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Quantitative evaluation of the usefulness and effectiveness of fleet shipboard guided-missile defense systems is essential for assessment of present and future fleet defense capability. The measure discussed, called equipment availability, is the average probability that an equipment will perform in a specified manner at some random point in time. Such variable parameters as equipment failure, repair time, practical operating conditions, and logistics are taken into account.

AVAILABILITY MODEL for Shipboard Equipment

Guided-missile weapon systems constitute a significant fighting force for defense of the fleet. Quantitative evaluation of the usefulness and effectiveness of these systems is important in long-range planning of future systems, in assessment of the status of current systems, and in assessment of improvements. In particular, a project engineer has a need to quantify his area of concern in terms of the ultimate intended use of a system so that various engineering changes and improvements can be compared in terms of cost/effectiveness. In this article, we develop a mathematical model that gives an effectiveness measure of shipboard equipments, taking into account practical operating conditions, equipment failure, and repair phenomena. During the development of the model, we show the effect of changes in parameters that determine effectiveness.

The measure developed in this article is called *equipment availability*. Availability is defined here as the average probability that an equipment will perform in a specified manner at some random point in time. The major interest is in what we shall term tactical availability; that is, the chance that an equipment will perform satisfactorily when involved in a tactical environment. Many authors have discussed at some time the term *availability* under the guise of reliability, readiness, dependability, or some other name. Many of the models developed under these names are useful for particular equipments that characteristically operate in peculiar ways. The availability model developed here is oriented toward tactical shipboard equipment operation. As we define availability, it does not take into account what is often referred to as "mission reliability" or the probability that an equipment or system will perform throughout an engagement period. Results of other availability models are liberally drawn upon insofar as they pertain to our model.[1,2]

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[1] R. E. Berlow and L. C. Hunter, "Mathematical Models for System Reliability, "*Sylvania Technologist*, Jan. 1960.

[2] P. M. Morse, *Queues, Inventories and Maintenance*, John Wiley and Sons, Inc., New York, 1958.

Measures of Availability

Our discussion begins by considering some practically desirable and simple measures of availability, pointing out the shortcomings in these measures. Ideally, if a particular tactical use of a weapon system were defined in terms of particular threat characteristics, one of the best measures of the success of the system against this particular threat could then be obtained by conducting repeated tests incorporating this threat against the weapon system. If we then ran the test n times, and the weapon system successfully achieved its goal s times, we could say that s/n, the fraction of times that the system was successful, is a measure of the usefulness of the system.

There are, of course, many possible types of threat that a surface-to-air guided missile weapon system may encounter; therefore, to measure adequately the usefulness of the system, many tests must be conducted. Unfortunately, organizing tests on this scale is difficult. In particular, the cost of such tests can often become as high as that of the system itself. Tests of this nature are really concerned with exercising the entire weapon system. Experience with complex systems has shown that it is in fact necessary to test the system as a whole rather than to try to combine the results of tests of its components.

On many occasions, equipments have been reported as capable of a satisfactory level of performance, but, when actually put to a system test, have been found to be unavailable. This difficulty is usually referred to as an interface problem. In this context, the interface problem means that even though an equipment is apparently operational by itself, its functions are impaired when used with the other required equipments in the system.

We are then faced with the following dichotomy. First, the cost of testing the complete system is prohibitive, and second, the lack of a complete system test fails to check equipment interfaces. Compromises can be made, however. The system can be broken down into subsystems, each consisting of perhaps several equipments that can be tested together, thus taking into account the interfaces between them. The subsystem test can often be done at reasonable cost. Our compromise consists, then, of a test that is within reasonable cost constraints and that gives a high confidence in the operability of the system at the end of the test, but which at the same time is not completely definitive. The model developed here takes this compromise into account, which leads to an incomplete test. Inputs to the model depend on a systematized failure and repair reporting system, as well as engineering analysis of the way in which equipments are monitored during normal operation.

Some authors use as a measure of equipment effectiveness the percentage of time an equipment is reported as operational. That is, effectiveness or availability is

$$\frac{Up\ Time}{Up\ Time + Down\ Time}. \tag{1}$$

For some equipments this is indeed a practical measure of effectiveness. For example, some long-range search radars used in continental air defense can be monitored in such a way that they are either operating or down for repair; no other conditions are possible. Simply recording these times, then, is sufficient to obtain a measure of effectiveness. Unfortunately, most shipboard equipments cannot be considered in this manner. Sometimes Eq. (1) is written as

$$\frac{MTBF}{MTBF + MTTR}, \tag{2}$$

where $MTBF$ is *mean time between equipment failures* and $MTTR$ is the *mean time to repair*. It should be understood here and in what follows that by failures we mean those phenomena that prevent the equipment from performing in some specified manner.

Basic Mathematical Development of Availability

We will now develop an expression for availability, which has been used by many authors,[1, 2] and show that the limiting value of this expression is simply that contained in Eq. (2). The method of derivation is by the use of difference equations. We define λ as the equipment failure rate (the reciprocal of the mean time between failures) and μ as the repair rate (the reciprocal of mean time to repair). Further, $P(t)$ is defined as the probability that an equipment will perform at time t. If λ and μ are independent of time, then in a *small time interval* Δt, the probabilities of failure and repair are respectively $\lambda \Delta t$ and $\mu \Delta t$. The expression for the probability that an equipment is up at time $t + \Delta t$ can then be written

$$P(t + \Delta t) = P(t)[1 - \lambda \Delta t] + [1 - P(t)]\mu \Delta t.$$

Rearranging terms and taking the limit as $\Delta t \to 0$ gives

$$P'(t) + P(t)(\mu + \lambda) = \mu,$$

where $P'(t)$ is the derivative of $P(t)$ with respect to time. The solution of this equation is

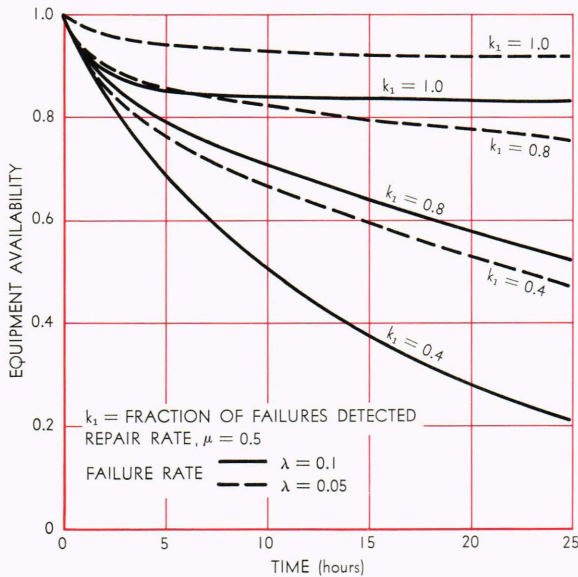$$P(t) = \frac{\mu}{\mu + \lambda} + c \exp\left[-(\mu + \lambda)t\right]. \tag{3}$$

Fig. 1—Equipment availability as a function of time, for selected values of equipment failure rate and repair rate.

To determine c, the constant of integration, it is assumed that the equipment is operating at the start of the interval. That is, $P(0) = 1$. With this assumption,

$$c = \frac{\lambda}{\mu + \lambda}.$$

Thus,

$$P(t) = \frac{1}{\mu + \lambda} \{\mu + \lambda \exp [-(\mu + \lambda)t]\}. \quad (4)$$

We will define availability as

$$A = \int_0^\tau P(t) \, dN(t).$$

In this expression, τ is the length of time under consideration and $N(t)$ is the distribution function for the time of need of the equipment. We will assume that the equipment is equally likely to be needed any time in the interval $0 \leq t \leq \tau$; that is,

$$N(t) = \begin{cases} 0 & (\text{for } t \leq 0) \\ t/\tau & (\text{for } 0 \leq t \leq \tau) \\ 1 & (\text{for } \tau \leq t). \end{cases}$$

This gives $A = \dfrac{1}{\tau} \displaystyle\int_0^\tau P(t)\,dt.$

Finally,

$$A = \frac{\mu}{\mu + \lambda} + \frac{\lambda}{\tau(\mu + \lambda)^2} \{1 - \exp [-(\mu + \lambda)\tau]\}.$$

The curves labelled $k_1 = 1.0$ on Fig. 1 (where k_1 is the fraction of failures detectable during normal operation) show $P(t)$ as a function of time for indicated values of μ and λ. Notice that if we take the limit of $P(t)$ as $t \rightarrow \infty$, then the availability becomes simply $\mu/(\mu + \lambda)$, which is equal to Eq. (2). Figure 2 shows availability as a function of $MTTR$ for several values of $MTBF$ for the steady-state case. This expression is particularly useful for equipments which have perfect monitoring; that is, equipments in which failures are discovered as they occur. The resulting expression is simple, and the mathematics used in the derivation is quite straightforward. The assumptions of time independence for λ have been proved by experience to be valid. However, experience indicates that the distributions of repair rates are log normal. The effect of this difference has not been evaluated.

Availability Problems

The problem that is most obvious to the analyst or engineer concerned with shipboard equipments is that such equipments can be in a non-operating state without the fact being apparent to operating personnel. This is, in fact, one of the major stumbling blocks in measuring availability. We have introduced this phenomenon into the availability model in the following way. Since some equipment failures may be seen in the normal course of operation and others not until periodic tests, we will consider the equipment as consisting
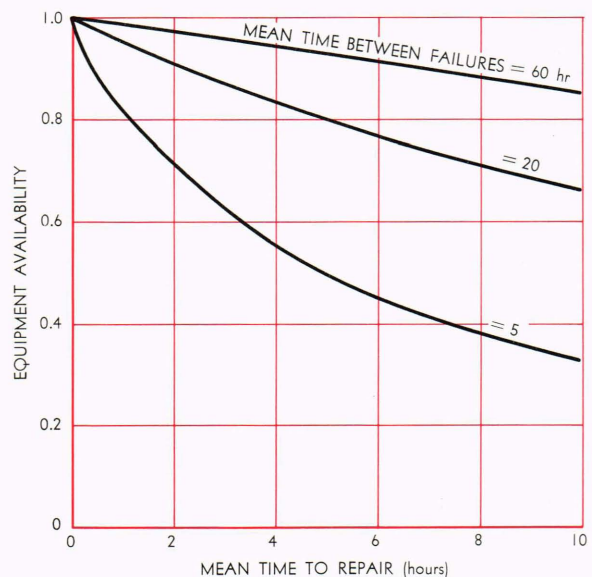

Fig. 2—Equipment availability as a function of mean time to repair, based on selected values of mean time before failure.

of two sections. In the first, failure of a part can be observed immediately. In the second, a part that fails is not observed until a system test is conducted. We will define the failure rate for the first section as $k_1\lambda$ and the failure rate for the second as $(1 - k_1)\lambda$. Consider first only observed failures. Let $P_o(t)$ be the probability that the first section of the equipment is up (in operating condition) at time t; then, using the same derivation as above, we find that

$$P_o(t) = \frac{\mu}{\mu + k_1\lambda}$$
$$+ \frac{k_1\lambda}{\mu + k_1\lambda} \exp\left[-(\mu + k_1\lambda)t\right]. \qquad (5)$$

Further, let $P_\mu(t)$ be the probability that the second section of the equipment is up at time t. Then,

$$P_\mu(t) = \exp\left[-(1 - k_1)\lambda t\right].$$

Finally, the probability that the equipment is up at time t is

$$P(t) = P_o(t)P_\mu(t).$$

This gives

$$P(t) = \frac{\mu}{\mu + k_1\lambda} \exp\left[-(1 - k_1)\lambda t\right]$$
$$+ \frac{k_1\lambda}{\mu + k_1\lambda} \exp\left[-(\mu + \lambda)t\right]. \qquad (6)$$

Figure 1 shows $P(t)$ as a function of t for some selected values of the parameters k_1, λ, and μ. Notice in this figure that the steady-state phenomenon that occurs as a result of taking the limit of Eq. (4) does not exist. This is because of the parameter k_1. Obviously, if there are undetected failures in the system, availability will approach zero. However, if a definitive test of the equipment is performed, or a trial of the real use of the equipment is performed, then the accrued failures during normal operation will be discovered and the equipment can be restored to a perfect condition. This indicates the necessity of introducing scheduled test procedures.

We will introduce test procedures by means of the parameter T. That is, if we schedule a definitive test every day, for example, then the equipment is restored on a daily basis to an availability of one. This satisfies the initial conditions assumed in the original derivation of $P(t)$. In order to simplify the mathematics, we will assume the duration of the test to be of zero length. During some of the

test periods there will be no failures in the equipment, and during others there will, of course, be failures that require repair. In order to take account of down time for repairs during tests, we will compute an expected down time T_s during each test, and define availability as

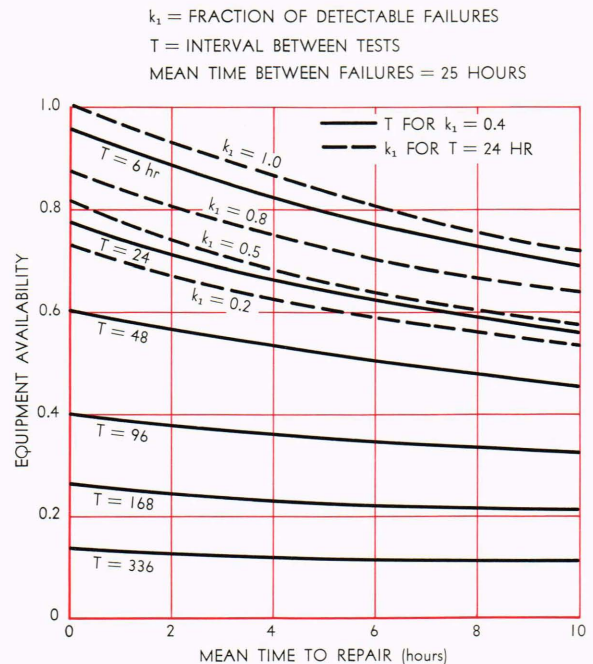$$A = \frac{1}{T} \int_0^{T-T_s} P(t)\, dt, \qquad (7)$$

where

$$T_s = \frac{(1 - k_1)\lambda T}{\mu + (1 - k_1)\lambda}.$$

The result of the integration in Eq. (7) gives

$$A = \frac{\mu}{(\mu + k_1\lambda)(1 - k_1)\lambda T}$$
$$\cdot\{1 - \exp\left[-(1 - k_1)\lambda(T - T_s)\right]\}$$
$$+ \frac{k_1\lambda}{T(\mu + k_1\lambda)(\mu + \lambda)} \qquad (8)$$
$$\cdot\{1 - \exp\left[-(\mu + \lambda)(T - T_s)\right]\}.$$

Figures 3 and 4 show availability computed from Eq. (8) as a function of $MTTR$. Figure 3 shows the effect of the time between system maintenance tests, as well as the effect of changing the

Fig. 3—Equipment availability as a function of mean time to repair, showing the effect of time between system maintenance tests and the effect of changing the fraction of detectable failures.
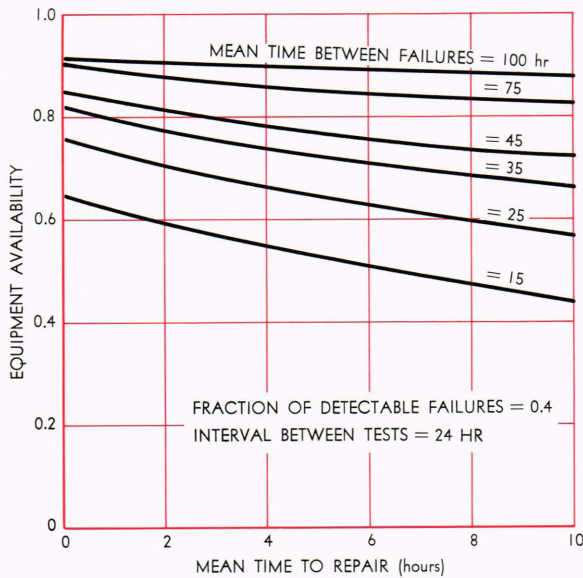
Fig. 4—Equipment availability as a function of mean time to repair, showing the effect of variation in the mean time between breakdowns.

fraction of detectable failures. Figure 4 shows the effect of a variation in the mean time between breakdowns.

With the parameter values chosen, notice from Fig. 3 that with weekly or biweekly tests the equipment can be available less than 25% of the time, and that availability is insensitive to $MTTR$. This is because during such a time period the expected number of unobserved failures is high and there is a good chance that the equipment will be down and unperceived by the operating personnel. As T decreases, the availability is more sensitive to $MTTR$.

The value of k_1 will vary considerably from one equipment to another and will depend particularly on the customary or normal operating mode. Figure 3 shows the effect of this parameter. If the equipment $MTTR$ is 8 hr, then increasing k_1 from 0.4 to 1.0 is equivalent to increasing simultaneously the mean time to failure from 25 to 45 hr and decreasing the repair time to 2 hr.

Figure 4 shows the effect of failure rate. A significant thing to notice here is that availability is more sensitive to a decrease in $MTBF$ than it is to the same percentage of increase in $MTBF$.

Availability-Test Problems

The next difficulty encountered in operating the system we have described is that the test procedures may not be complete. That is to say, because of either high cost or physical space limitations or a combination of these two factors, definitive ship-

board equipment tests can be difficult to achieve. To account for this fact in our model, we have introduced the parameter k_2—the percentage of failures detectable during a test. This means that when a test is conducted, instead of restoring the equipment to an availability of 1, the availability after tests cannot exceed $\exp[-(1 - k_2)\lambda t]$. Lacking a definitive test, we still have the problem that equipment availability can approach zero—although at a much slower average rate than without any test. There is, therefore, a need to perform somehow a definitive test that will in fact restore the equipment to an operating condition. Thus, we introduce the parameter n, the number of normal test cycles before a definitive test is conducted. A definitive test, as used here, is one that checks those parts of the equipment not tested in the normal test procedure. In the case of a missile system, a definitive test may be a missile firing. Figure 5A indicates the phenomenon just described. Here we see availability as a function of time. At time zero, availability is 1 and decays according to Eq. (6) until time $T - T_s$. At this time the normal test is performed. After an expected down time T_s, the equipment is restored to an operating condition with the probability $\exp[-(1 - k_2)\lambda(T - T_s)]$. In this figure, we have assumed that three test cycles precede a definitive test, that is $n = 3$. Thus, at time $3T$ the equipment is restored to full operating condition. Availability is defined as before in Eq. (7).
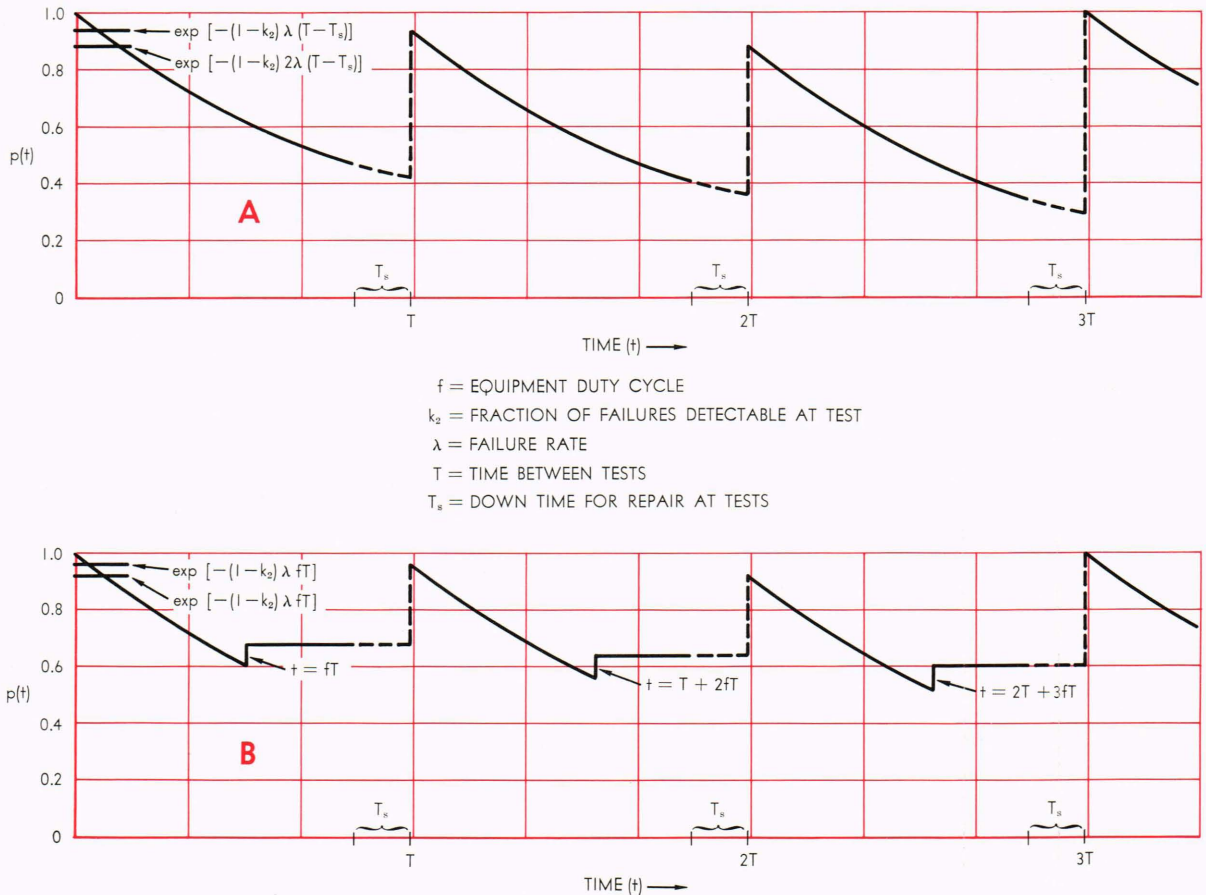
Thus,

$$A = \frac{1}{nT} \int_0^{nT} p(t)\, dt, \qquad (9)$$

where

$$p(t) = P(t) \text{ for } 0 \leq (T - T_s)$$

$$p(t) = 0 \quad \text{for } (T - T_s) < t < T$$

$$p(t) = \exp[-(1 - k_2)\lambda(T - T_s)]P(t)$$
$$\text{for } T \leq t \leq (2T - T_s)$$

$$p(t) = 0 \quad \text{for } 2T - T_s < t < 2T$$

$$p(t) = \exp[-2(1 - k_2)\lambda(T - T_s)]P(t)$$
$$\text{for } 2T \leq t \leq (3T - T_s)$$

$$\vdots$$

and

$$p(t) = \exp[-(n - 1)(1 - k_2)\lambda(T - T_s)]P(t)$$
$$\text{for } (n - 1)T \leq t \leq (nT - T_s)$$

$$p(t) = 0 \quad \text{for } nT - T_s < t < nT.$$

f = EQUIPMENT DUTY CYCLE
k_2 = FRACTION OF FAILURES DETECTABLE AT TEST
λ = FAILURE RATE
T = TIME BETWEEN TESTS
T_s = DOWN TIME FOR REPAIR AT TESTS

Fig. 5—Equipment availability as a function of time for incomplete tests (A) when the equipment oper-ates continuously, and (B) when the equipment is turned off for periods of time between tests.

Integration of Eq. (9) gives

$$A = \left(\frac{1}{nT}\right) \frac{1 - \exp\left[-(1 - k_2)\lambda n(T - T_s)\right]}{1 - \exp\left[-(1 - k_2)\lambda(T - T_s)\right]}$$

$$\cdot \left(\frac{\mu\{1 - \exp\left[-(1 - k_1)\lambda(T - T_s)\right]\}}{(1 - k_1)\lambda(\mu + k_1\lambda)}\right. \quad (10)$$

$$\left. + \frac{k_1\{1 - \exp\left[-(\mu + \lambda)(T - T_s)\right]\}}{(\mu + k_1\lambda)(\mu + \lambda)}\right).$$

Figure 6 shows the effect of k_2. Notice that it must be greater than k_1; that is, our test procedure must be at least as good as monitoring during normal operation.

Shipboard Operational Problems

The next phenomenon to be introduced occurs because many equipments in the fleet do not oper-ate continuously; that is, the stress on the electronic components occurs only at certain times during the day. For example, the diagram following indi-cates a typical equipment's operation for one day.

If we assume that stress on the equipment occurs only during the indicated ON periods, and that during all other periods the equipment will not fail, it is convenient for analysis to regroup the periods above as shown below.

Of course, the pattern and cycle for each day will be different. That is, the number of ON and OFF cycles, as well as repair periods, during both test and normal operation, will vary each day. These cycles, averaged over many days of opera-tion, are represented in terms of probability by Eq. (6). However, in this equation we have not taken OFF periods into account. An OFF period means that the equipment does not fail. The reason for regrouping the periods, with the OFF periods

together at the end of the day, is to simplify the mathematical treatment of nonstress periods.

In order to deal with OFF periods, we introduce the parameter f, the duty cycle of the equipment or the fraction of the time between test periods during which the equipment is subject to failures. Thus, assuming the equipment does not fail during nonoperative periods, we can reconstruct Fig. 5A in the form of Fig. 5B, in which we have again assumed that tests occur T cycles apart. The time from zero to fT represents the stressed period of the equipment. At time fT we will assume that the equipment is OFF. However, we will further assume that observed failures (which occur at the rate of $k_1\lambda$) will be fixed before turn-off. This accounts for the discontinuity and increase in availability at time fT. At time T, the same phenomenon as in Fig. 5A repeats again and continues through n cycles of tests. Using the definition of availability given in Eq. (9), and taking account of OFF periods, we find

$$A = \left(\frac{1}{nT}\right) \frac{1 - \exp\left[-(1 - k_2)\lambda n f T\right]}{1 - \exp\left[-(1 - k_2)\lambda f T\right]}$$

$$\cdot \left(\frac{\mu\{1 - \exp\left[-(1 - k_1)\lambda f T\right]\}}{(1 - k_1)\lambda(\mu + k_1\lambda)} \right. \tag{11}$$

$$+ \frac{k_1\lambda\{1 - \exp\left[-(\mu + \lambda)fT\right]\}}{(\mu + k_1\lambda)(\mu + \lambda)}$$

$$+ \left. [T(1 - f) - T_s] \exp\left[-(1 - k_1)\lambda f T\right] \right),$$

where, in this case,

$$T_s = \frac{(1 - k_1)\lambda f T}{\mu}.$$

Figure 7 shows the effect of the two parameters f and n. It is particularly relevant to notice that the duty cycle, the normal test cycle, and the definitive test cycle are three items that can be controlled by ship's operating doctrine; that is, f, k, and T are independent of failure rates, repair rates, and the monitoring procedures of equipments.

Equation (11) is a mathematical expression that takes into account most of the phenomena that occur in shipboard equipments. We have not included such important items as turn-on stress or test-equipment reliability. The former has not been included because of the difficulty in obtaining useful data. It is interesting that test-equipment reliability can be considered as a part of k_2. Thus, k_2 is defined as the percentage of failures detectable during a normal test. If

reliability of the test gear is r, then we could say that $k_2' = rk_2$ and simply replace k_2 in Eq. (11) by k_2'. In a similar way, if p is the probability that failure will occur at turn-on, then by knowing the average number of equipment turn-ons per day, m, we could say that $\lambda' = (mp/T) + \lambda$, and simply replace λ by λ' in Eq. (11).

Another usual shipboard occurrence is preventive maintenance events. Some preventive maintenance can occur without dismantling the equipment. Those scheduled preventive maintenance actions that require doing so can be included in the model if the average down time between test cycles is known. Let us call this time t_d; then,

$$T_s = td + \frac{(1 - k_1)\lambda f T}{\mu}.$$

Equation (11) is valid for the following range of parameter values: $0 < \lambda$; $0 \le \mu$; $0 < k_1 < 1$; $k_1 \le k_2 < 1$; $1 \le n$; $t_T \le T$; and $t_T/T \le f \le 1$, where t_T is the minimum time required to run tests. It is assumed in the model that the equipment is stressed during the time t_T, and minimum values for T and f are determined by the test time. Expressions for availability for limiting values of the parameters can be obtained easily by consideration of the basic derivation given at the outset. If we have a perfect test, that is $k_2 = 1$ and $n = 1$, then Eq. (11) reduces to Eq. (8). If, in addition, we have perfect monitoring of failures, that is $k_1 = 1$, then Eq. (11) reduces to the simple defini-
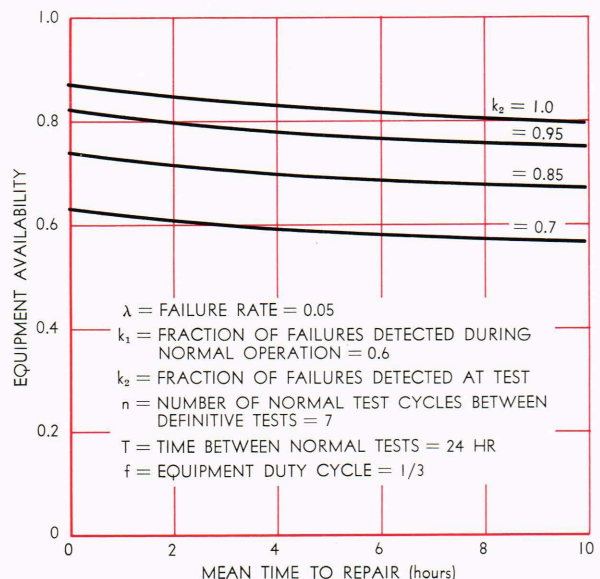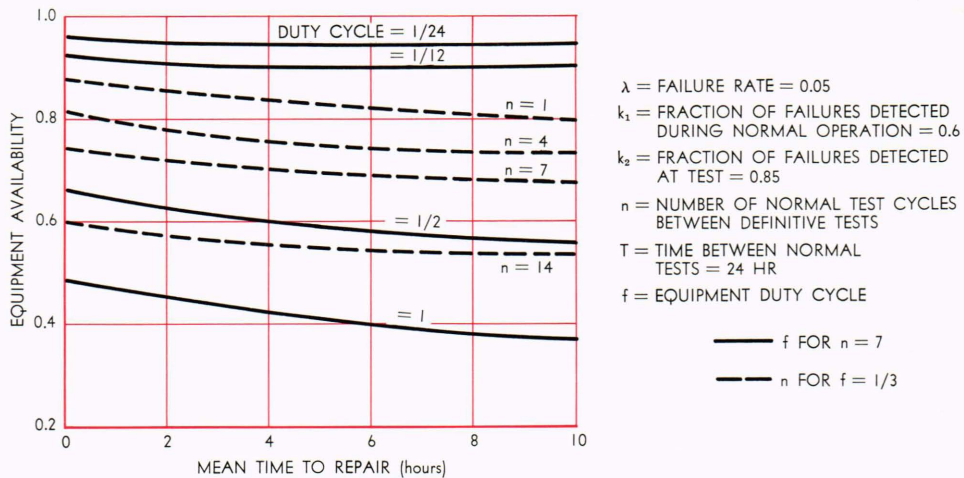

Fig. 6—Equipment availability as a function of mean time to repair, showing the effect of test completeness.

Fig. 7—Equipment availability as a function of mean time to repair, showing the effect of selected numbers of normal test cycles before definitive tests and of the duty cycles of the equipment.

tion of availability given at the outset in Eq. (2).

Some care should be exercised in using Eq. (11) in optimization of parameters, particularly with respect to the duty cycle. It would seem from the model that the greatest availability is obtained by not operating the equipment at all. This, of course, is due to the assumption of no failures during OFF periods. In any investigation of optima, the duty cycle is best treated as a constant.

The model as developed so far assumes that there are no logistic problems; that is, the availability is computed with the assumption that spare parts are in ample supply. If a given equipment has a logistic policy, the probability that the equipment is awaiting spares can be easily computed. Thus, from a model for logistics it is possible to compute the logistic availability, which we may call A_L. The overall availability of an equipment is then the product of A_L times the quantity A as computed in Eq. (11).

Engineering estimates of k_1 and k_2 require extensive effort. One method of obtaining input values for these parameters is to establish from a reporting system a list of failures for an equipment. This failure list is then analyzed by system engineers and operating personnel to determine (1) whether the failure would be observed in normal operation, and (2) whether it would be observed during system tests. When an equipment is in the design stage, the monitoring of particular equipment subassemblies should be considered in terms of theoretical failure rate and the cost of continuous monitoring.

This model for availability has been used to determine quantitatively the current level of effectiveness of guided-missile systems in the fleet, as well as to predict effectiveness levels on the basis of planned or proposed changes. Another useful output of the model is its capability for determining the direction that affords the best payoff in availability. For example, we have previously indicated the effect of changing the routine monitoring aspects of the equipment as compared to changing simultaneously the failure rate and the repair rate. Additionally, the model has proved useful to manufacturing contractors as a guideline toward new equipment design, which will give optimum equipment for the lowest cost.