The Constant-K lens is a microwave lens antenna formed of a homogeneous dielectric sphere and capable of generating beams in any direction. Its theory of operation, its limitations, and some manufacturing problems are discussed.

T. C. Cheston
E. J. Luoma

CONSTANT-K LENSES

Spherical dielectric lenses are used as microwave antennas because their inherent symmetry permits the formation of beams in any direction. The best known lens of this type is the Luneberg lens[1] (Fig. 1A), in which any plane wave incident on the lens is brought to a focus at a point on its surface. This is achieved by a gradation of the dielectric constant K as a function of the radius. Different general configurations of the Luneberg lens are possible where different laws of dielectric constant variations give rise to special properties; for example, the focal surface may be made to lie exterior to the physical surface.

Only little attention has been paid in the past to the spherical lens with an *unvarying* dielectric constant. This is the so-called "Constant-K" lens. This lens has focusing properties similar to those of Luneberg lenses, except that small aberrations—sufficiently small to be negligible for most practical applications—are present.[2,3] Figure 1B shows a Constant-K lens and indicates how the rays are brought to a focus. Such lenses are inherently of simpler mechanical design than Luneberg lenses, although some production problems have still not been solved for large precision lenses.

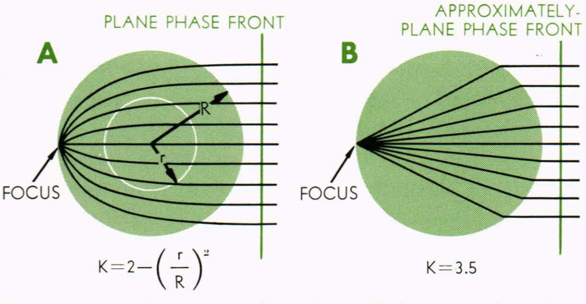

Fig. 1—Luneberg and Constant-K lenses, showing ray bending and ray refraction, respectively.

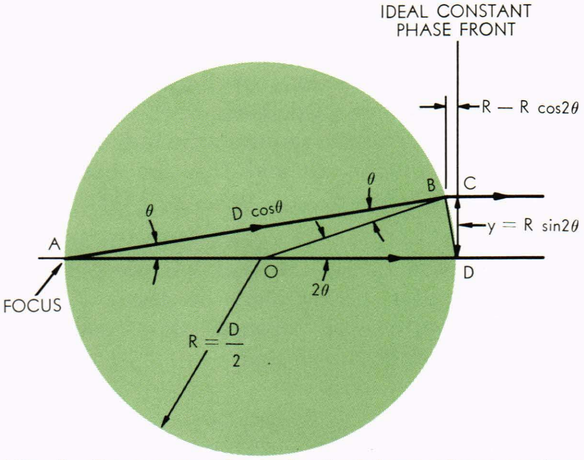

Fig. 2—Geometry of a spherical lens of unvarying dielectric constant—a Constant-K lens.

---

[1] R. K. Luneberg, *Mathematical Theory of Optics,* Brown University Press, Providence, R. I., 1944.

[2] G. Bekefi and G. W. Farnell, "A Homogeneous Dielectric Sphere as a Microwave Lens," *Can. J. Phys.* **34,** 1956, 790–803.

[3] R. N. Assaly, "Experimental Investigation of a Homogeneous Dielectric Sphere as a Microwave Lens," *Can. J. Phys.*, **36,** 1958, 1430–1435.

Theory of Constant-K Lenses

ABERRATIONS—With the parameters shown in Fig. 2, and with the focus on the lens surface, aberrations (phase errors) are found by calculating the difference between the electrical path lengths AD and $AB + BC$:

$$\text{Aberration } \Delta = AD \cdot n - (AB \cdot n + BC), \quad (1)$$

where n = refractive index = $\sqrt{K}$,

and K = dielectric constant.

Normalizing with respect to the lens diameter in wavelengths D/λ, the geometry gives

$$\frac{\Delta/\lambda}{D/\lambda} = n(1 - \cos\theta) - \frac{1}{2}(1 - \cos 2\theta) = 2n \sin^2\frac{\theta}{2} - \sin^2\theta, \quad (2)$$

and

$$\frac{y}{D} = \frac{1}{2}\sin 2\theta. \quad (3)$$

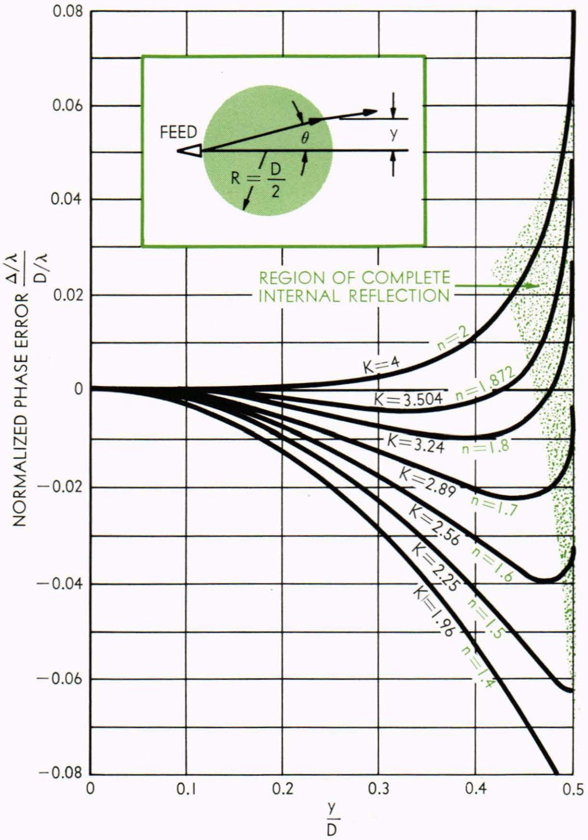

Fig. 3—Aberrations from a Constant-K spherical lens with feed at the surface. To obtain the phase error in wavelengths, multiply the ordinate by D/λ.

Figure 3 shows the aberrations over half the lens aperture as a function of dielectric constant. Since the lens has spherical symmetry, only spherical aberrations are present. The first approximation to Eq. (2) shows that aberrations are eliminated when $n^2 = K = 4$. However, as will be shown, a better compromise value of K is approximately 3.5. This compromise takes higher-order aberrations into account.

REFLECTIONS AT THE SURFACE—Complete internal reflections take place when

$$\sin\theta \geq \frac{1}{n} = \frac{1}{\sqrt{K}}. \quad (4)$$

The limits are shown in Fig. 3. The best compromise in dielectric constants is ≈ 3.5. The aberrations then have peak values of about $\pm 0.004\ D/\lambda$, and the available aperture diameter has been reduced by about 10%.

The sphere may be covered with a layer of dielectric material, giving a compromise match to reduce the reflection coefficient at the surface.

APERTURE AMPLITUDE DISTRIBUTION—The amplitude distribution at the final radiating or receiving aperture is modified by the presence of the lens.

The radiation pattern of the feed has increased directivity due to the dielectric, and it corresponds to a radiation pattern in air obtained from a feed aperture $\sqrt{K}$ times larger than its actual size. This effect increases the amplitude distribution taper by reducing the power density at the edge of the aperture.

Because of the spherical exit surface, there is a bunching or compressing effect that increases the power per unit area toward the edge of the aperture after refraction, relative to the power within the lens before refraction. This effect results in a power distribution factor of $\sec\theta \cdot \sec 2\theta$.

Reflections at the surface of an unmatched lens tend to reduce the radiated power near the outer bounds of the radiating aperture. It is relatively unimportant except close to the boundary of the radiating aperture, defined by Eq. (4), where complete internal reflections take place.

OFF-SURFACE-FOCUS LENS—If the dielectric constant of the Constant-K lens is greater than ≈ 3.5, then the focus of an incident plane wave will be *inside* the lens. As the dielectric constant is reduced from 3.5, the focus progressively moves externally away from the surface. An approximate law, proved experimentally, has been calculated using ray tracing techniques, giving

$$\frac{R'}{R} = \frac{1}{\sqrt{2}}\frac{K}{K-1}, \quad (5)$$

where R' is the focal radius and R is the radius of

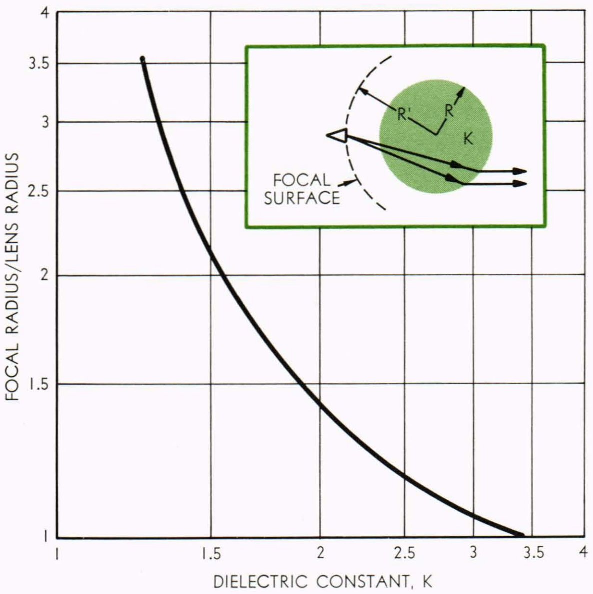

Fig. 4—Focal radius R' as a function of the dielectric constant K.

the lens. Figure 4 is a graphical presentation of Eq. (5).

Equation (5) differs from the general optical equation for thick lenses, $R'/R = n/[2(n - 1)]$, and takes into account that the optimum dielectric constant for a surface-focus lens is 3.5 rather than 4, as given by the first approximation.

LIMITATIONS OF APERTURE SIZE—As the aperture gets larger in terms of wavelength, the aberrations become more significant. For a 30-wavelength-diameter surface-focus lens of $K = 3.504$ (Fig. 2), the aberrations have peak values of $\pm\lambda/8$, normally acceptable for antennas. The nature of the aberrations, however, is such that appreciably larger diameters are possible. This is particularly true of lenses that focus off the surface ($K < 3.5$), although in this case ray-tracing techniques lose some rigor since the wave fronts are neither spherical nor plane. Theoretical investigations have shown that lenses with diameters well in excess of 100 wavelengths should still perform well, although a loss in efficiency due to beam widening would result.

Measurements and Experimental Results

Several Constant-K-type lenses have been built and tested, with dielectrics including silica sand, oils, and various natural and artificial solid materials. A cross-linked polyethylene (TPM-7) lens is shown in Fig. 5 being tested in the indoor range at Emerson and Cuming, Inc., where most of the experimental work was carried out.

Standard techniques were used to plot radiation patterns and to measure gain. The optimum focal position was judged to be that which gave the deepest nulls in the radiation patterns (corresponding to the smallest aberrations).

Electrical symmetry was measured at that angle where the radiation pattern was down by 15 db from its peak value. With the feed stationary at this −15-db point, the lens was rotated about its center in various planes, and the resulting variations in signal were recorded. This gives a sensitive measure of how uniformly constant the dielectric is maintained throughout the lens and of how accurately the lens shape is spherical about the point of rotation.

Representative of the measurements made, Fig. 6 gives the radiation pattern of an accurately built, homogeneous, low-loss, solid-dielectric lens. The pertinent parameters were:

Dielectric material	Emerson and Cuming Stycase, TPM-7
Frequency	9375 mc
Lens diameter	12.4 λ
Dielectric constant	2.34
$\frac{\text{Focal radius}}{\text{Lens radius}}$ measured:	1.24
calculated:	1.23
Feed-horn size	1.24 λ × 0.94 λ
3-db beam width	5.6°
Gain above isotropic radiator	29 db
Electrical symmetry	0.2 db at −15-db pattern level.

Fabrication

REQUIREMENTS—Constant-K lenses with nominal beam-forming characteristics are easily built from a variety of materials. However, because more than nominal performance is usually required, difficulties arise in the choice of material and in fabrica-

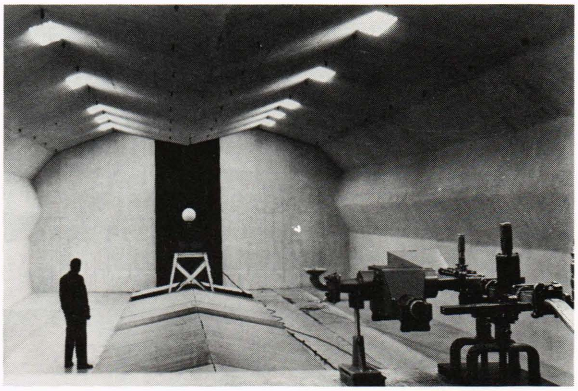

Fig. 5—A polyethylene lens undergoing test at Emerson and Cuming, Inc., Canton, Massachusetts.

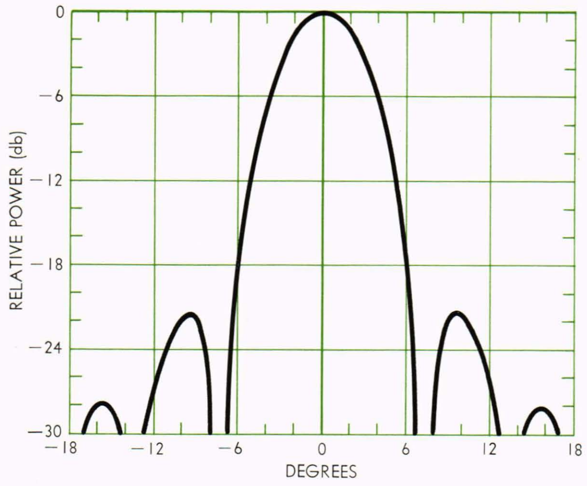

Fig. 6—Radiation pattern of a 12.4λ-diameter Constant-K lens. Feed size: 1.24λ × 0.94λ. Frequency: X-Band.

tion methods. The following factors have to be taken into account:

1. *Choice of dielectric constant*—The dielectric constant should be smaller than ≈3.5 so as not to focus within the lens, and normally not much smaller than ≈2.0 to keep the focal radius from getting too large.

2. *Electrical symmetry*—Electrical symmetry is a function of both the uniformity of the dielectric constant and the accuracy with which the spherical shape of the lens is maintained. The dielectric constant must be extremely uniform throughout the lens to maintain a constant focal radius and to avoid beam tilting and pattern deformation. For example, if the refractive index for the central ray of a 30λ-diameter lens changes by an amount of 0.01, then the phase at the aperture for that ray changes by 0.3 wavelength—a significant amount.

Similarly, the correctness of lens shape is important. If the electrical center of the lens is displaced to one side, due either to variations of the dielectric constant or to an aspheric shape, then the radiation pattern will be tilted. For example, a λ/20 sideways displacement of the electrical center will tilt the beam by 0.1 beamwidth.

3. *Loss Tangent*—The loss tangent, tanδ, must be kept small for an efficient lens. A good approximation for the energy dissipated in the lens is given by

$$\text{loss in db} \simeq 36\,\frac{R}{\lambda}\,\{K^{3/2} - (K-1)^{3/2}\}\,\tan\delta. \quad (6)$$

For example, with a 30λ-diameter lens of $K = 2$, a loss tangent of 0.001 gives rise to a loss of 1 db.

4. *Weight*—With natural dielectric materials, the weight of Constant-K lenses is considerable and, of course, increases as the cube of the diameter of the sphere.

Material Development

A development program has been carried out at Emerson and Cuming, Inc.,* with the aim being to produce precision Constant-K lenses. Considerable accuracy in solid lenses built of natural dielectrics has been achieved (Fig. 6). Present techniques have been shown to be adequate for producing natural dielectric materials of large size and, if necessary, for machining lenses with adequate accuracy.

Liquid dielectric lenses have also been built and tested, with good results, and it has been shown that a wide range of suitable liquids with small loss tangents are available. Liquid dielectric lenses have the potential of being able to handle large RF power, even without heat exchangers. The problem with liquids, however, is that the containers must be homogeneous and of sufficient strength so as not to deform when filled with liquid.

Because both solid and liquid dielectric lenses are heavy, development work has been concentrated on artificial dielectrics. These can be formed, for example, from lightweight polystyrene foam loaded with metallic particles that are small in comparison to a wavelength. The dielectric constant increases as the quantity of particles is increased. Production problems that are as yet only partially solved are due to relatively high loss tangents and lack of uniformity.

The loss tangent of these artificial dielectrics increases rapidly with both frequency and dielectric constant. However, loss tangents of about 0.001 at X-band can be achieved with dielectric constants below 2.0. The resultant lens would have a relatively long focal length. At lower frequencies the loss tangent would be smaller, permitting higher values of dielectric constant.

Difficulties in controlling and maintaining accurately an even distribution of the metallic particles within the foam gave rise to lack of uniformity of the dielectric. Even so, the electrical asymmetries which arose in experimental lenses were only a little greater than those obtained from state-of-the-art Luneberg lenses.

Conclusions

The Constant-K lens has properties similar to those of a Luneberg lens but is formed of homogeneous dielectric material. It has considerable potential in actual application although, with the present state of the art, precision lenses are heavy in weight. Future applications in the infrared, optical, and acoustical fields seem possible.

---

* Emerson and Cuming, Inc., Canton, Massachusetts.