SCALING *the effects of* AIR BLAST *on typical* TARGETS

The general scaling equations presented in this article are based on interactions between shock waves generated by explosions in air and targets that can be fully defined by two parameters in a simple mathematical model. Simplicity is achieved by ignoring many factors that complicate rather than illuminate the study of blast phenomena. These scaling equations, and the analytic techniques based on their use, are not intended to be precision tools for computing specific effects with maximum accuracy, even though they are often as accurate as far more complicated computational techniques. Their unique value lies in their use of a dimensionless, universal scaling parameter whose values span the entire spectrum of blast damage phenomena, from the effects of a few pounds of conventional high explosives to those of megatons of equivalent TNT. The reader is cautioned, however, not to expect real targets to behave precisely like the simplified models from which the scaling equations were derived, but he is encouraged to use the equations freely as a means of correlating isolated pieces of data with the basic analytic structure of blast damage relationships.

This analysis deals with targets and the interactions between them and the shock waves generated by the detonation of explosives. It limits its attention to that fraction of the total range of scaled distances within which a variety of actual targets, varying widely in toughness, have been destroyed by air blast. Within this limited range, an abundance of excellent experimental data show that shock waves behave in an orderly fashion allowing the parametric relationships that define their characteristics to be expressed mathematically by two simple, tractable equations.

Since the purpose of this article is to derive scaling equations for interactions between shock waves and targets that result in target "kills," these equations will be used to compute the effects of interactions only within the range of scaled distances at which such kills are possible. From the characteristics of shock waves it is possible to derive scaling equations of classic simplicity in which specific target characteristic data ensure that the equations will be valid for the range in which shock waves can and do kill such targets. The universal scaling parameter derived in this article allows the equations defining the blast-shock wave characteristics to be converted successfully into equations that scale the interactions between shock waves and targets.

Nature of Shock Waves

Detonation of a high explosive in air produces a sudden rise in pressure which propagates rapidly as a spherically expanding shock wave. When the shock wave arrives at some fixed point in its path, there is an almost instantaneous rise in pressure to a peak value, followed by a gradual decline until it reaches

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Interactions between shock waves, produced in air by detonation of explosives, and specific targets which they can destroy by air blast are described. A mathematical analysis is used to relate weights of explosives to the distances at which they can cause lethal damage over the entire range of blasts from a few pounds of conventional high explosive to kilotons or megatons of nuclear blast. Effects at sea level and higher altitudes are examined. In the analysis, typical targets are defined by two parameters for which specific numerical values can be established. A dimensionless scaling parameter relating a shock wave parameter to a target parameter is the key to the scaling relationships derived.

the original ambient pressure level at some later time which marks the end of the first positive impulse. The pressure then drops below ambient, producing a negative impulse, and then may be followed by a much weaker second positive impulse.

Shock wave data are usually given in terms of "free-air" or "side-on" values of overpressure (pressure above ambient), in which it is assumed that no obstacle impedes the free motion of the shock wave. When a solid surface presents an area normal or nearly normal to the direction of motion of the shock wave, the shock wave is reflected, producing a substantial increase in overpressure. For a very weak shock, including the limiting case of an acoustic wave, the overpressure for normal incidence is twice the "free-air" value. For very strong shocks in air ($\gamma = 1.4$), the ratio of the overpressure for normal incidence to the "free-air" value approaches an upper limit of eight.

Because this article deals with the interaction between shock waves and targets, we are not concerned with "free-air" conditions, but will deal with overpressures and impulses that act on typical target surfaces. It has been found that the overpressure near a surface reflecting a shock wave is substantially constant for angular orientations up to about 35° from the normal. Since a typical target, such as an aircraft, is irregular in shape and has elements of surface at all possible angular orientations, it is safe to assume that the target presents substantial areas with orientations within 35° of the normal for which the pressures acting on the target are essentially normally reflected overpressures. For this reason, the pressures, impulses, and other shock parameters discussed subsequently in this article will refer to face-on or normally reflected shock waves.

Shock Wave Characteristics

A shock wave whose pressure-time history is shown in Fig. 1 may be defined by the following parameters:

P = peak overpressure (psi); the difference between the absolute peak pressure and the ambient pressure.

T = time (msec) during which the pressure of the shock wave is continuously greater than the ambient pressure. T is called the duration of the positive impulse.

p = ambient air pressure (atm), i.e., $p = 1$ at sea level (14.7 psi).

$\underline{p}$ = overpressure of the shock wave at time t.

I = total positive impulse, given by $I = \int_o^T \underline{p} dt$.

This is sometimes termed the first positive impulse,

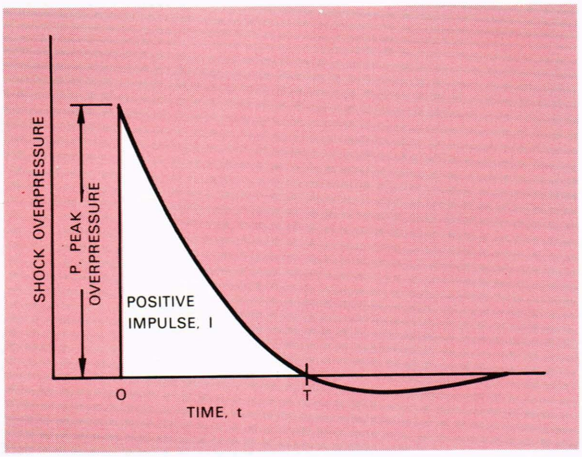


Fig. 1 — Pressure-time curve for a normally reflected shock wave.

since there may be a second, much weaker, positive impulse. In practice, the pressure variations beyond T are sufficiently small that they can be neglected in the analysis of blast damage.

The shock wave from a high explosive is a function of the distance from the detonation, the nature and size of the charge, and the pressure and temperature of the air through which the wave is propagated. It is possible to correlate the effect of different sizes of explosive charges on shock waves by using a scaled distance, Z, defined by

$$Z = \frac{R}{W^{1/3}} , \tag{1}$$

where:

- R = distance (feet) from the center of the detonation; and
- W = weight (pounds) of the explosive charge.

According to this scaling law, for a given value of scaled distance Z, the same peak overpressure would be obtained by the detonation of two different weights of a particular explosive; for example, the explosion of 10 pounds of TNT at a distance of 20 feet will produce the same peak overpressure as 80 pounds of TNT at a distance of 40 feet.

The relationships between peak overpressures, positive impulses, scaled distances, and atmospheric environments differ from one explosive to another. Since 50/50 Pentolite is a military high explosive which gives consistently reproducible results, it is often used as a standard explosive for evaluating the effects of air blasts. Furthermore, it is an explosive for which abundant and accurate data are available.[1]

[1] H. J. Goodman, "Compiled Free-Air Blast Data on Bare Spherical Pentolite," Aberdeen Proving Ground, Ballistic Research Laboratory, BRL, Report No. 1092, February 1960.

For these reasons this analysis has been built around the characteristics of 50/50 Pentolite.

The data for normally reflected peak overpressures for 50/50 Pentolite are shown as a function of the scaled distance in Fig. 2. The straight line fitted to these data points is given by

$$\frac{P}{p} = \frac{13{,}300}{[Zp^{1/3}]^3} . \tag{2}$$

In similar fashion the data for normally reflected positive impulses for 50/50 Pentolite are shown as a function of the scaled distance in Fig. 3. The straight line fitted to these data points is given by

$$\frac{I}{W^{1/3}p^{2/3}} = \frac{220}{[Zp^{1/3}]^{1.20}} . \tag{3}$$

These two analytically tractable equations, (2) and (3), define the characteristics of 50/50 Pentolite over the range of scaled distances with which this analysis is concerned. Anomalies which occur at scaled distances less than two or greater than 10 or 12 have no bearing on the accuracy and validity of these equations since they lie outside the range of parameters under consideration.

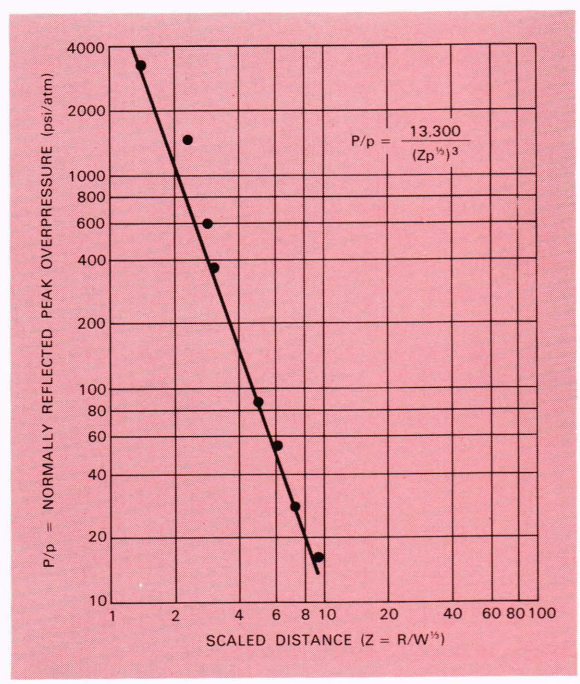


Fig. 2 — Normally reflected peak overpressure for 50/50 Pentolite spheres.

Equation (3) which is fitted to empirical data obtained at sea level (p = 1.00) indicates the qualitative effect of p on the impulse. At higher altitudes tem-

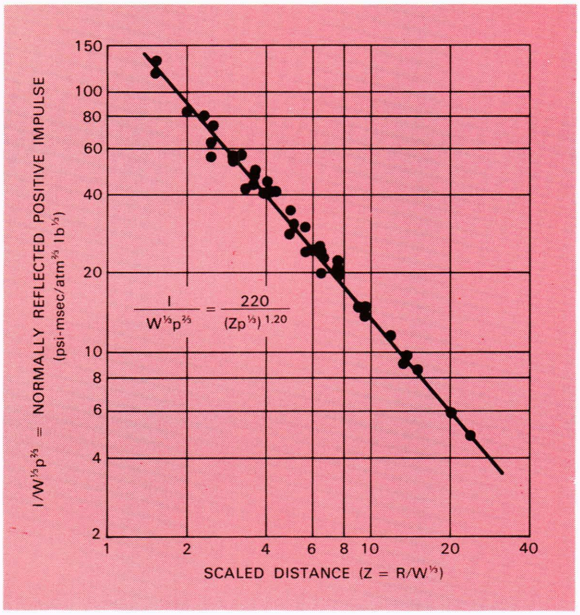


Fig. 3 — Normally reflected positive impulse data for 50/50 Pentolite spheres.

perature changes will affect the speed of sound and require a quantitative adjustment. These changes, which are independent of p, are accounted for by including the parameter, c/c_o, the ratio of the velocity of sound in air at the altitude to the velocity at sea level to obtain the impulse equation,

$$\frac{I\left(\frac{c}{c_o}\right)}{W^{1/3}p^{2/3}} = \frac{220}{[Zp^{1/3}]^{1.20}} . \tag{4}$$

Using Eq. (1) to eliminate Z from Eqs. (3) and (4), the following equations are obtained for the "normally reflected" peak overpressure and positive impulse:

$$P = \frac{13{,}300\ W}{R^3} , \tag{5}$$

and

$$I = \frac{220\ W^{0.733}p^{0.267}}{\left(\frac{c}{c_o}\right) R^{1.20}} . \tag{6}$$

Characteristics of a Target

Aircraft structures, which are typical targets for air blast, consist of superficial coverings of comparatively thin material supported by rather complex assemblies of ribs, stiffening webs, and braces of various sorts that are attached to the basic framework. Target damage severe enough to constitute a kill consists of substantial crumpling and distortion of the surface in the course of which it moves a considerable distance against the resisting forces offered by the structural elements which yield without breaking. The behavior of a target cannot be fully simulated, but a remarkably useful mathematical model can be defined in simple terms. Pressures below a certain minimum value will have no effect whatever on the target regardless of how long they are maintained. However, at some critical pressure the supporting structure will begin to yield, and if this pressure is maintained, the distortion will increase until the target is totally destroyed.

Therefore, it is possible to define a fundamental parameter representing target toughness as:

P_m = minimum pressure (psi) on the target surface which will initiate destructive distortion and which, if continued long enough, will cause target destruction.

The mathematical model used in this analysis assumes a constant value of P_m throughout the period of distortion up to the point of destruction. Many real targets approximate this behavior closely enough to consider P_m a statistically significant measure of target toughness.

A second target characteristic parameter which is important in the analysis is I_m, which is defined as the lower limit for the value of the impulse which can deform the target enough to destroy it. The parameter I_m is closely related to the total work done in deforming the target to the point of destruction.

Universal Scaling Parameter

This analysis is concerned with scaling the *interactions between shock waves and targets.* A scaling parameter for this purpose should be a dimensionless ratio between some distinctive characteristic parameter of the target and a similar distinctive characteristic parameter of the shock wave. The only characteristic common to both is pressure.

The significant pressure for the target is the *minimum* pressure, P_m, i.e., the pressure which causes target damage; for the shock wave it is the *maximum* pressure, P, which it can exert on the target. Hence, a universal scaling parameter, is defined by the equation,

$$\epsilon = \frac{P_m}{P} . \tag{7}$$

No target damage can occur if ϵ is greater than 1.00 and no real target, in which there is any finite resistance to deformation, can have an ϵ value of zero. The entire gamut of shock wave target combinations, which produce target kills, is covered by the open interval $0 < \epsilon < 1$. Within these limits, ϵ spans the entire spectrum of charge weights (and corresponding lethal distances) from a few pounds of conventional high explosive to nuclear devices whose yield is measured in kilotons or megatons of equivalent TNT.

In the scaling equations derived in this article, the characteristics of the explosive, the target, and the ambient atmospheric conditions (altitude and ratio of sound velocities) are all defined by appropriate numerical values of the parameters. Every set of conditions represented by these parameters corresponds to a unique weight of explosive, and the unique lethal distance associated with it. Therefore, ϵ is a universal scaling parameter for the interactions between shock waves and the targets that they are capable of killing when the shock waves are spherical and the targets are correctly defined by the parameters, P_m and I_m.

Interaction Between a Shock Wave and a Target

The target is defined by two parameters to which numerical values can be assigned. The target model accurately simulates the behavior of many real targets whose destruction by blast has been observed under test conditions. Associated with a target is a minimum pressure exerted on its surface below which threshold there will be no damage. It is assumed that when this critical minimum pressure is reached, the supporting elements will begin to yield and permanent deformation will be initiated. It is further assumed that as long as deformation (consisting of displacement of the surface relative to the more massive elements of the basic structure) continues, it will be resisted by a constant force per unit area of the same magnitude as the pressure which initiated destructive deformation. Every target will have to be deformed by some definite minimum displacement of the surface relative to the basic structure before its usefulness is destroyed, i.e., a certain minimum target distortion, or a certain minimum work per unit area, must be expended on the target to destroy it.

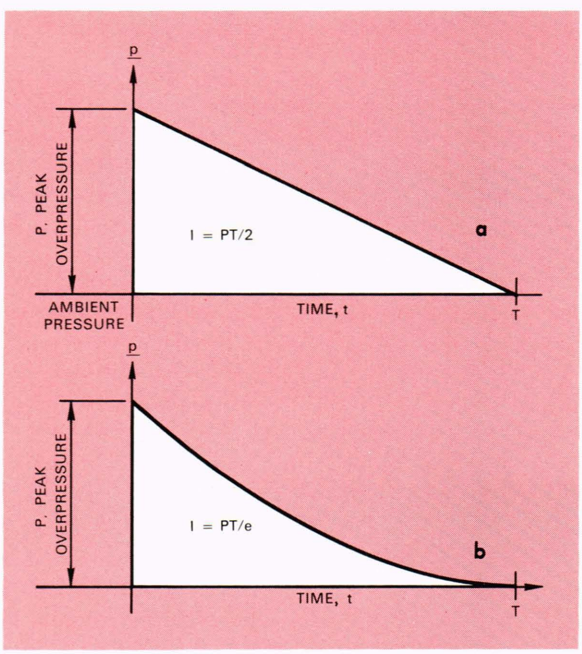


Fig. 4 — Pressure-time curves.

It is necessary then to derive functions for the total work per unit area any given shock wave can impart to the target, and to specify the characteristics of the shock wave which can impart just enough work to achieve target destruction.

It is now necessary to have an equation for the pressure-time profile of a shock wave acting against a target surface. This equation will give values of the overpressure as a function of time $(t < T)$. Two mathematical models for the pressure-time curves have been used, the linear profile shown in Fig. 4a with

$$\underline{p} = P\left(1 - t/T\right), \tag{8}$$

where

t = time from the first rise in pressure
$\underline{p}$ = overpressure at time $t\ (0 < t < T)$;

and the exponential profile shown in Fig. 4b with

$$\underline{p} = P\left(1 - t/T\right)e^{-t/T} \tag{9}$$

It can be shown that a completely self-consistent set of scaling equations can be derived from either of these pressure-time profiles and that the same relationships between charge weights and corresponding lethal distances are derivable from either equation. The linear relationship has been used in several investigations with satisfactorily consistent results. However, self-consistency within a mathematical model is not sufficient to assure that values of ϵ, as well as the values of such target parameters as P_m, portray realistic interaction characteristics.

The pressure-time relationships shown in Fig. 4 differ from one another in the relationship of I to PT. For the linear curve (Fig. 4a), the ratio is 1/2. For the exponential curve, it is $1/e$ or 0.368. The data for 50/50 Pentolite show that the ratio of I to PT is not constant but varies at a moderate rate with scaled distance. Furthermore, it was found that the ratio $1/e$ of the exponential function, Eq. (9), agrees

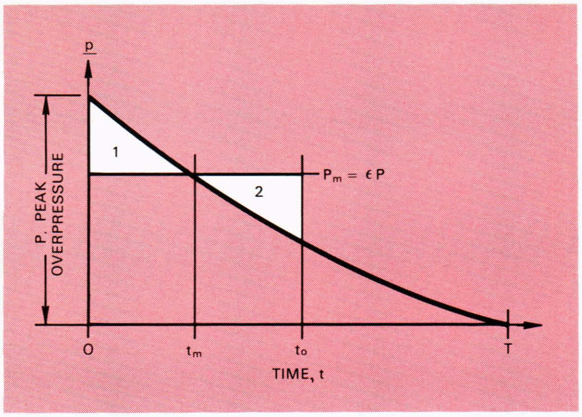


Fig. 5 — Pressure-time relationship for $\epsilon \geq 1/e$.

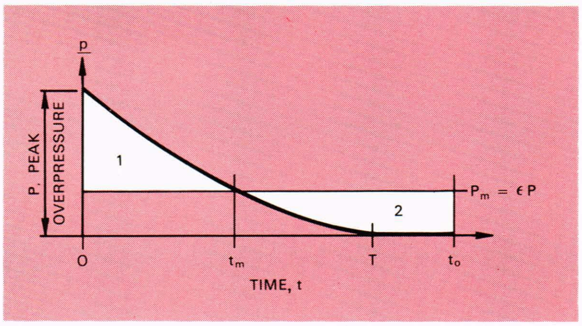


Fig. 6 — Pressure-time relationship for $\epsilon \leq 1/e$.

with the 50/50 Pentolite data in the midrange of scaled distances used in this analysis. Since an accurate mathematical formulation of the actual pressure-time profile of a shock wave produced by 50/50 Pentolite over the full range of scaled distances has not yet been found, the exponential function is the best representation within the range of scaled distances covered by this analysis.

On a pressure-time diagram, impulses are represented by areas. Figures 5 and 6 will show that areas above P_m, (designated by 1) are positive impulses which accelerate the mass, m, to a maximum velocity at time, t_m, and areas below P_m (designated by 2) are negative impulses which bring m to rest at some later time, t_o. Obviously the two impulses must be equal.

In setting up equations for the acceleration of the mass, m, (mass per unit area of target material which is moved relative to the basic target structure where destructive deformation takes place) one notes that between zero and T there is a constantly varying acceleration for which the pressure-time equation, Eq. (9), gives an analytical value, but beyond the time, T, the negative acceleration has a constant value.

Interactions which are completed within the period *(T)* of the positive impulse can be treated by a single set of equations and will be designated Case I (see Fig. 5). Interactions which continue beyond the end of the positive impulse will require two sets of equations, one with a constantly varying acceleration (up to time, *T*), and one with a constant acceleration (beyond time, *T*). This will be called Case II (see Fig. 6).

The results of some lengthy calculations[2] lead to the following equations for the impulse I of the shock wave up to time T, in terms of I_m, and the universal scaling parameter ϵ:

For Case I $\left(\epsilon \geq \frac{1}{e}\right)$:

$$I = \frac{I_m}{e\epsilon^{1/2}\left[2 - \epsilon - \epsilon(1 - \ell n\,\epsilon)^2\right]^{1/2}} \tag{10}$$

For Case II $\left(\epsilon \leq \frac{1}{e}\right)$:

$$I = \frac{I_m}{\sqrt{1 - 1.5315\epsilon}} \; . \tag{11}$$

Figure 7 relates the shock wave pressures and impulses to the minimum pressures and impulses which can damage a given target. This curve is the locus of all possible overpressure-impulse combinations which can just destroy (without overkilling) any given target characterized by P_m and I_m.

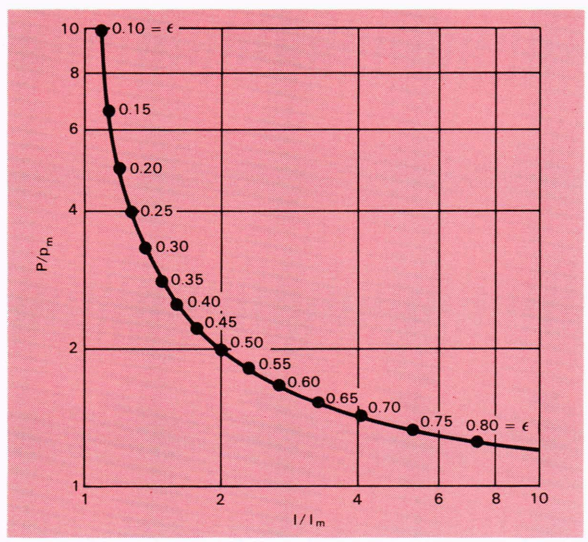


Fig. 7 — General pressure-impulse relationship.

[2] H. S. Morton, "Scaling the Effects of Air Blast on Typical Targets," APL/JHU Report TG-733, January 1966.

General Parametric Scaling Equations for Weight and Distance

Equations (5) and (6) can be solved simultaneously for weight, W, and distance, R, to obtain

$$W = \frac{0.0083 \left(\frac{c}{c_o}\right)^3 I^3}{P^{1.20}\, p^{0.80}} , \quad (12)$$

and

$$R = \frac{4.8 \left(\frac{c}{c_o}\right) I}{P^{0.733}\, p^{0.267}} . \quad (13)$$

When these are combined with Eqs. (7), (10), and (11) one obtains

$$W = 0.0083 \left[\frac{I_m^3}{P_m^{1.20}} \; \frac{\left(\frac{c}{c_o}\right)^3}{p^{0.80}}\right] F_W(\epsilon) , \quad (14)$$

and

$$R = 4.8 \left[\frac{I_m}{P_m^{0.733}} \; \frac{\left(\frac{c}{c_o}\right)}{p^{0.267}}\right] F_R(\epsilon) . \quad (15)$$

In this pair of equations, the final terms, $F_W(\epsilon)$ and $F_R(\epsilon)$ are restricted to ϵ values above or below $1/e$.

For $\epsilon \leq \frac{1}{e}$

$$F_W(\epsilon) = \frac{\epsilon^{1.20}}{(1 - 1.5315\epsilon)^{3/2}} ,$$

and

$$F_R(\epsilon) = \frac{\epsilon^{0.733}}{(1 - 1.5315\epsilon)^{1/2}} .$$

For $\epsilon \geq \frac{1}{e}$

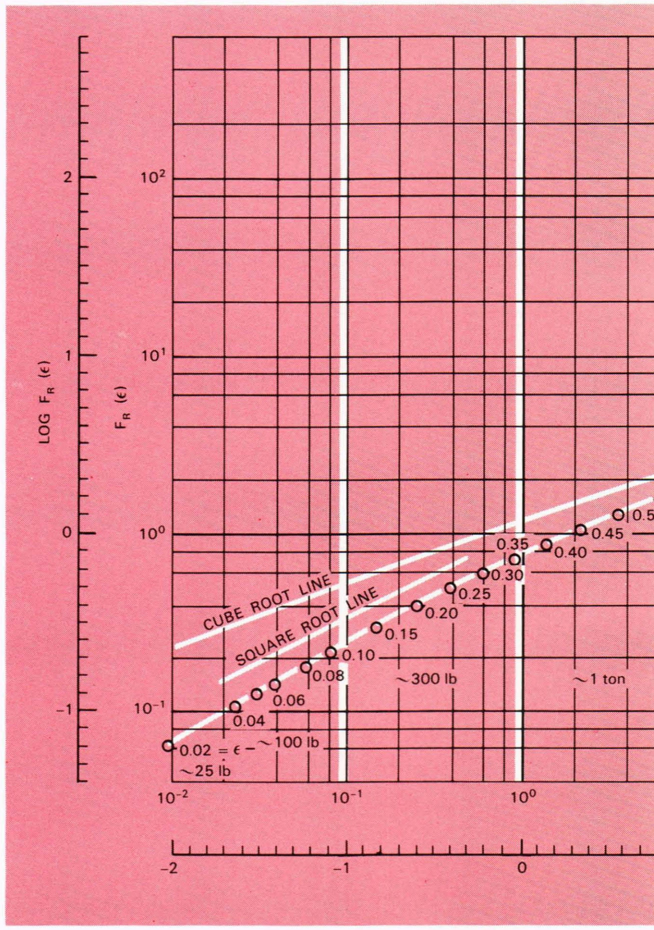
$$F_W(\epsilon) = \frac{1}{e^3 \epsilon^{0.30} [2 - \epsilon - \epsilon(1 - \ell n\epsilon)^2]^{3/2}} ,$$

and

$$F_R(\epsilon) = \frac{\epsilon^{0.233}}{e[2 - \epsilon - \epsilon(1 - \ell n\epsilon)^2]^{1/2}} .$$

These two pairs of functions of ϵ are related as:

$$F_R(\epsilon) = \left[F_W(\epsilon)\right]^{1/3} \epsilon^{1/3} .$$



This equation is valid for all values of ϵ.

The general parametric equations, (14) and (15), consist of four terms: the first is a numerical coefficient characteristic of the explosive (50/50 Pentolite) and would be different for different explosives; the second is a function of the target; the third is a function of the altitude; and the fourth is a function of the universal scaling parameter, ϵ. For any particular explosive, target and altitude, the first three terms are constants, i.e., only the functions of ϵ will vary.

The two general parametric equations, (14) and (15), for weight, W, and lethal distance, R, give all the necessary information with respect to interactions between spherical shock waves produced by explosive detonations and targets whose characteristics defined by P_m and I_m can be determined by experiment. The range of scaled distances must be sufficient to encompass the region in which targets of practical concern can be destroyed. These two equations epitomize the fundamental analytic struc-

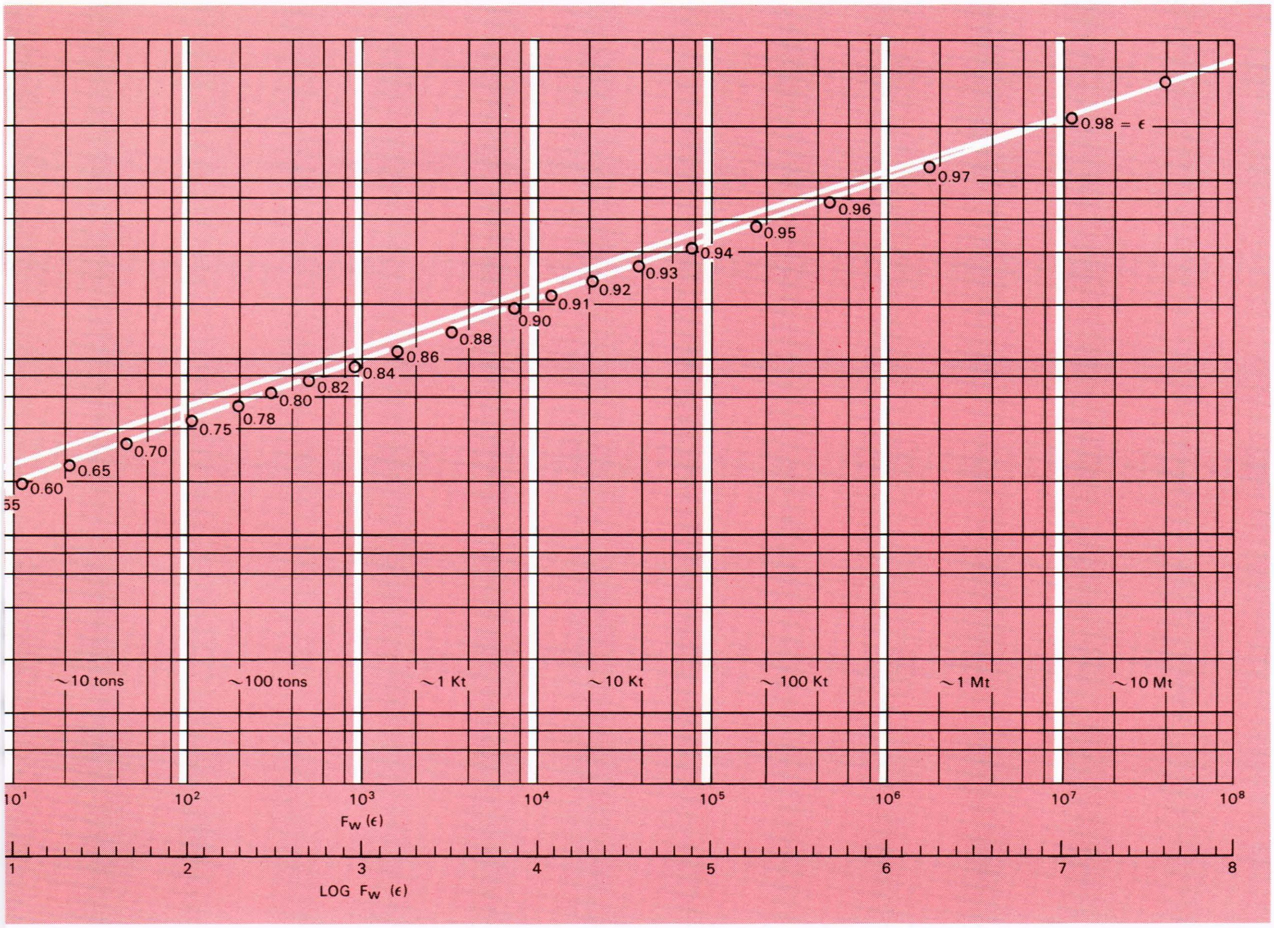


Fig. 8 — General distance-weight relationship.

ture of shock wave-target interactions, and are the most significant product of this entire article.

A graph of the $F_R(\epsilon)$ versus $F_W(\epsilon)$ values is shown in Fig. 8, with corresponding values of ϵ given for various points along the curve. A rough indication of the order of magnitude of the charge weight is shown on Fig. 8 from which it may be seen that over this range of ϵ the charge weights vary considerably—from 25 pounds to 10 megatons.

Two other lines are shown on Fig. 8. The first is a line of slope 1/3 which becomes tangent to the general curve as ϵ approaches 1.00 and $F_W(\epsilon)$ approaches infinity. This is consistent with the relationship $R \propto W^{1/3}$ which has been shown to be true for nuclear charges. The second is a line of slope 1/2 which is seen to be parallel to the general curve at values of ϵ around 0.20 to 0.25 and is consistent with the $R \propto W^{1/2}$ relationship known to apply to charges of a few hundred pounds. The change in the slope of the curve of log $F_R(\epsilon)$ versus log $F_W(\epsilon)$ is the most significant *scale effect* in relationships between charge weights and lethal distances.

Comments and Conclusions

This analysis has avoided the necessity of treating the full range of shock wave phenomena, and particularly the regions in which parametric relationships are far too complicated to fall into any simple pattern. By limiting its scope to the region in which parameter relationships can be expressed in simple analytic terms it fills a long-felt need for a direct and easy method of scaling blast effects on specific targets.

Fortunately a majority of targets of interest will fall well within the applicable range of the equations derived in this report. As soon as the parameters which define a target are known, it is a simple matter to determine the corresponding scaled distances and ascertain whether they are within the applicable range of the equations.

As a concluding statement, since the phenomena to be scaled are *interactions between shock waves and targets,* the parameter, ϵ, which relates a common characteristic of the two, is the most useful analytic tool for scaling these effects.