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BATTLE GROUP AIR DEFENSE ANALYSIS

Battle Group air defense is a complex problem involving many systems of different characteristics and capabilities. Coordination of these diverse elements potentially provides a total defense more effective than the sum of its parts. Analysis of some aspects of the total interaction among the weapon systems involved in Battle Group defense reveals the need for complementary zones of defense and coordination of weapons in order to achieve the required effectiveness.

INTRODUCTION: THE DEFENSE-IN-DEPTH CONCEPT

The concept of antiair warfare defense in depth recognizes that no antiair warfare system can singlehandedly defeat a concentrated attack on a Battle Group. Separate systems are normally applied to implement defense in depth in areas corresponding to the outer defense zone, the area defense zone, and the self-defense zone (Fig. 1). Effective antiair warfare defense requires the disruption of an attack and depletion of the enemy's resources at every opportunity.

The first line of defense, which must be initiated at maximum range, requires timely warning; in order to obtain it, one needs surveillance as well as information from command, control, and communication systems and early warning aircraft. The first line of engagement is conducted by manned combat aircraft and long-range surface-to-air missiles. In this outer defense zone, the coordinated use of weapons to reduce the number of standoff jammers and launch platforms prior to the launch of their antiship missiles is an essential part of the overall Battle Group defense. This diminishes the attack as seen by the area defense systems by (a) reducing the jamming levels so that radars can detect the attack earlier, (b) discoordinating the attack to increase its duration and lower its peak density, and (c) reducing the number of attackers that penetrate to the area defense zone. The outer defenses must also serve Battle Group needs by destroying loitering and retiring aircraft in order to reduce the intensity of follow-on attacks.

In addition, the outer defenses can cause hostile forces to reduce the exposure of their own valuable manned aircraft by forcing them to launch their weapons from increased ranges. This makes the targeting of surface ships more difficult for the attacker and consequently reduces the effectiveness of his attack. Conversely, the employment of airborne jamming by the defense can force the attacking aircraft to advance to much shorter launch ranges in order to target Battle Group ships, thus exposing the attackers to the outer air defenses for a longer period of time.

In the area defense zone, surface-to-air missiles are used to destroy antiship missiles, while electronic warfare systems are used to deceive them. The weapons of the area defense zone constitute the major defense of the Battle Group because they provide the most comprehensive multiaxis coverage, the highest rates of fire, and the greatest supply of ready ammunition.

Neither the outer defense weapons nor the area defense weapons — no matter how well employed — will ever be totally effective; some attackers will penetrate the defended area. Therefore, a self-defense zone constitutes the last line of defense. In the self-defense zone, engagement ranges are short. The penetrators from the outer zones must be handled by self-defense weapons that have an extremely high kill capability.

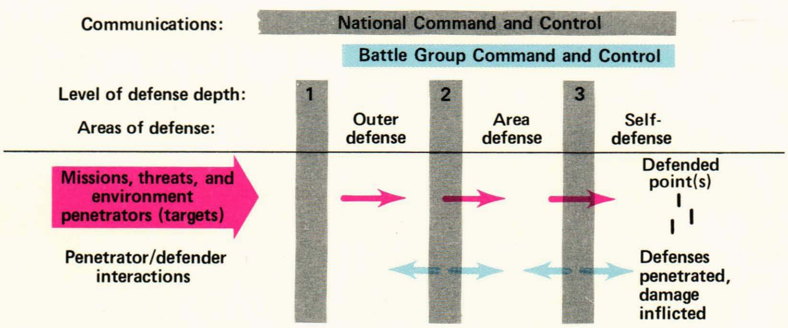

Figure 1 — The defense-in-depth concept is based upon three defensive zones. In the outer defense zone, manned aircraft intercept the attack at long range. Attackers surviving the outer zone are countered in the area defense zone by surface-to-air missiles. The survivors are then engaged by self-defense weapons in the last zone. Command, control, and communications provide coordination within and between zones.

Consideration must also be given to interactions among zones, either positive (e.g., passing information between zones) or negative (e.g., friendly aircraft operating within or transiting the surface-to-air missile defense zones and in the overlap between the area and self-defense zones). These factors must all be accounted for and coordinated.

Coordination of defense in all zones has substantial benefits, but the real-time coordination of area defense weapons is mandatory because of the limited time available to deal with penetrating targets, the distances between Battle Group ships, and the overlap of defensive weapon coverage. An uncoordinated response to a raid in the area surface-to-air missile zone could result in substantial (though inadvertent) overengagement of some targets, while other targets might be unengaged. Real-time weapons coordination across the Battle Group — i.e., dynamic allocation of ship and aircraft weapons to targets — alleviates this problem. It also provides the potential of mixing new, fast-reaction, high-firepower systems with older, slower systems having less firepower; the older systems reap added benefits from the superior data and control capabilities of the new units.

Systems deployed in each zone are functionally complementary, as shown in Table 1. Analyses lead to the conclusion that all elements are needed for effective Battle Group air defense. Furthermore, proposed improvements in the Navy's antiair warfare systems should be judged on the basis of their contribution to the total defense in depth.

ANALYSIS OF DEMANDS ON BATTLE GROUP ANTIAIR WARFARE

The demands placed on Battle Group antiair warfare depend upon many different factors. Insight into these demands can be obtained by considering, as an example, an attack by 60 enemy airborne vehicles, which then become targets for the defense. As a starting point, the defense must provide some level of assurance that the Battle Group can survive to complete its mission. Assuming that any target that penetrates the defense has a 0.5 probability of causing shipboard damage and that the mission success criterion is 0.8 probability of no shipboard damage, the Battle Group defense must attain a cumulative

Table 1

COMPLEMENTARY FUNCTIONS IN BATTLE GROUP AIR DEFENSE

	Outer Defense	*Area Defense*	*Self-Defense*
Sensors (range and position data)	Airborne long range	Sophisticated	Short range
Target capacity	Low capacity because of time on station and availability and number of platforms	Large capacity because of range and time windows for engagement	Inherent low capacity
Coverage	Sector coverage	Mutual support among escorts	Own ship
Threat	Engage enemy at long range (prior to enemy missile launch), counter enemy coordination, and jam enemy radars	Engage large, difficult threat	Engage residual threat
Cost	Substantial unit cost	Substantial unit cost	Moderate unit cost
Installation	Carriers	Major surface combatants	Every ship
Conclusion	**Insufficient by itself**	**Insufficient by itself**	**Insufficient by itself**

All are required to handle threat

kill probability per target (P_{E_T}) greater than 0.99 (Fig. 2). Note that cumulative kill probability per target is determined by the cumulative effect of all Battle Group elements that have an opportunity to engage the target. This result describes the demand placed on Battle Group antiair warfare. The equation relating these variables is

$$P_D = [1 - (1 - P_{E_T})P_{D/S}]^{N_T} , \qquad (1)$$

where P_D is the probability that no ship is damaged (assumed to be 0.8); P_{E_T} is the cumulative kill probability per target (calculated to be 0.99+); $P_{D/S}$ is the probability that a surviving target causes shipboard damage (assumed to be 0.5); and N_T is the number of targets (assumed to be 60). The cumulative kill probability per target includes the successful achievement of all functions, i.e., detection, coordination, and engagement, as well as system availability. This highly stringent demand is reasonably achievable only in the context of defense in depth.

The relationship between the number of engagement opportunities per target and the cumulative kill probability per target is shown in Fig. 3. At each engagement opportunity there is a probability of destroying an attacking target. The higher the kill probability, the better the system. However, weapon reliability sets an upper bound for this probability. Additionally, the higher the kill probability, the more costly the weapon system. On the other hand, weapons with low kill probabilities, although possibly not as costly, are not sufficiently effective. For the purpose of this example, 0.5 is assumed as a lower bound to an acceptable kill probability. The relationship between cumulative kill probability per engagement opportunity and number of engagement opportunities is

$$P_{E_T} = 1 - (1 - P_K)^{N_E} , \qquad (2)$$

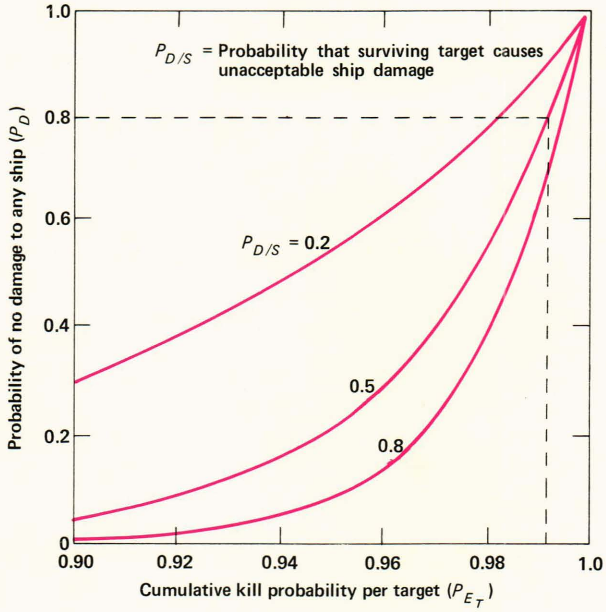

Figure 2 — The demands placed on Battle Group air defense are illustrated by considering an attack by 60 enemy airborne vehicles that then become targets for the defense. Assuming that a surviving attacker has a 0.5 probability of causing unacceptable shipboard damage and that Battle Group mission survival is a 0.8 probability of no shipboard damage, the cumulative kill probability per target must be 0.99. Cumulative kill probability per target is composed of the combined results of all engagements in all defensive zones against a given attacker.

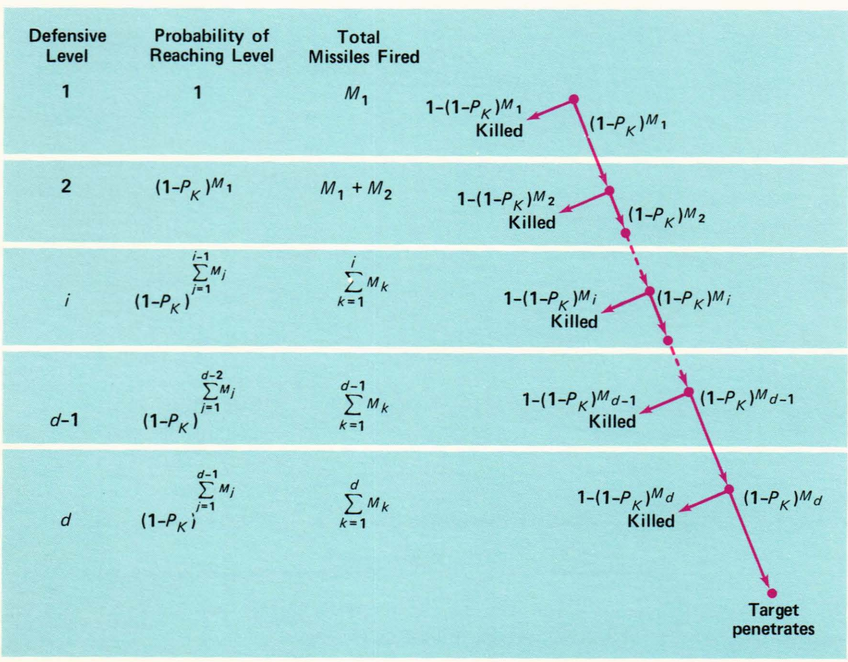

Figure 3 — A tree diagram of an engagement of a target by a defense that has a depth of fire d. If M_i missiles are fired at defensive level i, then the probability of surviving to the next defensive level is $(1 - P_K)^{M_i}$. The probability that the target is killed at the ith defensive level is $1 - (1 - P_K)^{M_i}$ given that it has survived the previous $i-1$ defensive levels. Note that the probability of firing $(M_1 + M_2 \ldots + M_i)$ missiles is the probability that the target survives to the ith defensive level. Using the results shown here, it can be shown that M, the average number of missiles fired, equals

$$\sum_{j=1}^{d} \sum_{i=1}^{j} M_i \prod_{k=1}^{j-1} (1 - P_K)^{M_k},$$

which reduces to

$$M_1 + \sum_{j=2}^{d} M_j \prod_{k=1}^{j-1} (1 - P_K)^{M_k}.$$

where P_K is the kill probability per engagement opportunity and N_E is the number of engagement opportunities. For a 0.5 kill probability per engagement opportunity, seven engagement opportunities per target are required to achieve a cumulative kill probability per target of 0.99, as illustrated in Fig. 4. The engagement opportunities per target necessary to attain the desired cumulative kill probability may be grouped together. If the defense has only a single chance at the attacker, then seven rounds must be fired together; however, if several chances are available, the engagement opportunities may be divided among them.

Considering engagement opportunities to be equivalent to the number of missiles fired, significant savings can be obtained by having several chances at a target, i.e., a depth of fire. The relationship among depth of fire, average number of missiles fired, and single-shot kill probability is shown in Fig. 5. For the example, a depth of 2 requires only 60% of the missiles needed for a depth of 1. As the depth of fire increases, fewer missiles per target are required on the average. The biggest payoff, however, is in going from a depth of 1 to a depth of 2.

This example indicates that a Battle Group antiair warfare combat system must achieve a high cumulative kill probability per target. By having several engagement opportunities against each attacker, a high cumulative kill probability can be attained with a moderate single-engagement kill probability.

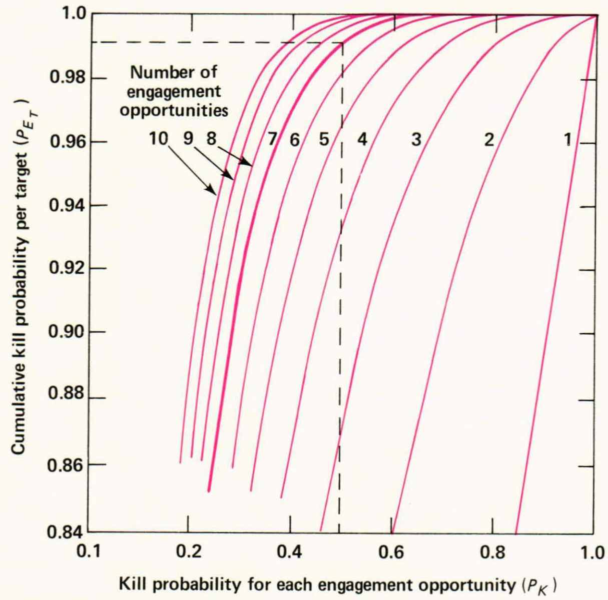

Figure 4 — Continuing the example of a 60-target attack, the number of engagement opportunities required per attacker is plotted against kill probability per opportunity. Assuming a single engagement kill probability of 0.5, seven engagement opportunities per target are required. These engagement opportunities may be grouped (i.e., taken simultaneously) or spread out in several defensive zones.

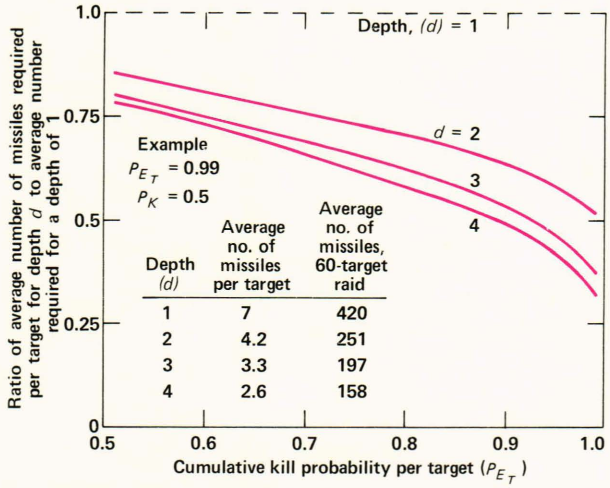

Figure 5 — Assuming that an engagement opportunity represents a single missile fired, the average number of missiles fired at an attacker for a cumulative kill probability of 0.99+ for various depths of fire d is shown. The ratio of missiles fired is plotted against cumulative kill probability for several depths of fire. As can be seen, a considerable number of missiles can be saved by having a depth of fire of 2 or greater. For the example of 60 targets, a depth of 2 requires only 60% of the missiles that a depth of one would need to obtain the same end result.

DEMAND ON AREA DEFENSE

The amount of demand put on area defense (the middle defense zone) is determined by the input from outer defenses and by the allowable leakage to the self-defenses. In this analysis, the premise is accepted that the function of the outer defenses is to establish a favorable defense environment; this includes disrupting attack coordination, destroying jammers, thinning the attack, and alerting other defenses.

As noted previously, air defense effectiveness can be increased by providing a greater depth of fire. Hostile jamming reduces depth of fire by reducing the range at which attacking missiles are detected. This reduction in depth can be severe. Outer defense weapons provide a vital service by destroying standoff jammers; this, in turn, increases the effectiveness of area and self-defense weapons.

Another aspect of the outer defense's effect upon area defense involves attack coordination. A weapon system is limited in the number of targets it can engage in a given amount of time. If an attacker coordinates his attack, he can saturate the defense by launching attacking missiles faster than they can be engaged.

The number of targets the defense can handle, N, is determined by

$$N = \frac{RTN_s}{\bar{e}} , \qquad (3)$$

where R is the engagement rate per ship (assumed to be 10 per minute); T is the time available for engaging targets, equal to the sum of the raid duration and the period of vulnerability (assumed to be one minute); N_s is the number of ships; and $\bar{e}$ is the average number of times a target is engaged. As an example, Fig. 6 shows the number of targets that can be handled by a given number of ships for various raid durations (the time interval over which all attacking missiles arrive in the area defense zone). It is assumed that each ship can carry out 10 engagements (salvos fired) per minute, has a depth of fire of 2 against every target, and fires dual missile salvos for every engagement with a missile single-shot kill probability of 0.5.

The average number of times a target is engaged is determined by the probability that a target is not destroyed at the first opportunity. For the conditions listed above, there is a 0.75 probability of destroying a target at the first opportunity with a dual-missile salvo and consequently a 0.25 probability that a second engagement will be required. Therefore, the average number of engagements per target is 1.25. Assuming the hostile force, if it is undisturbed, has the capability of launching all its missiles simultaneously, only a small number of attacking missiles can be engaged. As can be seen from Fig. 6, increasing the number of defensive ships to as many as five does not defeat the 60-target attack. However, if the outer defenses can harass the attackers and cause them to spread the attack out in time, a significant benefit can be attained.

The key input parameters for area defenses are thus the number and type of targets, their arrival distribution, their speeds and profiles, and the electronic countermeasures environment. These parameters can be highly variable, depending on offensive and defensive tactics. To arrive at the appropriate spectrum of conditions, many different battle situations must be considered. In each case, possible attacks and the conditions under which the defensive

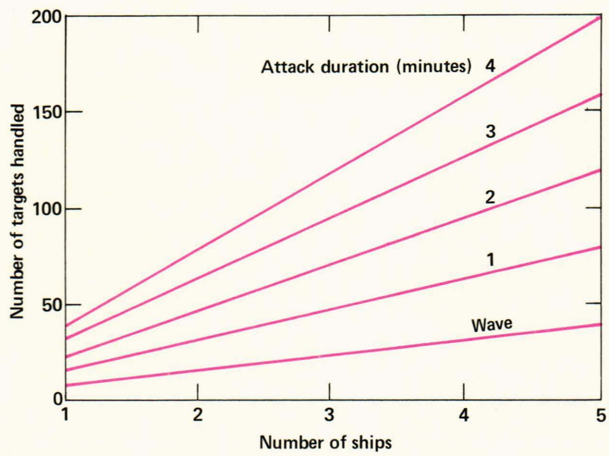

Figure 6 — The thinning or spreading out in time of the attackers penetrating to the area defense zone provides greater area defense effectiveness. Using the example of the 60-target attack, one ship with a defensive system that (a) can engage 10 attackers per minute, (b) has a single shot probability of 0.5, (c) fires dual-missile salvos, and (d) has a depth of fire of 2, can successfully engage eight targets when attackers arrive simultaneously. If the attack is spread out over a 4-minute duration, the ship can engage 40 targets successfully.

units are likely to fight are developed. The analytical results define the environments that exist at the start of these battles regarding such factors as warning time, jamming levels, ambient air and surface traffic density, and electromagnetic environment. The end result is not a single air battle with a specific outcome but rather a single class of air battles with a variety of possible outcomes.

The input to area defense requirements from the self-defense systems depends on the effectiveness of the self-defenses and the targeting of Battle Group ships by the surviving attackers. The contribution of self-defense weapons is illustrated by considering the 60-target attack mentioned earlier. It is assumed that two carriers and three major combatants are targeted by the attackers, with 18 attackers assigned to each carrier and the remaining 24 attackers evenly distributed among the three major combatants. The self-defenses of a carrier are assumed to have a cumulative kill probability of 0.98 against a maximum of five attackers; the self-defense zone of the major combatant is assumed to have a cumulative kill probability of 0.9 against a maximum of two attackers. For both ship types, it is also assumed that if the number of attackers penetrating to a ship's self-defense zone exceeds the maximum number, then all targets in excess of the maximum penetrate unopposed. For example, if eight attackers penetrate into a carrier's self-defense zone, three penetrate without being engaged.

The probability that a given ship receives no damage, Q_{D_i}, is the product of the probability that there are x penetrators through area and self-defense zones $(Q_i(N_{T_i}, x))$ and the probability that all penetrators fail to do damage. The probability function, $Q_i(N_{T_i}, x)$, where N_{T_i} is the number of targets aimed at the i-th ship, is

$$Q_i(N_{T_i}, x) = \sum_{x'=x}^{N_{T_i}} Q_{AD_i}(N_{T_i}, x') Q_{SD_i}(x', x) , \quad (4)$$

where $Q_{AD_i}(N_{T_i}, x')$ is the probability that x' out of N_{T_i} targets penetrate through the area defense zone and $Q_{SD_i}(x', x)$ is the probability that x out of x' targets penetrate through the i-th ship's self-defense zone. Assuming that P_{E_T} is the cumulative kill probability per target for the area defense zones and is the same for every ship, then

$$Q_{AD_i}(N_{T_i}, x') = \binom{N_{T_i}}{x'} \left(P_{E_T}\right)^{N_{T_i} - x'} \left(1 - P_{E_T}\right)^{x'} . \quad (5)$$

The function $Q_{SD_i}(x', x)$ cannot be represented in such a simple form since not all area defense penetrators can be engaged. The probability that x penetrators to the i-th ship cause no damage is simply $(1 - P_{D/S})^x$. The probability that the i-th ship suffers no damage is

$$Q_{D_i} = \sum_{x=0}^{N_{T_i}} \sum_{x'=x}^{N_{T_i}} \left[\binom{N_{T_i}}{x'} \left(P_{E_T}\right)^{N_{T_i}-x'} \right. \tag{6}$$

$$\left. \cdot \ (1-P_{E_T})^{x'} \ Q_{SD}(x', x) \ (1-P_{D/S})^{x} \right] .$$

For the Battle Group, the probability that no target ship is damaged, Q_D, is

$$Q_D = \prod_{i=1}^{N_S} Q_{D_i} \ , \tag{7}$$

where N_s is the number of targeted ships.

Given the foregoing assumptions, the probability that no ship receives damage, as a function of the average number of area defense penetrators, is shown in Fig. 7 for three levels of the probability that a survivor causes damage ($P_{D/S}$). Using the curve for $P_{D/S}$ = 0.5, the average number of area defense penetrators must be eight or less in order for the probability of no damage to any ship to be 0.8 or higher. In addition, the area defense weapons must attain a cumulative kill probability of 0.88 and have the capability of firing three to four missiles per attacker (Fig. 4) for a 0.5 single-shot kill probability. If the area defense is to engage the attack without benefit of self-defense weapons, the cumulative kill probability must be 0.99 and the number of missiles per target must be seven (Fig. 4).

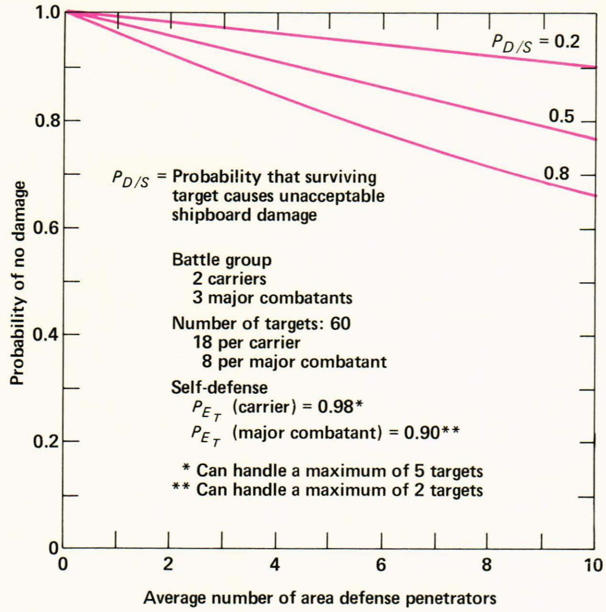

Figure 7 — The effectiveness of the self-defense zone determines the required area defense effectiveness. Assuming a 60-target attack and a 0.5 probability that a survivor causes unacceptable damage, under the conditions shown here the area defense can allow only an average of eight penetrators to the self-defense zone, for a 0.8 probability of no damage to any ship. This represents a cumulative kill probability per attacker of 0.88 in the area defense zone. This result, combined with the equation in Fig. 3, shows that a maximum of only three to four missiles per target is required as opposed to the seven required to achieve a cumulative kill probability per target of 0.99, as found previously.

CONCLUSION

The considerations that have been discussed here—attack coordination (number, type, speed, and arrival distribution of targets) and subsequent probability of damage to elements of the group — illustrate an approach to establishing goals for area defense surface-to-air missile systems. Attacking targets that penetrate the outer defense zone must be reduced in number by the area defense to a level the self-defenses can tolerate. Although many attackers are destroyed in the outer zone, the surviving attackers are able to launch numerous antiship missiles. Thus the greatest attrition must occur in the area defense zone. This requires that the area defense weapons attain a high cumulative kill probability per target, have the capability of engaging large numbers of targets, and be able to engage high-performance crossing targets (i.e., those not aimed at one's own ship). A ship designed to provide the Fleet with an area defense antiair warfare capability must achieve this goal in a variety of tactical situations against differing threats in unfavorable environments.

The continuing analyses being carried out by APL are intended to provide insights into some of the fundamental demands on antiair warfare systems from the perspective of the Battle Group. The results derived here reduce the complex interactive problems associated with many diverse and functionally different weapon systems to a simplified and manageable statement of demands for area defense antiair warfare systems. Among other things, this approach provides a basis for establishing top level requirements. Work is continuing, based on this fundamental approach, in the analytical formulation of requirements at the next level of detail, which includes factors such as range coverage, missile speed, and rate of fire.