Rocket motors sometimes break into acoustic oscillation of such amplitude that the consequences are devastating to the performance and even to the integrity of the motor. This phenomenon has led to a number of theoretical studies of the mechanics by which the energy of burning propellants is converted to high-amplitude sound. Analysis of the problem leads to the determination of analytical criteria for stability, whose satisfaction imposes certain restrictions on the properties of the propellant and on the motor configuration.

R. W. Hart

COMBUSTION INSTABILITY *in Solid Rockets*

To most casual observers whose closest contact with a solid propellant rocket has been a Fourth of July sky rocket, it might seem that the design of such a rocket motor ought to pose few problems. Admittedly, there will be the question of its trajectory, which will involve the exterior ballistician, but we are concerned here with the design of the motor itself and not with where it goes. The connection between rocket motors and acoustics is a vital and important one and of great concern to many whose responsibility involves the defense of our nation. Its explanation and understanding involve the focusing of several disciplines of science upon the central problem of how sound is amplified and attenuated in the hot and hostile environs of a rocket motor; thus, the central problem here must be attacked not only by the chemist, the fluid mechanician, the mechanical and chemical engineer, and the rheologist, but also, in a vital way, by the acoustician.

By way of historical interest, and to put the problem in context, it has been asserted that solid-propellant rocket instability made its first dramatic reappearance in recent times during the last war. Certain British rockets occasionally exploded "without cause" very soon after firing. Since the rockets were designed for release from under the wings of planes, and since the aircraft were not designed to survive flying through the fragments of their own rockets, this was a serious problem. Throughout the subsequent years unstable burning, in varied guises, has been a frequent nemesis to the rocket designer. In fact, it may not even be an exaggeration to say that most rockets have been unstable at one time or another during their initial development.

From the practical point of view, the development of rocket motors still remains, in many respects, an art rather than a science. Thus, when a motor is found occasionally to be unstable during the course of its early development, various cut-and-try procedures have customarily been attempted in order to eliminate the instability. Such procedures are costly, and the need for a fundamental understanding of unstable burning is evident.

Much progress has been made in the development of a basic, and at the same time practically useful, understanding of the instability problem, but much remains to be accomplished. In the following, we shall examine the present status of this understanding, with emphasis on its acoustic aspects. We shall especially attempt to focus attention on many of the unsolved areas of the problem which fall within the domain of acoustics.*

The Nature of the Stability Problem

When unstable motors such as the one mentioned above are encased in heavy steel chambers used for ground testing purposes, pressure-time traces similar to those of Fig. 1 are obtained. In view of the high overpressures which these traces show, the fracturing of the relatively lightly encased flight motors is not too surprising.

The essential clue to the most frequently occurring mechanism producing unstable burning is provided by the response of microphones especially constructed to withstand the hostile environment within the motor chamber. As shown in Fig. 2, the high overpressures are typically

* Most of the work reported here has been carried out under the direction of F. T. McClure and in collaboration with J. F. Bird. Some of the very recent work has been done with the collaboration of R. H. Cantrell.

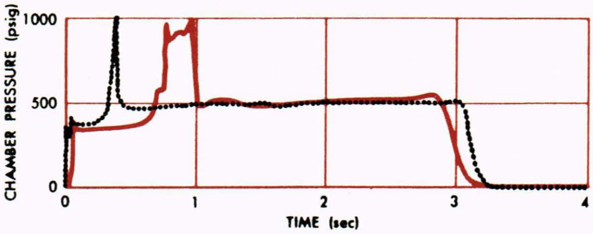

Fig. 1—Pressure-time traces characteristic of an unstable rocket motor. The two curves correspond to two firing temperatures. (From W. G. Brownlee.)

accompanied by a rather severe acoustic pressure oscillation in the motor cavity. This oscillatory pressure is often comparable to the mean pressure (which is typically 30 to 70 atm.). Sound fields of this magnitude are almost without equal, and it is not surprising that they can do serious damage to the motor. (Surprisingly enough, the acoustic field outside the motor often shows little of the ferocity contained within, a point which we shall touch on later.) The frequency-analyzed output of a microphone placed in a cylindrical motor with an internally burning charge (cf. Fig. 8) is shown in Fig. 3; the amplitudes of the various frequency components are indicated by the brightness of the oscilloscope trace.

It was easily recognized that the frequencies observed were associated with the acoustic modes of the gas-filled part of the motor cavity. Thus, the low-frequency mode shown in Fig. 3 is the lowest axial mode, with its frequency rising slowly with time as the grain shortens due to end burning. The higher frequency modes are transverse modes of the chamber, which decrease in frequency as the propellant burns radially outward. Thus, it becomes clear that the unstable rocket motor is, in fact, an acoustic oscillator.

There are two obvious and diverging directions which the interior ballistician might take. On

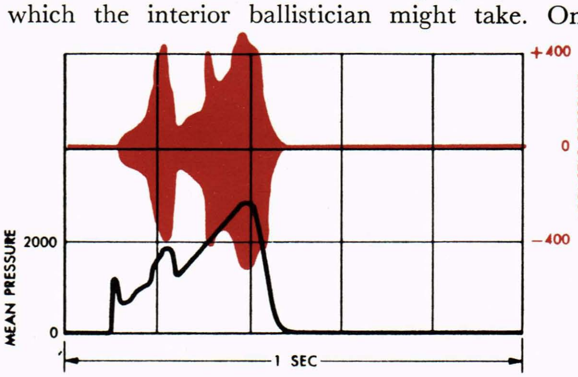

Fig. 2—Oscilloscope trace showing pressure. The upper trace is the oscillatory pressure component, and its magnitude is read off on the right-hand ordinate scale. The lower trace is the mean pressure, as indicated by the left-hand scale. (From E. W. Price.)

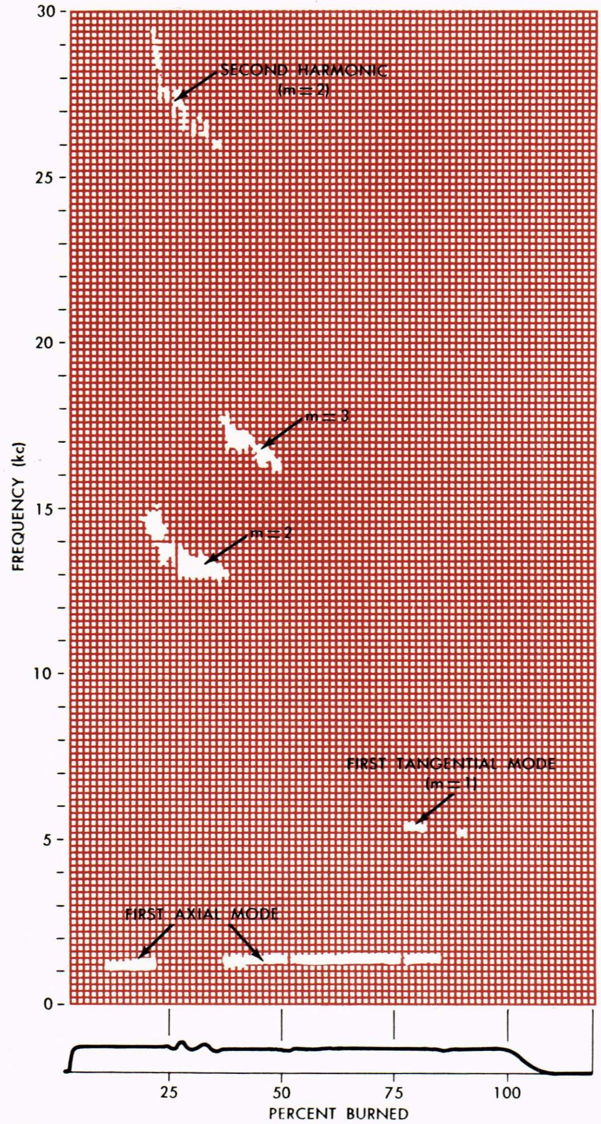

Fig. 3—Frequency versus percent of web burned, illustrating the usual occurrence of oscillations in acoustic modes and, for very large amplitude cases, in harmonics of the acoustic modes. (From T. Angelus.)

the one hand, his intellectual curiosity may lead him toward the explanation of the properties of unstable motors in terms of the large-amplitude acoustic fields. On the other hand, he may pursue the central question which concerns the reasons why such fields may build up in the first place, and thereby develop an understanding of how they may be avoided. This latter choice is the one of greatest immediate significance, and is the one which would lie nearest the concern of the acoustician. Thus, we shall consider mainly the question of whether or not a rocket motor will or will not be acoustically stable in the presence of arbitrarily-small disturbances. This will restrict our attention to the relatively simple, but still very complex, domain of ordinary linear acoustics.

In general, the question of stability or instability of an acoustic system is to be resolved by considering the balance of acoustic gains and acoustic losses. If the losses are sufficiently great compared to the gains, then stability will result. Thus, the one ingredient which is absolutely essential to the existence of acoustic instability will be an acoustic amplifier. Where will we find a source of energy for the acoustic field in a solid propellant motor? The answer to this question is to be found in the thin burning region at the propellant surface.

Acoustic Amplification at the Burning Surface

A very delicate balance exists in the burning region between the rate of efflux of gasified solid and the rate at which the flame front eats away at this gas (cf. Fig. 4). If the solid should begin to gasify at too fast a rate, the flame front will be pushed away from the solid, the flow of heat from the flame to the solid will thereby be diminished, and the gasification rate will be reduced. A similar compensating action applies if the gasification rate fluctuates downward or if the flame velocity itself fluctuates. Stable burning will ordinarily result as long as the combustion zone has time to equilibrize between the disturbances. Since the steady-state burning rate of a propellant depends on pressure, it is not obvious just what will happen as a result of the rapid pressure variations characterizing a sound wave. One must have recourse to a physico-chemico-mathematical analysis of the response of the combustion zone to pressure disturbances.

What should be the major specific goal of such an analysis? From the acoustical point of view, the response of a surface is frequently characterized by a "specific acoustic admittance,"

$$Y \equiv \frac{\text{amplitude of acoustic velocity directed into the surface}}{\text{acoustic pressure amplitude}}.$$

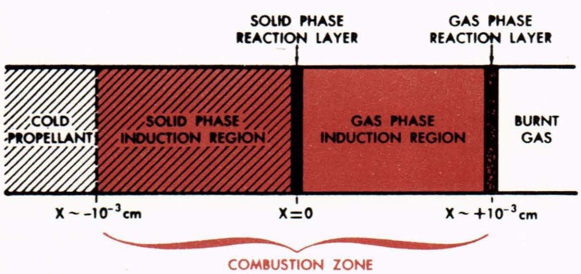

Fig. 4—Schematic illustration of the combustion region.

The real part of this admittance describes the extent to which sound is amplified or attenuated. If the real part is negative, amplification occurs, whereas attenuation will occur when the real part is positive. Thus, the determination of the specific acoustic admittance of the burning propellant surface will be one item of major concern.

The ideal propellant, of course, would be one which would be incapable of acting as a source of an acoustic field. In the absence of a flow field, the condition that the admittance of the propellant have a positive real part would insure this feature. In the presence of flow, however, energy is transferred not only by a mechanical wave at the speed of sound, but also by convection at the mean flow speed. If the energy transported convectively away from the propellant should be converted back into sound somewhere downstream, then there could be a net input of energy into the acoustic field even though the burning surface did not quite succeed in amplifying sound. It turns out that such a conversion does occur for axial modes in an end-burning, center-vented chamber (a "T" motor).[1] In that case, the condition that the real part of the admittance be greater than zero, while sufficient to insure attenuation rather than amplification of the mechanical wave at the propellant surface, will not be quite sufficient to insure stability. Under these conditions, it turns out that for typical geometries the criterion for stability reads

$$\text{Real of } Y_p \gtrsim v_p/\gamma\bar{P},$$

where Y_p is the propellant specific acoustical admittance, v_p is the speed of the hot product gas leaving the burning zone, γ is the specific heat ratio, and $\bar{P}$ is the mean pressure.

In order to illuminate the nature of the admittance of the burning surface, it will be helpful first to relate it to the mass rate of burning of the propellant. We note that the mass flow rate at the hot boundary of the burning zone is related to the local density ρ and velocity v by the relationship $m = \rho v$. Upon perturbing this, and assuming that the ratio of fluctuating density to pressure is given by the usual adiabatic relationship ($1/\gamma$, where γ is the usual specific heat ratio) characteristic of a sound field, we would find

$$Y = \frac{-v_p}{\bar{P}}\left(\frac{\tilde{\mu}}{\tilde{\epsilon}} - \frac{1}{\gamma}\right). \tag{1}$$

Here, the tilde indicates the Fourier amplitude

[1] F. T. McClure, R. W. Hart, and R. H. Cantrell, "Interaction Between Sound and Flow—Stability of T-Burners," The Johns Hopkins University, Applied Physics Laboratory, TG 335-12 (to be published).

in the usual $e^{i\omega t}$ notation, the bars indicate time average values, and $\bar{\mu}/\bar{\epsilon}$ is the ratio of the fractional increment in burning rate $(m - \bar{m})/\bar{m}$ to the fractional increment in acoustic pressure amplitude, $\bar{\epsilon}$. Thus, we would expect amplification at the propellant surface if $\bar{\mu}/\bar{\epsilon}$ should exceed $1/\gamma$.* An appreciation of the size of the burning rate response, as measured by the quantity $\bar{\mu}/\bar{\epsilon}$, can be obtained by considering the limiting case of zero frequency, where considerable information is directly available. For zero frequency, the effect of pressure on burning rate is usually written in the form

$$\bar{m} \propto (\bar{P})^n, \tag{1a}$$

where n is the so-called pressure index. This pressure index is always less than unity, and it is commonly between about 0.2 and 0.8. For the so-called mesa propellants, n may be zero and even somewhat negative at some pressures. Comparison of Eq. (1a) with Eq. (1) shows that in the zero frequency limit we expect $\bar{\mu}/\bar{\epsilon} \to n$. It follows that since n is ordinarily less than $1/\gamma$, propellants are not expected to amplify sound of very low frequency.

It is clear, then, that one of the important experimental tasks concerns the measurement of the burning surface admittance, and that one of the important theoretical tasks concerns the illumination of the structure of this quantity. Much recent progress has been and is now being made in this area.

In view of the fact that the chemical reactions and the rates at which they occur are not known in detail for solid propellants—nor are they likely to be known in the near future—the problem of calculating the acoustic amplification or attenuation experienced by a sound wave at the burning surface might seem to be a formidable one indeed. Fortunately, however, many chemical reactions are intrinsically very fast. It is the relatively slower physical processes of mass and heat transport which seem to be mainly responsible for the failure of the combustion zone to maintain its static equilibrium properties when subjected to an acoustic disturbance. This means that the acoustic response of the burning layer can often be calculated by neglecting the fact that the chemical reaction rates themselves are actually finite. The calculation becomes tractable if a few simplifying assumptions are made.

The acoustic properties of the burning zone are to be found by solving the time-dependent equations expressing conservation of energy and mass in the solid and gas-phase induction zones.[2] (The pressure drop across these thin zones is very small and is neglected. We also neglect diffusion.) It becomes necessary to solve the following set of two coupled partial differential equations for the gas and solid phases:

$$\frac{\partial}{\partial x}\left[\lambda \frac{\partial T}{\partial x} - mC_p T\right] = \frac{\partial}{\partial t}(C_v \rho T), \tag{1b}$$

and

$$\frac{\partial m}{\partial x} = -\frac{\partial \rho}{\partial t}, \tag{1c}$$

where T is the temperature, λ is the thermal conductivity, and C_p and C_v are the usual specific heats. Since we are here interested in the effect of small (acoustic) perturbations, these equations are to be perturbed about their steady-state solutions, and all second and higher order terms discarded. A further simplification can be effected by regarding the solid as rigid in comparison with the gas, so that Eq. (1c) may, in effect, be discarded in the solid where $m = \bar{m}$. The effect on the response of the burning surface of making this approximation has been found to be small.[2]

The associated boundary conditions must also be specified for $x \to \infty$, for the solid-gas interface and for the boundary between the gas-phase induction zone and the gas-phase combustion zone (see Fig. 4).

Since the reactions themselves are assumed "fast," both solid-phase and gas-phase combustion zones are collapsed to bounding surfaces across which there are discontinuities in heat flux and temperature gradient appropriate to the energy release, but no discontinuity in mass flow rate, m. The boundary equations for these boundaries each have the general form

$$\left(\lambda \frac{\partial T}{\partial x} - mC_p T\right)_{\text{hot side}} - \left(\lambda \frac{\partial T}{\partial x} - mC_p T\right)_{\text{cold side}} = -mq\left(T, \frac{\partial T}{\partial x}\right), \tag{1d}$$

where q is the heat release rate per unit mass in the collapsed combustion zone. For small (acoustic) perturbations, the function q will evidently have the general form

* In actual fact, the simple acoustic relationship between density and pressure is not, in general, valid near the burning zone because thermostating action of the flame zone tries to maintain isothermal rather than adiabatic conditions there. As a result, Eq. (1) is not quite correct, as we shall see later in more detail.

[2] R. W. Hart and F. T. McClure, "Combustion Instability: Acoustic Interaction with a Burning Propellant Surface," *J. Chem. Phys.*, **30**, June 1959, 1501–1514; also, J. F. Bird, L. Haar, R. W. Hart, and F. T. McClure, "Effect of Solid Propellant Compressibility on Combustion Instability," *J. Chem. Phys.*, **32**, May 1960, 1423–1429.

$$q = \bar{q} + A(T - \bar{T}) + B\left[\left(\frac{\partial T}{\partial x}\right) - \overline{\left(\frac{\partial T}{\partial x}\right)}\right], \quad \text{(1e)}$$

where the parameters A and B are determinable, at least in principle, from the unknown details of steady-state chemistry. (In practice, it is convenient to relate the A and B describing the gas-phase combustion zone to phenomenological, but empirically determined, parameters, such as the pressure index n and an analogous temperature index j which describes the effect of firing temperature of the propellant on burning rate. The A and B for the solid-phase combustion zone may be inferred from the solid-phase activation energy and the zero-order type of chemistry usually used to characterize solid-phase reactions.) Insofar as the boundary condition at the solid is concerned, it is merely that

$$T \rightarrow \bar{T}_c, \quad \text{(1f)}$$

the firing temperature of the propellant.

The mathematical nature of the response of the burning surface has now been specified in terms of the partial differential equations and the associated boundary conditions. The solution of these equations has been carried out elsewhere,[2] and the general nature of the results is shown in Fig. 5. Here, we note that the real part of the mass response term ($\tilde{\mu}/\tilde{\epsilon}$) typically increases from its static zero frequency value, passes through a rather broad maximum, and then declines. Subject to the assumptions incident to Eq. (1), we would expect acoustic amplification to occur over the broad band of frequencies where $\tilde{\mu}/\tilde{\epsilon}$ lies above $\sim 1/\gamma \sim 0.8$. Substitution of appropriate values for burning rate and pressure into Eq. (1) leads to the conclusion that solid propellants should be expected to be characterized by acoustic admittances having real parts of the order of 10^{-4} to 10^{-6} Rayls.

The measurement of the admittance has become

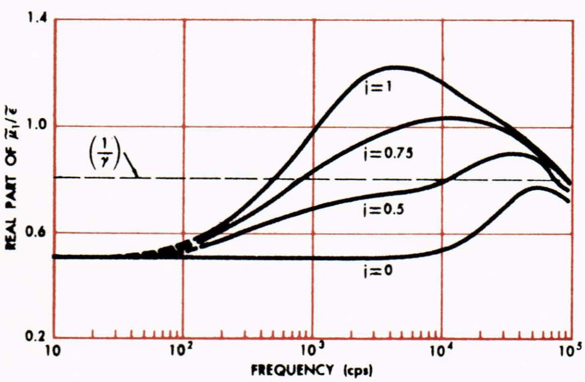

Fig. 5—Pressure response of four hypothetical propellants, each having a pressure index n = 1/2, but having four different temperature indices (j = 0, 1/2, 3/4, 1).

a paramount challenge to the experimental researcher. There are many difficulties, primarily due to high temperature, flow, and high pressure, but the first crude results are now being obtained at several laboratories, and they seem to support the general predictions of theory. It should be emphasized that theory will probably remain incapable of dealing accurately with the calculation of burning surface response for a long time. Thus, the final decision as to whether a given propellant will be a stable one will necessarily remain in the hands of the experimenter who measures the admittance of that propellant.

The Normal Modes

Now that the burning surface of the propellant has been established as a sometime amplifier of sound, it will be useful to look toward evaluating the acoustic gain-loss balance.

From an acoustic point of view, a rocket motor is an acoustic cavity (cf. Fig. 8) somewhat like an organ pipe and possesses similar "normal modes" in which it may resonate. In order to decide whether one or more of these modes will be excited, it is necessary to consider the net flux of acoustic power associated with each mode. It is usually convenient to assume tentatively that a mode exists and then to determine whether the net flux of acoustic power is directed out of or into the cavity. If the net flux vector is directed outward, there is a net loss of acoustic energy, and the mode will not be excited.

The realistic determination of the normal modes presents many problems, several of which have not yet been solved. Let us consider the information which would have to be available to us in order to obtain the general solution.

First of all, it would be necessary to know the speed of propagation of shear and dilatational waves in the solid propellant. At first thought, one might suppose that since the solid is so massive and rigid with respect to the gas, the motion of the solid could safely be ignored and the solid-gas boundary could be regarded as an acoustic velocity node. That this is not always the case is borne out both by theory and by direct measurement. But perhaps none of the direct measurements has had quite the impact of the first controlled experiments designed to establish the importance of the participation of the solid in the determination of the modes. These experiments occurred as a result of the following considerations.

It is clear that if the elastic motion of the solid is important, then the question of whether the outer surface of the propellant is tightly constrained or is free to move will be important. Theoretical

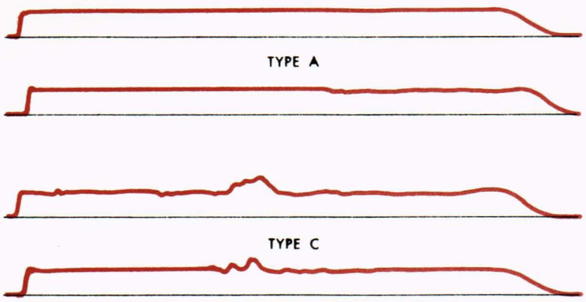

Fig. 6—The effect on stability of tightness of fit of the propellant grain is shown here by mean pressure traces at 70°F. Type A fit is the tight fit, and Type C is the controlled loose fit. (From T. Angelus.)

study showed that looseness corresponding to only a few thousandths of an inch between the motor case and the propellant would be sufficient to constitute a loose rather than a constrained boundary condition for transverse modes. This simple observation is seen to have considerable practical importance when we recall the disturbing unpredictability of the instability phenomenon and at the same time note that the grain dimensions were not customarily controlled with such accuracy. In order to see whether these considerations could be an important source of test unreliability, it was suggested that experiments be carried out using two sizes of grains:

1. grains which were oversize by a few thousandths of an inch, and could be contracted by cooling in order to be slipped into the motor and then rewarmed to achieve a tight fit, and
2. grains which were undersize by an accurately controlled amount to achieve a reproducible loose fit.

Such experiments were carried out at Allegany Ballistics Laboratory by T. Angelus, who found the striking differences in stability properties shown in Fig. 6.[3] Subsequently, it has become well recognized that in order to obtain reproducible data it is necessary to control the fit of the grain with high accuracy.

Secondly, it would be necessary to determine the mean flow field of the gases, since the propagation of sound depends upon the velocity of the medium in which it travels. The determination of the mean flow field is primarily a problem in gas dynamics. Insofar as the flow in the propellant channel is concerned, it is frequently rather slow and seems to have only a minor effect on the acoustic field distribution, at least for relatively high frequency modes. Nevertheless, it may have an important effect on the overall stability.

Finally, it would be necessary to know the acoustic admittance characterizing the sonic nozzle of a rocket (and its associated cavity if there is one). This nozzle problem has been attacked theoretically by at least two fluid dynamicists,[4,5] but the approximations are such that it is difficult to have great confidence in the results except perhaps for quite low frequencies. Experimental studies of nozzle admittance are almost nonexistent, and that area still constitutes essentially virgin acoustic territory. Fortunately, the transverse modes are relatively insensitive to nozzle admittance, which is primarily a boundary condition on the axial component of the motion. Thus, for some purposes, it will be permissible to ignore the "end effect" arising from the finite length of the propellant, and pretend that the motor is infinitely long.

The Long Rocket Motor

Considerable insight into the stability problem has been achieved by considering the limiting case of infinite length. In this limit, and with the small mean flow velocity neglected, the acoustic modes can be found with little difficulty. It is necessary only to solve the vector wave equation appropriate to the solid and join the solution properly to the solution of the simpler scalar wave equation for the gas. This problem has been studied in detail only for rather simple boundary conditions at the outer surface of the propellant.[6, 7] For example, suppose that the outer surface is rigidly constrained, both radially and tangentially, and that the viscosities can be ignored. Then we may easily find the frequencies of the various fundamental modes and their overtones. A typical result is shown in Fig. 7. The two families of dotted lines are the results of approximate calculations which aid in the interpretation of the result. The "quasi-gas" mode family is obtained by regarding the solid as infinitely rigid compared with the gas, and the "quasi-solid" mode family is obtained by regarding the gas as infinitely elastic compared with the solid. (The quasi-solid modes are not ex-

[3] T. A. Angelus, "Unstable Burning Phenomenon in Double-Base Propellants," *Progress in Astronautics and Rocketry*, **1**, Academic Press, Inc., New York, 1960, 527–559.

[4] L. Crocco, "Transversal Admittance of de Laval Nozzles" (unpublished notes).

[5] F. E. C. Culick, "Stability of High Frequency Pressure Oscillations in Gas and Liquid Rocket Combustion Chambers" (Doctoral dissertation, Massachusetts Institute of Technology, 1961).

[6] F. T. McClure, R. W. Hart, and J. F. Bird, "Acoustic Resonance in Solid Propellant Rockets," *J. Appl. Phys.*, **31**, May 1960, 884–896; also, J. F. Bird, R. W. Hart, and F. T. McClure, "Vibrations of Thick-Walled Hollow Cylinders: Exact Numerical Solutions," *J. Acoust. Soc. Am.*, **32**, Nov. 1960, 1404–1412; also, J. F. Bird, "Vibrations of Thick-Walled Hollow Cylinders: Approximate Theory," *ibid.*, 1413–1419.

[7] O. J. Deters, "Effects of Gas Phase and Solid Phase Damping on Instability of Low Frequency Modes in Solid Propellant Rockets," *J. Am. Rocket Soc.*, **32**, Mar. 1962, 378–384.

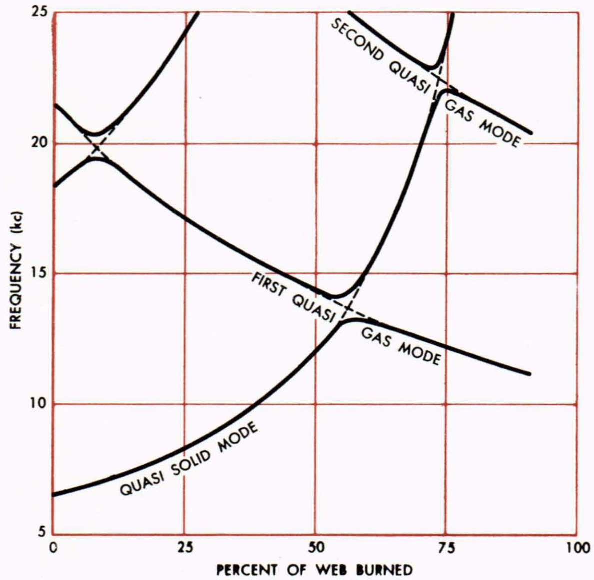

Fig. 7—Frequencies of the lowest transverse mode and its first two overtones for a cylindrical grain.

cited because they have a pressure node at the burning surface.) Because of the great difference in elastic moduli of the solid and the gas, both points of view are accurate for most frequencies. It is only in the neighborhood of the degeneracies of these quasi-modes that neither will be accurate, and where we must have recourse to rigorous theory. Thus, we may note the downward trend of the quasi-gas modes, which reflects the fact that the gas region increases in diameter as the web burns, and of the upward trend of the quasi-solid modes which occurs as the diameter of the solid decreases. The true modes split apart in the neighborhood of the quasi-mode crossings, as indicated by the unbroken curves.

One significant result follows immediately from these considerations, if we recall that it is the pressure amplitude at the burning surface which the propellant can amplify. Thus, a mode will be stable whenever it is characterized by a pressure node lying sufficiently close to the burning surface, and these stable configurations occur in the immediate neighborhood of the degeneracy points. If the motor of Fig. 7 were oscillating in the first quasi-gas mode, for example, we would expect those oscillations to decay for web thicknesses of approximately 10% and 60% of the total web. The experimental data of Fig. 3 show this type of intermittency which is typical of many propellants.

Acoustic Losses

When the acoustic fields have been determined, we must consider the acoustic energy balance in order to decide whether or not the acoustic losses will be sufficient to insure stability. The rocket motor, with its possible sources and sinks of acoustic energy, is illustrated schematically in Fig. 8. Each source or sink may be characterized for acoustic purposes by appropriate viscosities or admittances.

The question of when or whether sound is damped or amplified in the body of gas is in some respects a mixed acoustic and chemical problem. Thus, the chemist has been concerned with the effect on the acoustic field of residual reactions in the propellant channel. But the measurement techniques lie predominantly in the domain of acoustics, and there have not yet been any research studies directed explicitly at measurement of the acoustic damping length in the hot propellant gases. Even the question of the relaxation losses to be expected in such gases remains unresolved. There is, however, one treated aspect of gas phase attenuation which is relevant to the rocket problem, whose application leads to some useful insight, namely acoustic damping due to particles in the gas.

Propellant gases are rarely really smokeless, and often contain appreciable amounts of various solids larger than "smoke." These particles impede the acoustic motion of the gas, and act to some extent like thermal sinks for the fluctuating temperature, which is a part of the acoustic field. The theoretical description of the attenuation produced by such particles was developed several years ago, and it is rather easily applied to the rocket motor problem.[6]

Little can be said quantitatively about the other acoustic loss mechanisms. It appears that nozzle loss is likely to be relatively low for the relatively high-frequency transverse modes, and thus one

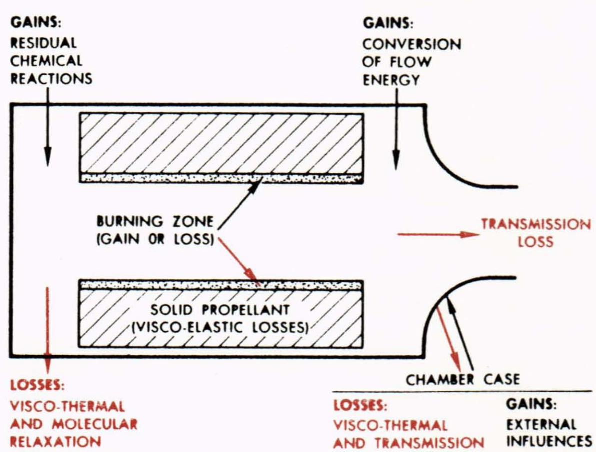

Fig. 8—Schematic representation of a rocket motor showing possible sources of acoustic gains and losses.

can perhaps explain why such modes may be highly excited within a rocket motor without being dramatically audible to an outside listener. However, the nozzle draws attention to itself not only as an acoustic loss, but also as a possible acoustic source. In this connection, we recall that oscillatory motion is transported not only via the acoustic field at the speed of sound, but also via convection at the speed of the mean gas motion. As a result, the possibility of conversion of convectively transported energy into sound must be considered. It has been shown theoretically that this conversion must occur under those circumstances[4, 5] for which the nozzle is characterized by an admittance having a negative real part. Under such circumstances, even the condition that $\tilde{\mu}/\tilde{\epsilon}$ be less than $\sim 1/\gamma$ is not necessarily sufficient to insure acoustic stability.

The thermal and viscous losses at the motor walls should receive consideration, although relatively little wall surface is exposed in motors of practical geometry. Theoretical calculation of such losses is straightforward in systems for which the mean temperature of the walls and the mean temperature of the gas are equal. This condition is not met in the usual rocket motor, however. In fact, one can hardly be certain that the effects will correspond to losses, since there is the possibility of conversion of convectively transported energy into sound in the boundary layer.

Finally, in order to specify the losses in the solid, at least the frequency-dependent shear and dilatational viscosities are required, and some data on the shear viscosities of propellants have started to appear very recently.[8] There has been almost no application of predominantly acoustic techniques to this problem.[6, 7]

Stability Determination

The question of stability or instability has been discussed quantitatively in terms of a rather complicated equation which expresses the net of the gains and losses as the sum of all of the various contributions from the various surfaces, and from the volume of the gas and the solid.

Since, as has been discussed, many of the parameters necessary for the evaluation of the stability criterion are not yet known, it is possible to discuss the problem analytically only for certain special cases. For illustrative purposes, let us consider the stability in the first transverse mode of a long motor in which gas phase relaxation or particle

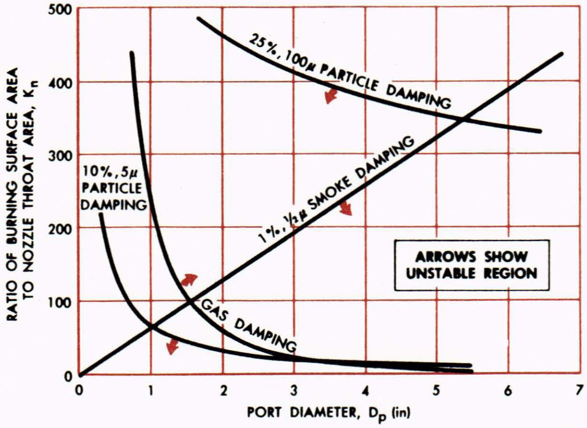

Fig. 9—Neutral stability contours corresponding to various kinds of damping mechanisms. The arrows point into the unstable region.

damping might be the major sources of attenuation. Then it is possible to estimate the loss terms. Further, we shall confine our attention to a relatively narrow band of frequencies wherein the admittance of the burning surface might be regarded as a constant. The result is shown by Fig. 9.[9] We should note that at the instant of firing, the initial values of port diameter and burning surface area define a starting point on the K_n, D_p chart. As the propellant burns, both D_p and K_n increase linearly with time, and the instantaneous configuration of the motor describes a straight line passing through the initial point and through the origin. On the other hand, if smoke damping were predominant, and if the initial K_n, D_p were to lie below the smoke-damping line, the motor would be unstable throughout its firing, whereas it would remain stable if its geometry placed its initial chart-point above this line. It is interesting that precisely such a straight neutral stability line was observed in the experiments of Brownlee and Marble.[8] On the other hand, if damping by particles were predominant, the motor would be unstable initially but would become stable when its chart-point crossed the corresponding curve.

Further Considerations

In the attempt to gain an overall picture of rocket instability, many interesting and important aspects of the problem have necessarily been omitted. The mean flow field, for example, has associated with it a number of effects other than its influence on orifice loss which has already been mentioned. Here are two other consequences of mean flow which should be considered.

[8] W. G. Brownlee and F. E. Marble, "An Experimental Investigation of Unstable Combustion in Solid Propellant Rocket Motors," *Progress in Astronautics and Rocketry*, 1, Academic Press, Inc. New York, 1960, 455–494.

[9] J. F. Bird, F. T. McClure, and R. W. Hart, "Acoustic Instability in the Transverse Modes of Solid Propellant Rockets," 12th International Astronautical Federation Congress, Washington, D. C., Oct. 1–7, 1961.

Entropy Waves—It has already been mentioned that the slow mean flow in the propellant channel will usually have little effect on the dynamical field of the relatively high-frequency transverse modes. Perhaps surprisingly, the state of affairs is quite different at low frequencies because of a boundary layer effect. We will recall that sound waves are isentropic disturbances, and thus the local gas temperature fluctuates in accordance with the adiabatic relationship

$$\frac{\delta T}{T} = \frac{\gamma - 1}{\gamma}\frac{\delta \rho}{\rho}. \quad (2)$$

However, the boundaries tend to be rather more isothermal than adiabatic, so that the dynamical field in the immediate neighborhood of a boundary is not describable in terms of the usual sound field. As a result, there is an acoustic boundary layer which, in effect, separates the actual boundary from the purely isentropic sound field (cf. Fig. 10). Thus, when a sound wave is incident on the boundary, not only is a reflected sound wave produced, but also a thermal "wave" or front. In the usual acoustic circumstances, this thermal front disappears in the immediate vicinity of the boundary. (The boundary layer thickness in a nonflowing gas is associated with a characteristic length for conduction of heat into the boundary, and the layer is very thin.) This length is $\sqrt{\lambda/2\omega C_v \rho} \sim 10^{-3}$ cm for thermal conductivity $\lambda = 5 \times 10^{-4}$ (cal/sec cm°K), $\omega = 2\pi \times 1000$ cps, ρ = gas density $\sim 6 \times 10^{-3}$ gm/cm³, and C_v = heat capacity $\sim 1/3$(cal/gm°K). However, in the presence of a mean flow out of the surface, the boundary layer can become quite thick. For typical flow velocities, its thickness is indicated by the characteristic length $\gamma C_v \rho v^3/\lambda\omega^2 \sim 1$ meter at a frequency of 200 cps. In this low frequency domain,

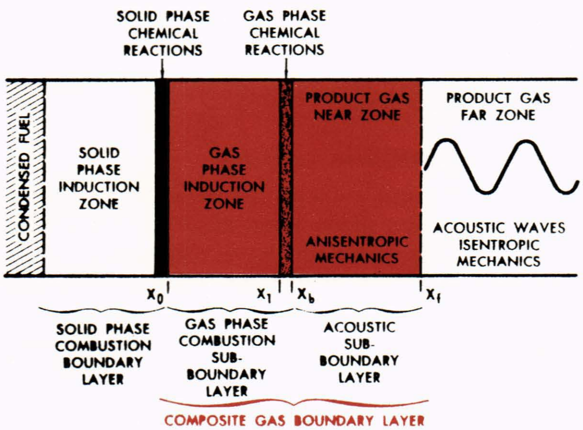

Fig. 10—Schematic representation of the gas-solid boundary region.

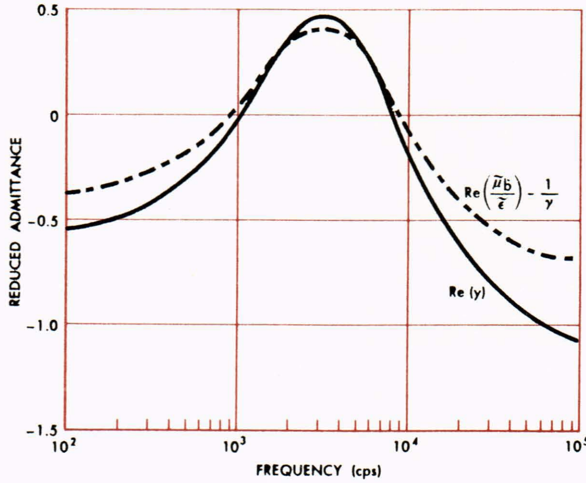

Fig. 11—The effect of entropy wave generation on amplification by the burning propellant. Amplification occurs above the X-axis.

the entire rocket motor channel may be included within the acoustic "boundary layer!" The thermal "front" has been modified by flow to become an entropy wave which propagates at essentially the speed of the mean flow. Since its speed is perhaps one-thousandth that of sound speed in the gas, the wavelength of these entropic waves is very short. These waves are characterized by oscillations primarily in temperature and density, and thus tend to be observable visually in rocket motors as thin striations in brightness which propagate away from the propellant surface with the flow speed.

Even in the moderately high-frequency regime where the acoustic boundary layer is thin, its effects do not vanish. We find that the relationship between the mass response of the propellant and the acoustic admittance is not really given by so simple a relationship as that of Eq. (1). The correct relationship is not difficult to obtain,[10] but it involves frequency, flow rate, and temperature gradient at the boundary in a rather complicated way. However, the expected effect of the acoustic boundary layer on the acoustic admittance of the burning layer is shown by the sample calculation in Fig. 11. In this figure, the quantity y represents the dimensionless reduced specific admittance obtained by dividing out the factor $-v_p/\bar{P}$. As we should expect, the low frequency values could be obtained by replacing the $1/\gamma$ in Eq. (1) by unity (the dot-dash curve corresponds to Eq. (1)). The domain of interest is the Real of $y \geq 0$, so we see that, at least in this example, the inclusion of the acoustic boundary

[10] R. W. Hart and R. H. Cantrell, "On Amplification and Attenuation of Sound by Burning Propellants," The Johns Hopkins University, Applied Physics Laboratory, TG 335-11, April 1962.

layer would have a minor effect on the frequency region in which instability might occur.

Erosion and Some Nonlinear Acoustics—The burning rate of propellants is affected not only by pressure but also by the component of velocity parallel to the surface. In early experiments, it was found that the presence of such a parallel velocity component increased the steady-state burning rate, hence the terminology "erosion" and the reference to this velocity as the "erosive velocity" (v_e). In general, a linear relationship between burning rate (m) and erosive velocity (v_e) exists over a fairly wide velocity range, so that the steady-state burning is often described by the equation

$$\bar{m} = (\text{function of pressure})\left\{1 + K\frac{|v_e|}{c}\right\}, \qquad (3)$$

where c is the sound speed, and K is a dimensionless constant (usually found to be less than about 5) called the erosion constant. The absolute value sign describes the fact that the effect of erosive velocity is independent of its direction.

Now it is clear that the acoustic response of the burning surface to erosive velocity should be studied in order to be able to describe further the ability of the propellant to amplify. From the experimental point of view, the measurement of this response seems to fall in the domain of the acoustician, although none have so far tackled this problem. From the theoretical point of view, the erosive response calculation appears to be fundamentally two-dimensional and thus should prove to be more difficult than the essentially one-dimensional calculation of the pressure response which we have already considered.

It seems instructive to evaluate the possible importance of erosive effects on acoustic stability. This can readily be done if we carry over the linear form of Eq. (3) into the time domain and introduce a frequency-dependent erosion constant.[11]

One conclusion becomes evident immediately: where the mean flow velocity is zero, or where the erosive component of acoustic velocity is perpendicular to the mean flow (as it would be for transverse modes in the motor of Fig. 8), the absolute value sign indicates that rectification occurs, and thus no erosive response would occur at the fundamental frequency. Since no acoustic energy would be erosively coupled into the normal mode, erosion would then only be a producer of harmonics (and changed mean burning rate).

[11] F. T. McClure, J. F. Bird, and R. W. Hart, "Erosion Mechanism for Nonlinear Instability in the Axial Modes of Solid Propellant Rocket Motors," *J. Am. Rocket Soc.*, **32**, Mar. 1962, 374–378.

But for an axial mode, the acoustic velocity adds and substracts from the axial mean flow, and for arbitrarily small acoustic velocities, the absolute magnitude sign may be removed. In this case, erosion could influence linear acoustic stability. In order for such an influence to occur, however, there must be a component of response in phase with pressure. In the usual single-ended rocket motors, the mode symmetry is such that erosion produces amplification in alternate quarter-wave segments and attenuation in the intermediate segments, so that the net effect tends to cancel out.

These considerations suggest an interesting consequence of *finite* amplitude, however, where rectification may occur in only that part of the motor where the mean flow is small. Amplification or attenuation would pertain elsewhere. Thus, if the forward half of the motor were damping for infinitesimal fluctuations, and the rear half were an erosive amplifier, one would find a net amplification at amplitudes high enough that rectification occurred in the front end only.

Fortunately, this is one type of nonlinear problem which is not difficult to treat. The reason stems from the fact that the boundary equation (3) introduces the nonlinearity as a term of order $v/\bar{v}_e$, rather than as a term of order v/c. This means that the acoustic wave equation for the gas, wherein the nonlinear terms are much smaller

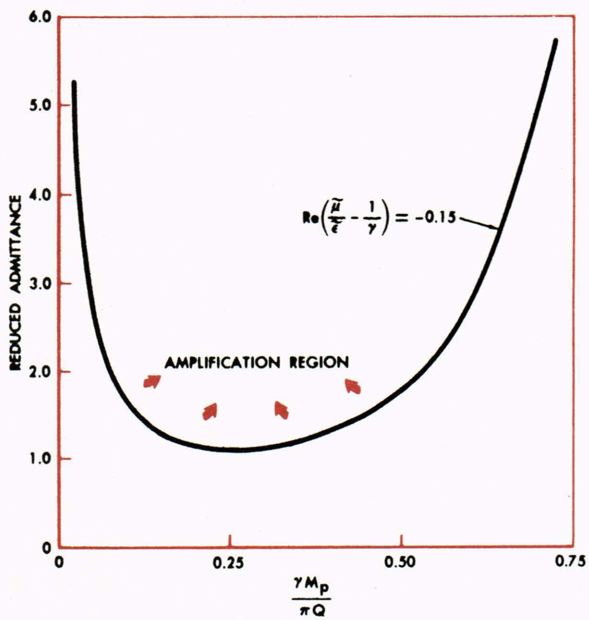

Fig. 12—The effect of finite amplitude and erosion on stability in axial modes for a hypothetical propellant. The ratio of acoustic pressure amplitude to the mean pressure is Q, and the port Mach number is M_P.

TABLE I

IMPORTANT PARAMETERS REQUIRED FOR STABILITY DETERMINATION*

<table>
<tr><th>Quantity</th><th>Mechanism</th><th colspan="2">Function of</th></tr>
<tr><td>Admittance of burning layer</td><td>Amplification or attenuation at the burning surface</td><td>Frequency, pressure, propellant temperature, erosive velocity</td><td rowspan="3">Propellant composition, curing time, method of cure, etc.</td></tr>
<tr><td>Gas-phase damping length or attenuation coefficient</td><td>Amplification or attenuation in the gas phase</td><td rowspan="2">Frequency, pressure, propellant temperature</td></tr>
<tr><td>Solid-phase viscoelastic constants</td><td>Contributes to determining mode frequencies and to attenuation in the solid phase</td></tr>
<tr><td>Nozzle admittance</td><td>Contributes to determining mode frequencies and to gain or loss at the nozzle plane</td><td colspan="2">Frequency, mode, mean flow distribution, nozzle size and shape, sound velocity in gas, and the sound field itself</td></tr>
</table>

* Other parameters as well, such as those describing wall losses, resonant rod losses, inputs due to aerodynamic screaming, etc., may occasionally be important.

(of order v/c), may correctly be regarded as linear. Because of this, it is permissible to ignore the perturbing effect of erosion on the normal mode and to evaluate the net flux of acoustic power by essentially the same procedure that was applicable to the infinitesimal amplitude case. Focusing our attention on the burning surface response to erosion by ignoring other gains and losses, we calculate the behavior shown in Fig. 12.[11] This figure shows, for example, that if the pressure response of the burning surface were given by $\tilde{\mu}/\tilde{\epsilon} - 1/\gamma = -0.15$ and if the imaginary part of the erosion constant were $-1.8/c$, then the surface would amplify and the motor could become unstable if the quantity $\gamma M_P/\pi Q < \frac{1}{2}$, i.e., if it were subjected to a disturbance having a maximum fractional pressure amplitude $Q > 2\gamma M_P/\pi$, where M_P is the port Mach number. For $M_P = 0.025$, $\gamma \sim 1.2$, we would expect instability if the disturbance pressure amplitude were greater than 2% of the steady-state pressure. Perturbations of this order of magnitude may sometimes be encountered in practice, and there is some qualitative experimental support for the pertinence of these calculations. Quite obviously, however, there are still too many unresolved pieces of the complete stability problem to permit really quantitative calculations to be made.

Resume

It will be apparent from the preceding description that theoretical studies have met with an encouraging degree of success in discerning and illuminating critical features of rocket motor instability. Concurrently, experimental studies have progressed in elegance and in their pertinence to the basic description of instability. The scope of the problem is becoming fairly well described, but it is by no means solved. There are many critical unknowns and a spectrum of associated aspects which have not been studied under conditions such as exist in rocket motors.

Some of the factors which are clearly important are collected for review in Table I. Their relevance to the field of acoustics is quite evident, and they may provide problems of considerable challenge.

There are other factors which are perhaps of less immediate importance, but which in any case provide puzzling acoustic problems. For example, there is the problem of the accurate determination of acoustic modes, taking into account realistic mean flows and the entropy waves. The theoretical study of finite amplitude effects in rocket motors is still virtually untouched. In this connection, it has been reported that steel rods have "burned" away at the rate of 1.0 in. per sec in the high-amplitude acoustic environment, but the mechanism is not yet understood. It might even be useful merely to clarify quantitatively how it is that virtually monochromatic acoustic fields with fluctuating pressure amplitudes of the order of 50 psi sometimes exist when in more reasonable environments such waves quickly degenerate as wave form distortion and harmonic generation set in.

Hopefully, in addition to presenting a picture of the progress in our understanding of acoustic instability in solid propellant rockets, we have also conveyed the impression that here is a new and important field open to acousticians.