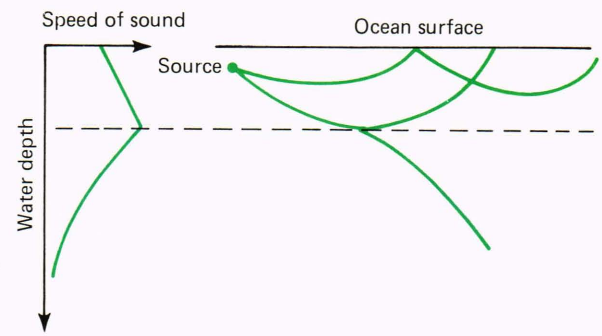


Figure 4—Surface duct propagation.

If an acoustic source is placed in the layer that has the positive sound-speed gradient, a cone of rays will be completely trapped in the layer because of upward refractions. Some typical rays are shown in Fig. 4. Because sound is trapped in the upper layer, it is referred to as a surface duct. In the ray theory approximation, all the energy that is put into the cone of rays trapped in the duct will remain in the cone as it propagates along the duct.

However, a full wave solution shows that energy can "leak" out of the duct due to diffractive effects as a wave propagates along the duct. The amount of leakage depends mainly on the frequency, duct thickness, and the sound-speed gradient in and below the duct.

In addition to its inability to treat diffractive leakage out of the duct, a second deficiency in using ray theory in a surface duct is the inability of ray theory to account for caustics or focal surfaces within the duct. A detailed description of caustics in ocean propagation can be found in a previous *Digest* article.[21]

A March North Atlantic profile that exhibits a 400-meter-deep surface duct is shown in Fig. 3 and a March North Pacific profile is shown in Fig. 5. In these northern waters, the duct extends the entire distance to the bottom, which can be 3000 to 4000 meters. The change in the sound-speed gradient at about 100 meters is due to salinity effects.

Figure 6 shows a ray trace for the North Pacific profile. The source is at a depth of 100 meters. Only rays exiting the source in a ±7° cone are shown; these are the ones that refract or turn over in the water column. Rays that leave the source at an angle greater than 7° will strike the bottom. Note that multiple surface interactions are apparent.

EXAMPLE OF SURFACE DUCT PROPAGATION WITH A ROUGH BOUNDARY

Now let us consider an example of surface duct propagation in a waveguide with a rough boundary. The example here will not use a randomly rough boundary, but rather a sinusoidal sea surface of a single wavelength Λ. Our purpose here is not to discuss a realistic case of ocean propagation, but to illustrate as simply as possible some of the features of coupled mode propagation. The features can best be discussed with a simple rough surface rather than a random surface.

We will consider the case of a sea being driven by a wind whose speed is 30 knots. The Pierson-Moskowitz spectrum is shown in Fig. 7 for a wind speed of 30 knots. From this figure and the dispersion relation given by Eq. 17, we see that the maximum power occurs at a surface wavelength of 200 meters. For this reason, we will use 200 meters as our surface wavelength. The wave amplitude is determined by a slight generalization of Eq. 18, namely,

$$h = \sqrt{2 \int_0^\infty P(\Omega')d\Omega'} = 1.8 \text{ meters.}$$

Thus, for this example, we are allowing the full bandwidth of the sea surface spectrum to determine the wave amplitude.

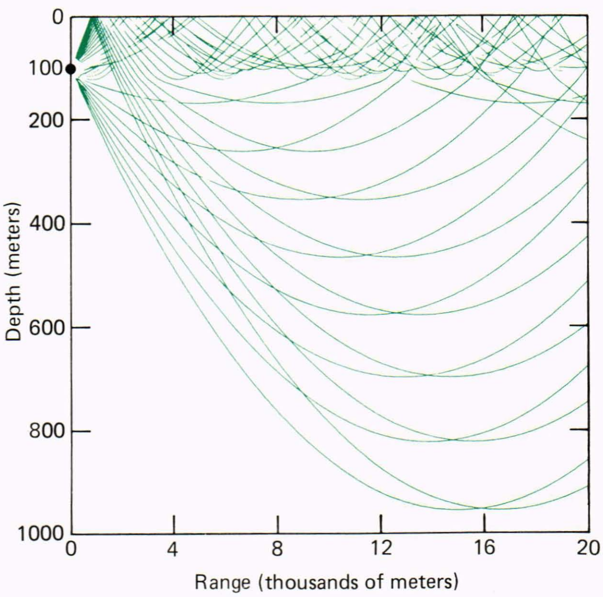


Figure 6—A ray trace for the North Pacific profile with the source at 100 meters.

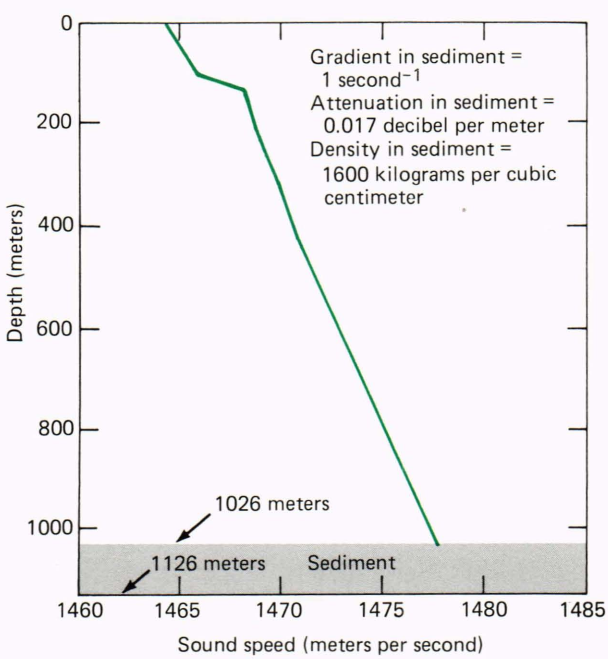


Figure 5—A North Pacific sound-speed profile in March.

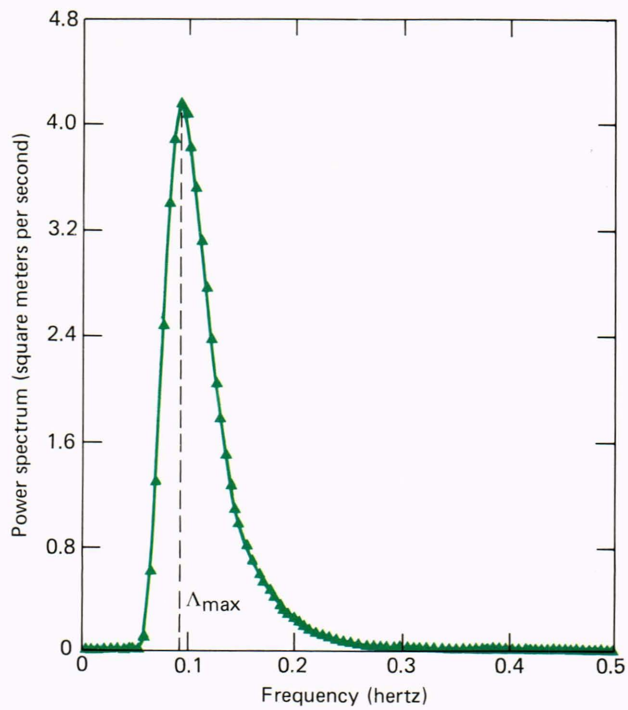


Figure 7—Pierson-Moskowitz spectrum for a 30-knot wind speed.

Let us put aside the waveguide problem for a moment and consider a plane wave traveling through an isospeed halfspace and impinging on a sinusoidal boundary. The principal scattering directions are given by the well-known equation from grating theory,[22]

$$\cos \Theta_{2m} = \cos \Theta_1 + m \frac{\lambda}{\Lambda} \quad (m = 0, \pm 1, \pm 2, \ldots), \tag{20}$$

where Θ_1 is the grazing angle of the incident wave, Θ_{2m} is the grazing angle of the mth order scattered wave, and λ is the acoustic wavelength. The geometry is shown in Fig. 8.

The "grating equation" (Eq. 20) is obtained in grating theory by considering a grating of, say, parallel wires (originally fine rulings on glass, or molecules of crystals), spaced at intervals Λ (Fig. 8), excited by a wave incident at a grazing angle Θ_1. If the phase delay between the scattered waves A and B is calculated from the geometry of the figure, and all angles, Θ_{2m}, for which this delay equals $2m\pi$ (i.e., the angles for which the scattered waves will be in phase) are determined, then Eq. 20 is obtained. In our case, the physical meaning of Eq. 20 is quite similar: Θ_{2m} are the directions for which the waves scattered from individual periods of the surface will be in phase and will reinforce each other, giving rise to the peaks of the scattering diagram.

Thus the scattering directions, θ_{2m}, given by Eq. 20, correspond to the maxima of the lobes of the scattering pattern. To each integer m, there corresponds a scattered mode propagated in a direction θ_{2m}. The total number of possible modes is limited by the condition

$$| \cos \theta_{2m} | < 1. \tag{21}$$

The mode with $m = 0$ is seen from Eq. 20 to correspond to specular reflections from the surface. The modes $m = \pm 1$ lie on either side of the specular direction, as shown in Fig. 9. They thus continue to lie on either side of the specular mode until the last modes that will still satisfy Eq. 21 are reached.

If λ/Λ is small, it follows from Eq. 20 that m will run through a large number of integral values before Eq. 21 is violated, so that, if the wavelength of the incident radiation is small compared to the period or wavelength of the surface, the incident wave will be broken up into many scattered waves. On the other hand, one would conclude from Eq. 20 that for $\lambda/\Lambda \gg 1$ there will be only the mode $m = 0$, so that for $\lambda \gg \Lambda$, any periodic surface will reflect specularly regardless of its roughness.

Keeping m in Eq. 20 constant and increasing λ, we find that the direction in which this selected mode is propagated becomes more nearly horizontal until it becomes a surface wave and the limit of condition Eq. 21 is reached. It is then known as the "cutoff mode" for that particular frequency.

The grating equation can be obtained also from the more rigorous application of wave theory. Scattering according to the grating equation is sometimes called Bragg scattering.

While the grating equation will not hold exactly for the waveguide problem, it at least gives us some feel for the scattering process.

Returning now to the waveguide problem, we assume the waveguide is characterized by the sound-speed profile shown in Fig. 5. Further, we assume there is a 50 hertz point source at a depth of 100 meters and a point omnidirectional receiver at a depth of 100 meters. We want to examine transmission loss as a function of range.

Transmission loss is a measure of the loss in intensity of sound between a point 1 meter from the source and a receiver at some arbitrary distance from the source. If I_0 is the wave intensity at 1 meter from the source and I is the intensity at the receiver, the transmission loss TL between the reference distance of 1 meter and the distant receiver is

$$TL = 10 \log (I_0/I) \text{ (decibels)}. \tag{22}$$

Figure 10 shows transmission loss over a range of 100 nautical miles. The black curve is for a waveguide with a smooth sea surface and the red curve is for a

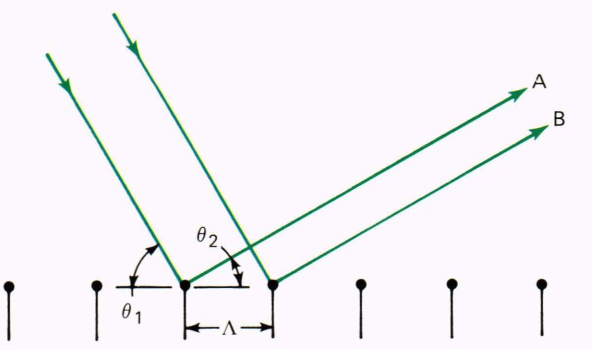


Figure 8—Derivation of the grating equation by phase delay.

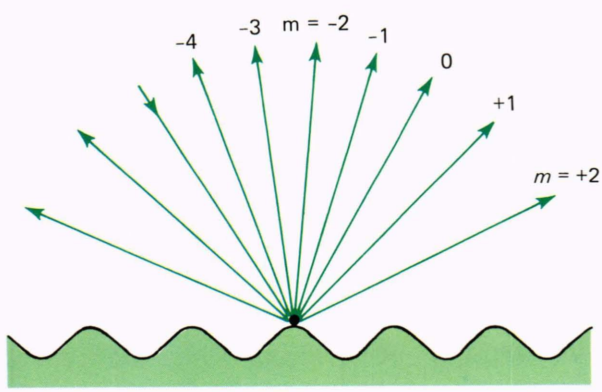


Figure 9—The directions in which various modes, m, are scattered by a periodic surface or directions of the sidelobes of the scattering diagram.

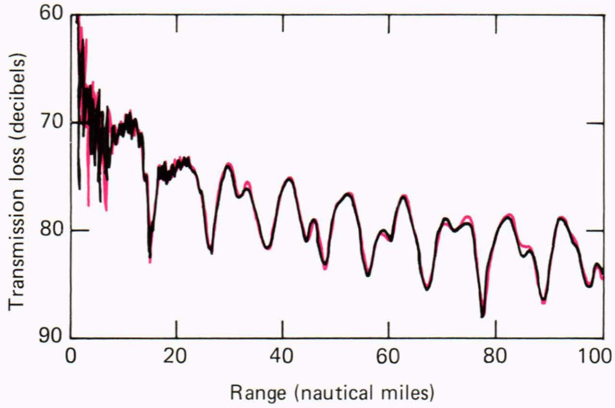


Figure 10—Comparison of transmission loss for a waveguide with a smooth sea surface (black curve) with one having a rough, sinusoidal sea surface (red curve).

waveguide with the rough, sinusoidal surface described above. For these conditions, we see that after a few nautical miles the rough surface has very little effect. This can be explained in terms of Bragg scattering. In order to calculate the Bragg angles from Eq. 20, recall that for our case, Λ equals 200 meters and λ equals 30 meters. The latter value arises from the fundamental relationship

$$\lambda = c/f \; , \tag{23}$$

where the standard value of c equals 1500 meters per second was used, and f is the wave frequency, where $f = \omega/2\pi$.

In the following discussion, we will talk in terms of rays because they are easier to visualize than modes. However, always keep in mind that the solution presented is a full wave solution and all aspects of it cannot be accounted for properly by ray theory alone.

Recall from Fig. 6 that the rays that were confined to the water column exited the source in a cone of $\pm 7°$. Consequently, let us look at the Bragg scattering angles for an incident wave angle of 5°. From Eq. 20, we see that the first Bragg angle is 31.8°, the second Bragg angle is 45.3°, and the third is 56.2°. While there are more Bragg angles, examining just the first three gives us a good idea of the nature of the scattering.

Since the first Bragg angle is considerably greater than the 7° needed to confine the scattered energy to the water column, all the scattered energy penetrates into the bottom sediment layer.

Since in our model there is considerable absorption in the sediment layer, all the energy that penetrates into that layer (scattered and nonscattered) propagates only a few miles before it is totally absorbed. There are many more fluctuations in the transmission-loss curve in the first 20 nautical miles than subsequently. Before 20 nautical miles, the rays from 7° to 90° contribute to the total field. The interference from these many paths causes rapid fluctuations. However, by 20

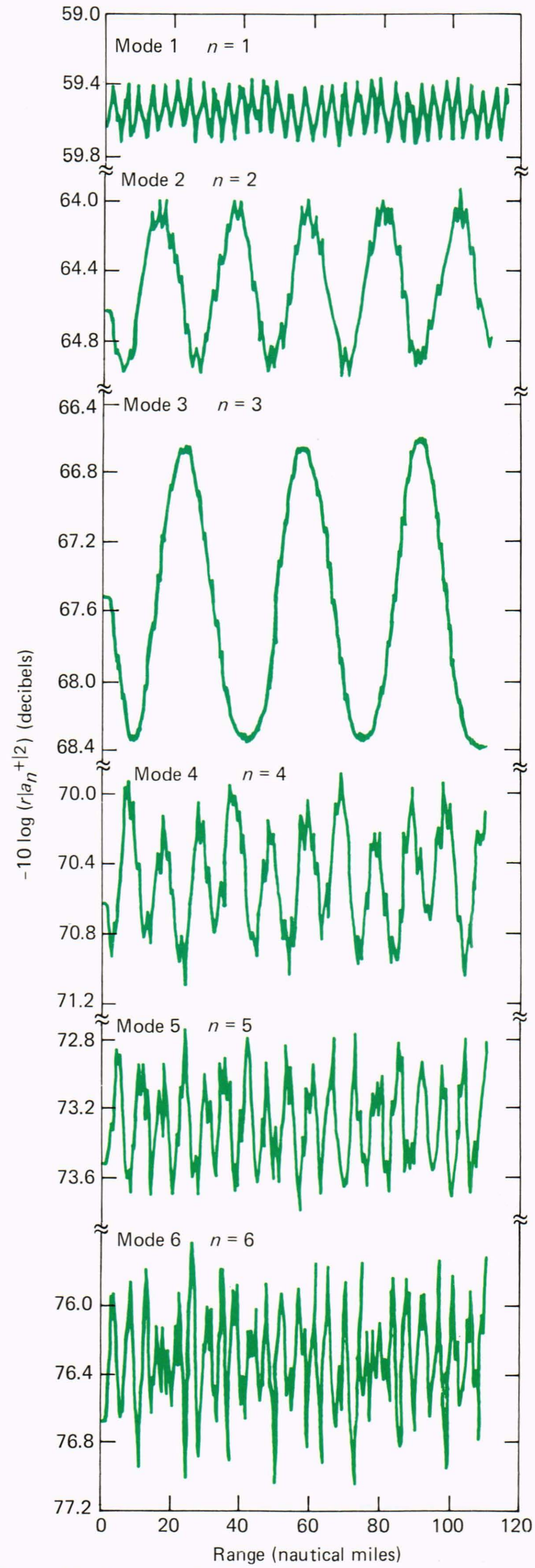


Figure 11—Square of the amplitude of the six radially propagating modes.

nautical miles, all these bottom interacting paths have been absorbed, and from 20 to 100 nautical miles only the energy confined to the water column contributes. The reduced number of paths yields fewer fluctuations.

For two reasons there is very little difference between the two curves from 20 to 100 nautical miles. First, even for the surface roughness, very little energy is scattered. Second, the scattering is at angles that do not involve the modes trapped in the water column, which means that since the first Bragg angle is greater than 7°, the scattered energy penetrates into the bottom and is absorbed. It does not reenter the water column to interfere with the energy that is trapped there. Thus, the fluctuation pattern is almost identical to that of the smooth surface case.

Figure 11 shows plots of the radial amplitudes, $|a_n|^2$, as a function of range for the six modes trapped in the water column. Note that we have taken out the cylindrical spreading term, $1/\sqrt{r}$. For a range-independent normal mode solution, these curves would be constant. For the case of a rough surface, we see that the energy in each mode fluctuates as a function of range. For this case, the fluctuations are only about 1 decibel, but energy is continually being transferred from one mode to the other even if there is very little net loss.

REFERENCES

[1] R. J. Urick, *Principles of Underwater Sound,* Chaps. 1 and 2, McGraw-Hill, New York (1975).

[2] G. Neumann and W. J. Pierson, Jr., *Principles of Physical Oceanography,* Prentice-Hall, Englewood Cliffs, N.J. (1966).

[3] E. Y. Harper and F. M. Labianca, "Perturbation Theory for Scattering of Sound from a Point Source by a Rough Surface in the Presence of Refraction," *J. Acoust. Soc. Am.* **57**, 1044-1051 (1975).

[4] F. G. Bass, V. D. Freulicher, and I. M. Fuks, "Propagation in Statistically Irregular Waveguides – Part I: Average Field," *IEEE Trans. Antennas Propag.* **AP-22**, 278-288 (1974).

[5] F. G. Bass, V. D. Freulicher, and I. M. Fuks, "Propagation in Statistically Irregular Waveguides – Part II: Second Order Statistical Moments," *IEEE Trans. Antennas Propag.* **AP-22**, 288-295 (1974).

[6] F. G. Bass and I. M. Fuks, *Wave Scattering from Statistically Rough Surfaces,* Pergamon Press, New York (1979).

[7] F. I. Kryazhev, V. M. Kudryashov, and N. A. Petrov, "Propagation of Low-Frequency Sound Waves in a Waveguide with Irregular Boundaries," *Sov. Phys. Acoust.* **22**, 211-215 (1976).

[8] W. A. Kuperman and F. Ingenito, "Attenuation of the Coherent Component of Sound Propagating in Shallow Water with Rough Boundaries," *J. Acoust. Soc. Am.* **61**, 1178-1187 (1978).

[9] S. T. McDaniel, "Comparison of Coupled Mode Theory with the Small Wave Height Approximation for Sea Surface Scattering," *J. Acoust. Soc. Am.* **70**, 535-540 (1981).

[10] L. B. Dozier, "PERUSE: A Numerical Treatment of Rough Surface Scattering for the Parabolic Wave Equation," *J. Acoust. Soc. Am.* **75**, 1415-1432 (1984).

[11] C. A. Boyles, *Acoustic Waveguides: Applications to Oceanic Science,* Chap. 7, John Wiley and Sons, New York (1984).

[12] L. B. Dozier, "Numerical Solution of Coupled Mode Equations for Rough Surface Scattering," *J. Acoust. Soc. Am.* **73** (Supplementary), 596 (1983).

[13] P. M. Morse and H. Feshback, *Methods of Theoretical Physics: Part I,* Chap. 5, McGraw-Hill, New York (1953).

[14] M. Born and J. R. Oppenheimer, *Ann. Physik* **84**, 457-484 (1927).

[15] A. D. Pierce, "Extension of the Method of Normal Modes to Sound Propagation in an Almost-Stratified Medium," *J. Acoust. Soc. Am.* **37**, 19-27 (1965).

[16] L. Fortuin, "The Wave Equation in a Medium with a Time-Dependent Boundary," *J. Acoust. Soc. Am.* **53**, 1683-1685 (1973).

[17] F. M. Labianca and E. Y. Harper, "Connection Between Various Small-Waveheight Solutions of the Problem of Scattering from the Ocean Surface," *J. Acoust. Soc. Am.* **62**, 1144-1157 (1977).

[18] See Boyles, Ref. 11, pp. 197-201.

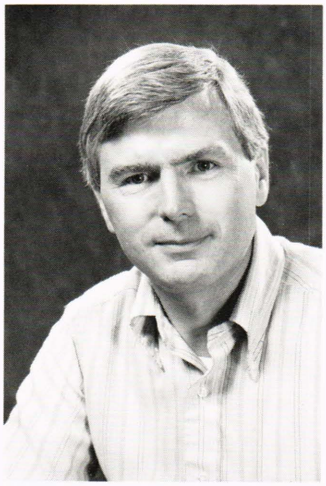
[19] See Harper and Labianca, Ref. 3.

[20] K. V. Mackenzie, "Nine Term Equation for Sound Speed in the Ocean," *J. Acoust. Soc. Am.* **70**, 807-812 (1981).

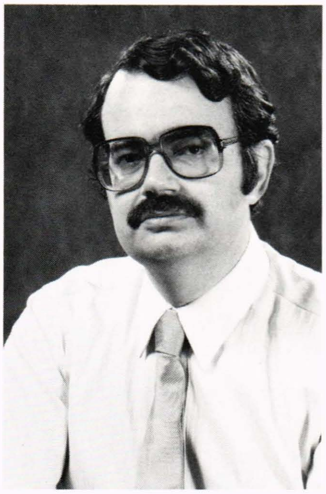
[21] C. A. Boyles and G. W. Joice, "Comparison of Three Acoustic Transmission Loss Models with Experimental Data," *Johns Hopkins APL Tech. Dig.*, **3**, 67-76 (1982).

[22] P. Beckman and A. Spizzichino, *The Scattering of Electromagnetic Waves from Rough Surfaces,* Chap. 4, Pergamon Press, New York (1963).

THE AUTHORS



C. ALLAN BOYLES was born in Columbia, Pa., in 1936. He holds B.S. and M.S. degrees in physics from the Pennsylvania State University and has completed the course work required for a Ph.D. Mr. Boyles worked at the Ordnance Research Laboratory at Penn State (1964-67). From 1967 to 1970, he was with TRACOR, where he continued his work on spherical acoustic lenses and worked on scattering of acoustic pulses from random volume scatterers and the transmission of electromagnetic waves in water. Mr. Boyles joined APL's Submarine Technology Department in 1970. From 1970 to 1976, he worked on the theory of spherical acoustic Luneburg lenses and helped design and conduct several sea tests to measure transmission loss and coherence in the ocean. From 1976 to the present, his main effort has been in developing the mathematical theory of acoustic wave propagation in the ocean, including a coupled mode propagation model that gives an exact solution of the wave equation for an oceanic waveguide with a random sea surface and range-dependent bathymetry and sound speed. Mr. Boyles is author of the book, *Acoustic Waveguides: Application to Oceanic Science.* He is a member of the Principal Professional Staff and is supervisor of the Propagation and Noise Analysis Section.



LEWIS B. DOZIER is a senior applied mathematician at Science Applications International Corporation (SAIC), McLean, Va., specializing in the modeling of underwater acoustic propagation. He was born in Rocky Mount, N.C., in 1947. After receiving his B.S. in mathematics from Duke in 1969, Dr. Dozier joined the Bell Telephone Laboratories, where he was involved in the development of large-scale computer simulations of magnetohydrodynamics, with applications to advanced ballistic missile defense systems. Meanwhile, he earned his M.S. in mathematics from the Courant Institute of New York University in 1972. In 1973, while still at Bell Laboratories, he became involved with ocean acoustics and was responsible for designing the logic of a computer simulation to evaluate system performance. In 1974, Dr. Dozier took a leave of absence from Bell Laboratories to do doctoral research at the Courant Institute. His thesis concerned a Monte Carlo numerical simulation to evaluate a theory of acoustic fluctuations due to internal waves.

In 1977, Dr. Dozier joined the staff of SAIC, where he has continued to pursue his interest in acoustic fluctuations due to the random ocean medium and boundaries. He has been principal investigator of several research efforts in that field, including considerable collaboration with APL. A novel technique for numerical computation of scattering from a rough ocean surface was published in 1984.

GERALDINE W. JOICE was born in Raleigh, N.C., in 1950. She completed her undergraduate studies in mathematics at Bennett College (1972) and obtained the M.S. degree in numerical science from The Johns Hopkins University (1974). She has also studied physics and mathematical physics at the University of Maryland and The Johns Hopkins University.

Mrs. Joice joined APL in 1973 as a computer specialist. In 1977, she transferred to the Acoustics Group of the Submarine Technology Department. Since then, she has worked primarily in the areas of underwater acoustic propagation modeling, sea test analysis, and threat assessment. She also serves as a consultant to other users of the various propagation models available at APL.