NONLINEAR FRICTION IN SERVOMECHANISMS

PRECISION TRACKING SYSTEMS with error specifications in the arc second range have created new interest in more accurate representations of friction in servomechanism design. The conventional nonlinear friction model,[1] which assumes that friction forces can be represented by appropriate static, coulomb, and viscous components, represents a significant improvement over linear theory but does not adequately predict performance for arc second tracking in direct-drive servomechanisms.

To improve prediction, the nonlinear model was extended at APL by B. F. Hoffman and R. W.

[1] J. Tou and P. M. Schultheiss, "Static and Sliding Friction in Feedback Systems," *J. Appl. Phys.* **24**, 1953, 1210–1217.

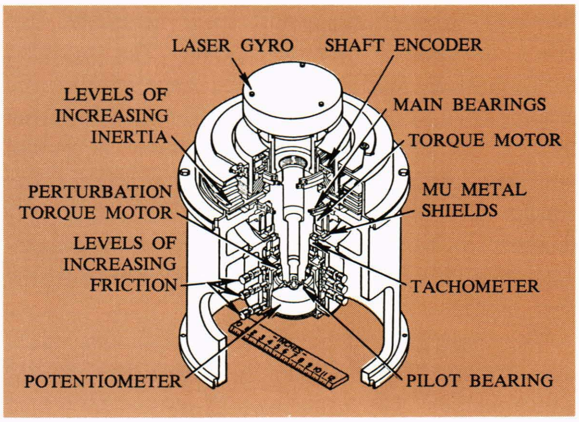

Fig. 1—Cutaway view of experimental hardware.

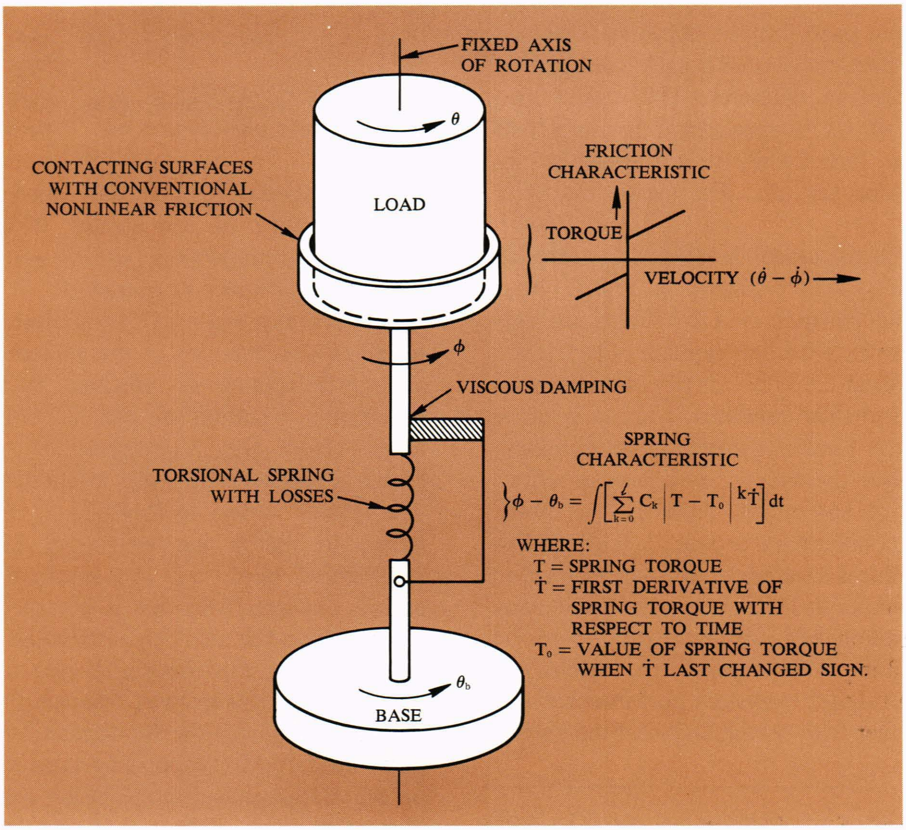

Fig. 2—Physical representation of mathematical model.

Fowler* to include elastic deformation effects in the bearings and other friction-producing elements of the mount. This resulted in a high-performance tracking loop that gave smoother and more precise tracking than was predictable using the conventional nonlinear friction model techniques. Although bearing engineers have recognized for many years the existence of elastic deformation of bearings, control engineers have neglected this effect in control system models. The updated mathematical model developed thus retains the nonlinear differential equation for conventional friction but also incorporates a second empirical nonlinear differential equation to represent the elastic deformation phenomenon. Excellent correlation has been obtained between observed hardware behavior and predicted performance using the new model.

The experimental hardware used in this study was designed specificially to investigate mount nonlinearities. The experimental mount shown in the cutaway view in Fig. 1 is a single axis, direct-drive mechanical test fixture. The main drive electrical torque motor and tachometer are of standard pancake designs with brushes. The motor has a maximum torque capability of 22 lb-ft. The primary rotor support consists of a preloaded pair of bearings 9 inches in diameter. The mount also contains a set of six nonpreloaded bearings that may be engaged individually or collectively to introduce additional friction. The inertia may also be varied.

The mathematical model developed to represent the mechanical system is shown in Fig. 2. Basically this model consists of a rotating load supported on a base by a friction element. The friction element is represented by a pair of contacting surfaces with conventional nonlinear friction in series with a nonlinear torsional spring. The nonlinear spring represents the elastic deformation effect. When this spring has infinite stiffness, the model reduces to the conventional model for friction. For the infinite stiffness condition, torque applied to the rotating member at levels below the breakaway level of the friction will not cause motion of the load. The resulting "dead space" has serious implications with respect to system performance and stability. However with the inclusion of the spring,

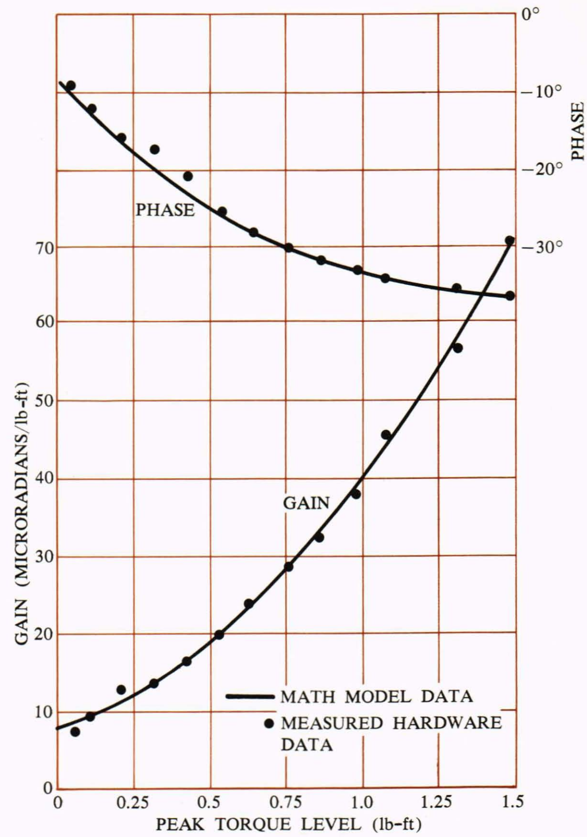

Fig. 3—Sinusoidal steady-state phase and gain response versus peak torque level (frequency = 3 Hz).

torque applied to the load will cause motion owing to compliance of the spring. Thus the "dead space" is circumvented, and smoother, more accurate performance is obtained.

The windup angle of the nonlinear spring was found to be a function of the absolute value of the change of torque, as indicated by the nonlinear differential equation of Fig. 2. The transfer resulting from this representation exhibits a hysteresis characteristic with attendant energy loss. The power series coefficients C_k have been selected to match the measured data from the mechanical hardware. An additional small hysteresis characteristic that is not accounted for by this equation has also been attributed to the spring.

The steady-state sinusoidal response of the experimental hardware was measured using a servo analyzer to extract the fundamental component of shaft motion. Corresponding data were obtained on the mathematical model using describing function techniques. Samples of the calculated and measured results for applied torque less than the

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breakaway friction level are compared in Fig. 3. These data show excellent correlation between model predictions and measured hardware data over a wide range of torque levels. The conventional friction model, which neglects elastic deformation, would yield no motion for these torque levels. The model has also been verified by comparison with measured hardware data over a wide range of frequencies (1 to 1000 radians per second).

PUBLICATIONS

Compilation of principal recently published books and technical articles written by APL staff members.

A. G. Lubowe (Bell Tel. Labs.) and R. E. Jenkins, "Numerical Verification of Analytic Expressions for the Perturbations Due to an Arbitrary Zonal Harmonic of the Geopotential," *Celestial Mechanics* **2**, No. 1, May 1970, 21-40.

E. L. Cochran, F. J. Adrian, and V. A. Bowers, "Electron Spin Resonance Study of Elementary Reactions of Fluorine Atoms," *J. Phys. Chem.* **74,** No. 10, May 14, 1970, 2083–2090.

E. B. Dobson, "Measurement of the Fine-Scale Structure of the Sea," *J. Geophys. Res.* **75,** No. 15, May 20, 1970, 2583–2586.

J. R. Kuttler, "Finite Difference Approximations for Eigenvalues of Uniformly Elliptic Operators," *SIAM J. Numer. Anal.* **7,** No. 2, June 1970, 206–232.

Y. J. Alloucherie, "Light Conversion Efficiency of Calcium Tungstate Using a Light Intensifier Tube," *Appl. Optics* **9,** No. 6, June 1970, 1403–1409.

F. W. Schenkel, "Design Consideration for Infrared Imaging in the 10–12.6 Micron Band from a Synchronous Earth Satellite," *J. British Interplanetary Soc.* **23,** June 1970, 423–440.

M. H. Friedman, "Free Swelling of Biological Tissue: The Corneal Stroma," *Chem. Eng. Symp. Series* **66,** No. 99, 1970, 33–42.

L. R. Gieszl, "Continuous Flow Charts," *Simulation* **14,** No. 6, June 1970, 281–289.

R. E. Walker and B. F. Hochheimer, "Inversion-Rotation Emission Spectrum of Thermally-Excited NH_3 in the 60–200-cm^{-1} Region," *J. Molec. Spectroscopy* **34,** No. 3, June 1970, 500–515.

P. M. Bainum, P. G. Fuechsel, and D. L. Mackison, "Motion and Stability of a Dual-Spin Satellite with Nutation Damping," *J. Spacecraft and Rockets* **7,** No. 6, June 1970, 690–696.

J. M. Whisnant, D. K. Anand, V. L. Pisacane, and M. Sturmanis, "Dynamic Modeling of Magnetic Hysteresis," *J. Spacecraft and Rockets* **7,** No. 6, June 1970, 697–701.

S. M. Krimigis, "Alpha Particles Trapped in the Earth's Magnetic Field," *Particles and Fields in the Magnetosphere*, edited by B. M. McCormac, Dordrecht, Holland: D. Reidel Publishing Co., 1970.

ADDRESSES

Principal recent addresses made by APL staff members to groups and organizations outside the Laboratory.

W. Seamone, "Control of an Externally Powered Upper Limb Prosthesis," *IEEE Washington Chapter*, McLean, Virginia, May 4, 1970.

W. H. Avery, "Transportation," *The Johns Hopkins University, School of Hygiene Seminar*, Baltimore, May 5, 1970.

W. H. Avery, "An Overview of the Basic Technology and the Economics of Advanced Transportation," *Carnegie-Mellon Institute, Transportation Research Institute Conference on Urban Transportation Systems*, Pittsburgh, May 26, 1970.

W. H. Guier, "Cardiovascular Sensors in the Clinical Environment," *National Bureau of Standards Transducer Conference*, Gaithersburg, Maryland, May 1970.

The following two lectures were presented at the *Polytechnic Institute of Brooklyn, Phased Array Symposium,* Brooklyn, New York, June 2–5, 1970:

E. V. Byron, "A New Flush Mounted Antenna Element for Phased Array Application;"

T. C. Cheston, "Beam Steering of Planar Phased Arrays."

M. H. Friedman, "Computer Experiments on the Cornea, "*Summer Computer Simulation Conference*, Denver, June 12, 1970.

B. E. Tossman, "Low Nutation Rate Dampers," *Fifth Aerospace Mechanisms Symposium*, NASA Goddard Space Flight Center, Maryland, June 15, 1970.

APL COLLOQUIA

May 1—"The Present Status and Future Plans of the N.A.L.," by T. L. Collins, National Accelerator Laboratory.

May 8—"Air Pollution and Global Climatic Change," by J. M. Mitchell, Environmental Sciences and Services Administration.

May 15—"Technical and Economic Evaluation of Urban Transportation Systems," R. A. Makofski, Applied Physics Laboratory.