Criteria for radiation patterns, cross-over gain, and modulation sensitivity are analyzed for conical-scan-type tracking antennas using amplitude- or phase-comparison methods. It is shown that the radiation patterns of interest are not, as generally believed, the individual static patterns, but their RF vector sum and difference. A method of measuring these sum and difference patterns is suggested.

T. C. Cheston

Criteria for CONICALLY-SCANNED TRACKING ANTENNAS

Tracking antennas often obtain high angular resolution by a time-sequential lobe-comparison method. The radiation pattern is scanned (nutated) about the boresight direction in such a way that it traces out a cone, a method called conical scanning. Figure 1 shows two radially opposite positions of the radiation pattern. When the target is in the boresight direction, a continuous-wave (CW) signal is obtained. When the target is off boresight, the signal is amplitude-modulated at the nutation frequency; the magnitude and phase of that modulation define the direction of the target. In this discussion, the tracking antenna will be considered as a receiving antenna, with the target separately illuminated.

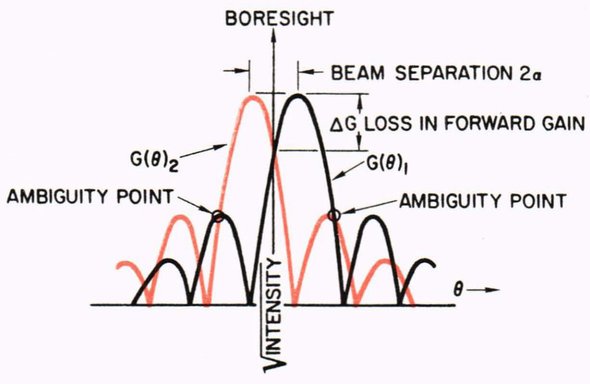

Fig. 1—Lobe comparison—static patterns.

Introduction

Radiation Patterns—The antenna tracks after having acquired the target. In this way, the target is always, at least approximately, in the boresight direction. If the only signal at the tracking antenna is an undistorted plane wave front arriving from a single target, then the only antenna pattern characteristics of interest are gain and modulation sensitivity in the boresight, or cross-over, region. Such other characteristics as sidelobe structure are irrelevant under these conditions.

The complete radiation pattern characteristics are of interest only if one or more additional signals from secondary sources in other directions arrive simultaneously at the antenna. These secondary sources may be real or virtual and may or may not be phase-coherent with the signal from the target. Only coherent secondary sources will be considered here, though it can be shown[1] that similar arguments apply if the secondary sources are not phase-coherent.

The signals from the target and secondary source considered here are thus of the same frequency. Virtual secondary sources may arise if the target signal is received from two or more directions because of reflections from the ground or from obstacles or because of distortions of the plane phase front. Radomes give rise to both reflections and phase-front distortions, and the aberrations of a radome may be regarded as errors due to virtual secondary sources. Real secondary sources can result from the presence of additional targets moving with the same radial velocity. Small, incoherent variations in relative radial velocity may, in such cases, be regarded as noise.

Since the antenna tracks the primary target, a constant RF signal from that target is always present; its amplitude is determined by the cross-over gain. The return from the secondary source, modulated at the nutation frequency, is added *vectorially* to that signal. Phase is meaningful since both the primary and secondary signals co-exist at the same output terminal. Therefore, the modulation that results from the secondary source, that gives erroneous directional information, arises from the *vector* difference of the static patterns in the direction of that secondary source as a function of time. Similarly, a component is obtained from the vector sum of the static patterns, which gives erroneous amplitude information. The radiation patterns of interest are, therefore, the sum and the difference patterns obtained by the *vectorial addition and subtraction* of the static patterns; these are exactly equivalent to the familiar sum and difference patterns of monopulse-type antennas. In fact, the conically-scanned tracking-type antenna may be regarded, in this respect, as a monopulse antenna operating on a time-sharing basis.

The vector sum and difference patterns, as opposed to static patterns, correspond to a secondary source that has the worst possible phase relationship with the primary target signal. Under real conditions, this phase relationship can have any value and there can be no control over it. The worst case must therefore be taken for assessing antenna performance.

A simple example shows how misleading it can be to assess antenna performance only from static patterns. Figure 2A shows ideal sidelobe-free sum (Σ) and difference (Δ) patterns. Figure 2B shows the equivalent static patterns that each contains one large, but desirable, sidelobe. Figure 8 shows how, in the case of a phase-comparison system, a large sidelobe in the static pattern is inherently present but is in no way detrimental.

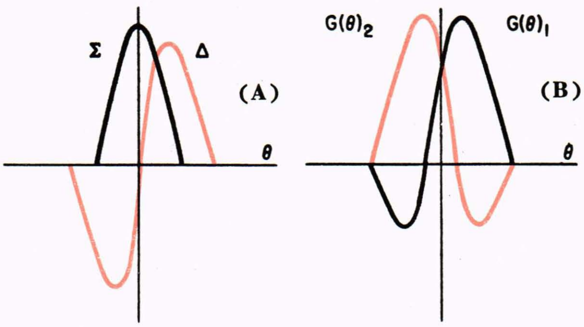

Fig. 2—(A) Ideal sum and difference patterns, and (B) static patterns for ideal Σ and Δ.

Cross-Over Gain and Modulation Sensitivity—Forward, or cross-over gain, is the gain of the radiation pattern in the boresight direction and is defined by the loss in gain (ΔG) relative to the peak gain (Fig. 1). Modulation sensitivity is a measure of the amount of modulation obtained for a small, given, off-set of the target from the boresight direction and may be defined as

$$S=100\frac{d}{d\theta}\left(\frac{G(\theta)_1-G(\theta)_2}{G(\theta)_1+G(\theta)_2}\right)_{\theta=0}\ \%\text{ per unit angle.} \tag{1}$$

Alternatively, it may be defined as

$$S'=\frac{d}{d\theta}\left(20\log\frac{G(\theta)_1}{G(\theta)_2}\right)_{\theta=0}\ \text{db per unit angle.} \tag{2}$$

Referring to Fig. 1, an increase in beam separation 2α is accompanied by a decrease in forward gain and an increase in modulation sensitivity. The minimum permissible cross-over gain is determined from the overall system requirements and characteristics and must be adequate to give an acceptable signal/noise ratio for the weakest expected target returns. Increasing the gain beyond that minimum value will not normally lead to system improvements. Increasing the modulation sensitivity, on the other hand, continuously improves the direction-finding and tracking capabilities, up to limitations imposed by other parts of the system. Excess cross-over gain may, therefore, be usefully traded for increased modulation sensitivity.

[1] T. C. Cheston, "Criteria for Conically-Scanned Tracking Antennas," The Johns Hopkins University, Applied Physics Laboratory, CM-992, Feb. 1961.

Similar to monopulse systems, an antenna radiation pattern may be conically scanned by the equivalent of an amplitude- or phase-comparison method. The general criteria for both these methods will be analyzed.

Amplitude Comparison

In the amplitude-comparison system of conical scanning, the beam is displaced by a tilted phase front that rotates to scan the beam conically. A typical example of such a system would be a nutating feed in the focal plane of a reflector or lens-type antenna.

Optimum Beam Displacement for Small Sidelobes—In the following analyses, the problem is treated one-dimensionally. Referring to Fig. 3,

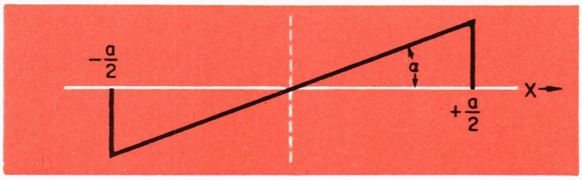

Fig. 3—Aperture phase distribution.

the antenna aperture extends from $x=-a/2$ to $x=+a/2$ and has an amplitude distribution $F(x)$. The phase distribution across the aperture is linear and is given by $k\alpha x$, where $k=2\pi/\lambda$ and α is the beam tilt angle.

The radiation pattern is then given by the Fourier transform of the amplitude and phase distribution

$$G(\theta)=\int_{-a/2}^{+a/2} F(x)\ e^{-jk\alpha x}\ e^{-jk\theta x}\ dx.$$

The sum radiation pattern is obtained from two amplitude distributions with opposite phase slopes:

$$\begin{aligned} G(\theta)_\Sigma &= \frac{1}{2}\int_{-a/2}^{+a/2} F(x)\ e^{-jk\alpha x}\ e^{-jk\theta x}\ dx \\ &+\frac{1}{2}\int_{-a/2}^{+a/2} F(x)\ e^{+jk\alpha x}\ e^{-jk\theta x}\ dx \qquad (3) \\ &= \int_{-a/2}^{+a/2} F(x)_\Sigma\ e^{-jk\theta x}\ dx, \end{aligned}$$

where

$$F(x)_\Sigma = F(x)\ \cos(k\alpha x).$$

Similarly, the difference pattern is

$$G(\theta)_\Delta = \int_{-a/2}^{+a/2} F(x)_\Delta\ e^{-jk\theta x}\ dx, \qquad (4)$$

where

$$F(x)_\Delta = jF(x)\ \sin(k\alpha x).$$

If the aperture had been excited with constant phase, its radiation pattern would have been

$$G(\theta)_{cp} = \int_{-a/2}^{+a/2} F(x)\ e^{-jk\theta x}\ dx. \qquad (5)$$

Thus, the sum radiation pattern is derived from an amplitude distribution that equals the original (constant-phase) amplitude distribution multiplied by $\cos(k\alpha x)$. Similarly, the difference radiation pattern is derived from an amplitude distribution that equals the original (constant-phase) amplitude distribution multiplied by $j\sin(k\alpha x)$.

With constant amplitude distribution, the smallest sidelobes would be obtained in the sum pattern when $\cos(k\alpha x)=0$, with $x=\pm a/2$. In the difference pattern, the smallest sidelobes would be obtained when $\sin(k\alpha x)=0$, with $x=\pm a/2$, but this amount of beam tilt would cause the sum pattern to split, having zero forward gain.

The maximum value of the beam tilt angle is thus limited by considerations of the sum pattern. Low sidelobes in both sum and difference patterns, however, may still be obtained by suitably tapered amplitude distributions.

Other considerations may be used to define optimum beam tilt angles. Rhodes[2] suggests that the product of the sum signal and the slope of the difference signal should be a maximum. This maximum is broad and corresponds approximately to a 3-db loss in forward gain. It agrees reasonably with the optimum requirement for low sidelobes.

Modulation Sensitivity—All usual radiation patterns of an antenna may conveniently be described, with sufficient accuracy for present purposes, in terms of 3-db beamwidth, BW, of the static pattern, by substituting $\frac{\frac{8}{9}}{BW}$ for a/λ, giving

$$G(\theta)=\frac{\sin\left(\frac{P}{BW}\ \theta\right)}{\frac{P}{BW}\ \theta},$$

where

$$P=8\pi/9.$$

[2] D. R. Rhodes, "Introduction to Monopulse," McGraw-Hill Book Co., New York, 1959.

The static patterns of Fig. 1 are then

$$G(\theta)_1 = \frac{\sin\left\{\frac{P}{BW}(\theta-\alpha)\right\}}{\frac{P}{BW}(\theta-\alpha)},$$

and

$$G(\theta)_2 = \frac{\sin\left\{\frac{P}{BW}(\theta+\alpha)\right\}}{\frac{P}{BW}(\theta+\alpha)}, \qquad (6)$$

where α is the beam tilt angle.

Equations (1), (2) and (6) give the modulation sensitivity for the cross-over region as

$$S = 279\left\{\frac{1}{P\frac{\alpha}{BW}} - \cot\left(P\frac{\alpha}{BW}\right)\right\} \% \text{ per } BW, \qquad (7)$$

$$S' = 48.5\left\{\frac{1}{P\frac{\alpha}{BW}} - \cot\left(P\frac{\alpha}{BW}\right)\right\} db \text{ per } BW. \qquad (8)$$

Figure 4 shows the modulation sensitivity as a function of beam separation, and Fig. 5 shows it

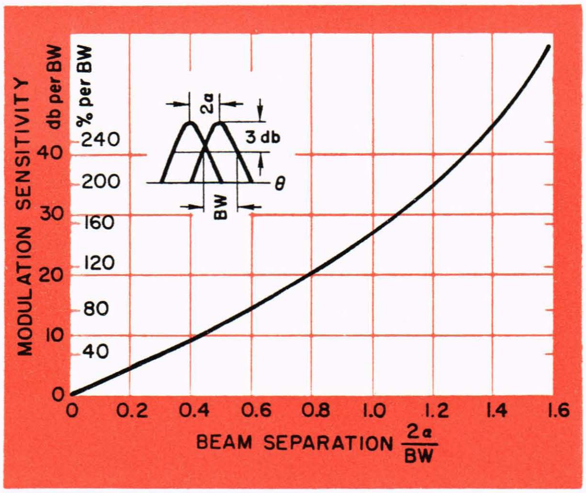

Fig. 4—Amplitude comparison—modulation sensitivity versus beam separation.

as a function of ΔG, the loss in forward gain at cross-over, where

$$\Delta G = -20 \log\left(\frac{\sin\left(P\frac{\alpha}{BW}\right)}{P\frac{\alpha}{BW}}\right) \text{db}. \qquad (9)$$

When ΔG is smaller than about 2 db, then Eq. (9), combined with Eq. (7) or (8), gives approximately

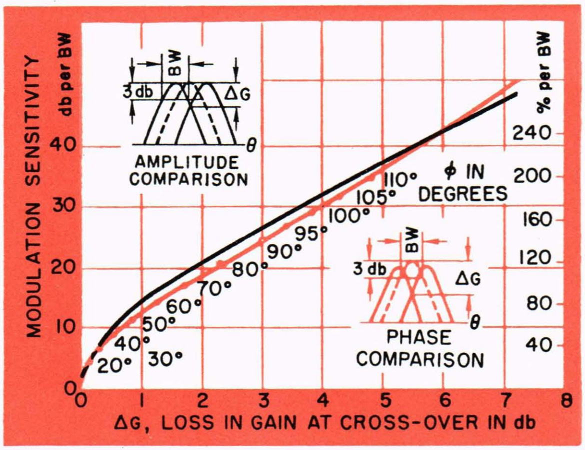

Fig. 5—Modulation sensitivity versus cross-over gain.

$$S \simeq 77\sqrt{\Delta G}/BW \qquad \% \text{ per degree},$$

$$S' \simeq 13.5\sqrt{\Delta G}/BW \qquad \text{db per degree},$$

where ΔG is expressed in db and BW in degrees. These results agree with published data.[3]

Phase Comparison

In a phase-comparison system, the aperture is divided into separate parts. For example, for a line-source system, the aperture may be divided into parts A and B (Fig. 6). The phase distribution across A and B is constant, and the two radiation patterns have maxima pointing in the same direction. The two apertures are displaced in space; hence, off-boresight radiation received by A will differ in phase from that received by B. For a monopulse system, the signal received by

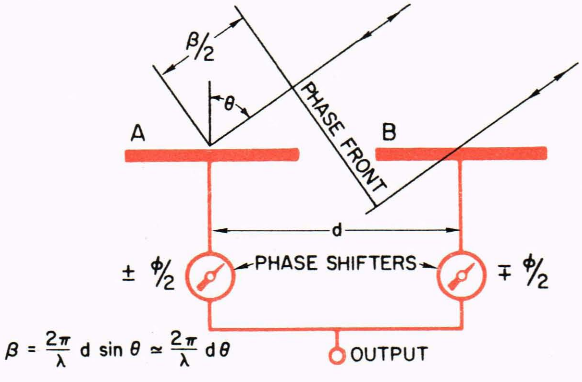

Fig. 6—Phase comparison.

[3] J. B. Damonte and D. J. Stoddar, "An Analysis of Conical Scan Antennas for Tracking," *I.R.E. National Convention Record,* **4,** 1, 1956.

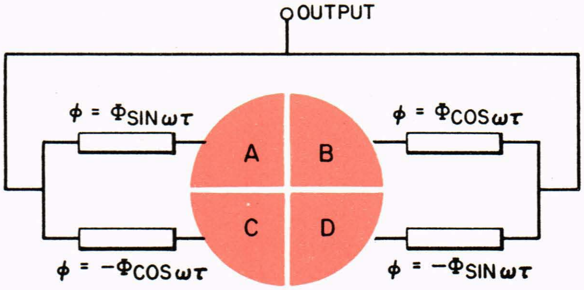

Fig. 7—Conical scan with phase comparison.

A and B may be combined in a hybrid, such as a magic-T, whose outputs are the sum and difference channels. The two apertures are thus excited in phase for the sum pattern and in antiphase for the difference pattern. For scanning, apertures A and B may be coupled to a common output through variable phase shifters, as shown in Fig. 6. In a two-dimensional system, a conically-scanned beam can be obtained from apertures A, B, C, and D, with four phase shifters as shown in Fig. 7. The insertion phase for each aperture varies sinusoidally with time, adjacent apertures being in phase quadrature and diagonally opposite apertures being in phase opposition. The beam tilt angle and modulation sensitivity are defined by the amount of phase shift.

Radiation Patterns—Let A and B denote the radiation patterns of apertures A and B, respectively, as shown in Fig. 6. Let $G\theta_1$ be the combined radiation pattern (static) when the phase shifts to apertures A and B are $+\phi/2$ and $-\phi/2$, respectively, and let $G(\theta)_2$ be the equivalent pattern (static) with phase shifts of $-\phi/2$ and $+\phi/2$.

The radiation patterns $G(\theta)_1$ and $G(\theta)_2$ are given by

$$G(\theta)_1 = \frac{1}{\sqrt{2}} A\, e^{j\beta/2}\, e^{j\phi/2} + \frac{1}{\sqrt{2}} B\, e^{-j\beta/2}\, e^{-j\phi/2}, \tag{10}$$

$$G(\theta)_2 = \frac{1}{\sqrt{2}} A\, e^{j\beta/2}\, e^{-j\phi/2} + \frac{1}{\sqrt{2}} B\, e^{-j\beta/2}\, e^{+j\phi/2}, \tag{11}$$

where β is the difference in phase of the incoming signal as received by apertures A and B.

The sum $G(\theta)_\Sigma$ and difference $G(\theta)_\Delta$ radiation patterns are then obtained by adding and subtracting $G(\theta)_1/2$ and $G(\theta)_2/2$ vectorially. Where the apertures A and B are adjacent and have identical amplitude distributions, i.e., when $A \equiv B$ and $d = a/2$, this gives

$$G(\theta)_\Sigma = \sqrt{2}\, A \cos\left(\frac{\pi}{2}\frac{a}{\lambda}\theta\right) \cos(\phi/2), \tag{12}$$

$$G(\theta)_\Delta = \sqrt{2}\, A \sin\left(\frac{\pi}{2}\frac{a}{\lambda}\theta\right) \sin(\phi/2), \tag{13}$$

and the ratio

$$\frac{G(\theta)_\Sigma}{G(\theta)_\Delta} = -\tan\left(\frac{\pi}{2}\frac{a}{\lambda}\theta\right) \tan(\phi/2). \tag{14}$$

The shape of both sum and difference radiation patterns are seen to be entirely independent of the value of ϕ. This is also true when the apertures are neither adjacent nor identical. The pattern shapes are determined only by the amplitude and phase distribution across the apertures and the spacing d. The distribution of power between sum and difference patterns, and hence the modulation sensitivity, is determined by ϕ. Figure 8 shows how static patterns and sum and difference gain patterns vary with ϕ. Static patterns $G(\theta)_1$ and $G(\theta)_2$ change with ϕ from the sum pattern, when $\phi = 0$, to the difference pattern, when $\phi = \pi$. For intermediate values of ϕ, such as $\phi = \pi/2$, large "sidelobes" will, therefore, appear in the static patterns. As previously discussed, these are not undesirable.

Equations (12) and (13) show that $\phi = \pi/2$ gives a cross-over gain $(\theta = 0)$ which is 3 db down from the gain of the apertures excited in phase $(\phi = 0)$.

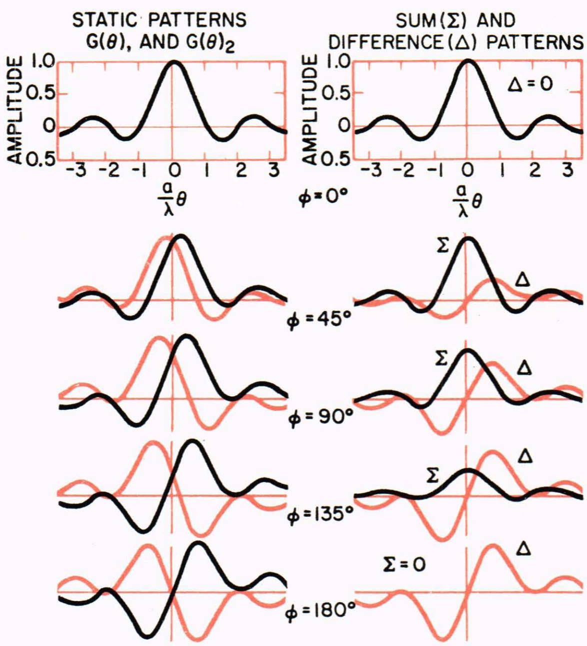

Fig. 8—Phase comparison—effects of varying ϕ. Apertures are adjacent and have a constant distribution.

If the amplitude distribution is constant and the two apertures are adjacent and of equal size a/λ, then

$$G(\theta)_\Sigma = \sqrt{a}\ \frac{\sin\left(\pi \dfrac{a}{\lambda}\theta\right)}{\pi \dfrac{a}{\lambda}\theta} \cos(\phi/2), \qquad (15)$$

and

$$G(\theta)_\Delta = \sqrt{a}\ \frac{1 - \cos\left(\pi \dfrac{a}{\lambda}\theta\right)}{\pi \dfrac{a}{\lambda}\theta} \sin(\phi/2). \qquad (16)$$

These patterns, $G(\theta)_\Sigma$ and $G(\theta)_\Delta$, will be recognized as the radiation patterns from a constant-amplitude distribution aperture excited, respectively, in phase and in anti-phase.[4]

The normalized sum and difference radiation patterns obtained with amplitude- and phase-comparison methods are compared in Fig. 9. In both cases, the amplitude distribution is constant and the loss in forward gain is 3 db. The amplitude-comparison method gives a wider sum pattern and a narrower difference pattern beamwidth than the phase-comparison method.

MODULATION SENSITIVITY—From Eqs. (1), (2), and (14), the modulation sensitivity for the cross-over region is with adjacent and equal apertures:

$$S = 50\ \pi \frac{a}{\lambda} BW \tan(\phi/2) \quad \%\ \text{per}\ BW, \qquad (17)$$

$$= 2.74 \frac{a}{\lambda} \tan(\phi/2) \quad \%\ \text{per degree}, \qquad (18)$$

and

$$S' = 8.7\ \pi \frac{a}{\lambda} BW \tan(\phi/2)\ \text{db per}\ BW, \qquad (19)$$

$$= 0.47 \frac{a}{\lambda} \tan(\phi/2) \quad \text{db per degree}. \qquad (20)$$

Equations (18) and (20) show that with two identical apertures, the modulation sensitivity per degree is entirely independent of radiation patterns, beamwidth, or amplitude distribution. This is also true if the two apertures are not adjacent. This result agrees with Rhodes' data.[2]

The above results may be expressed in terms of beamwidth to yield good approximations, for adjacent apertures which are not identical, by

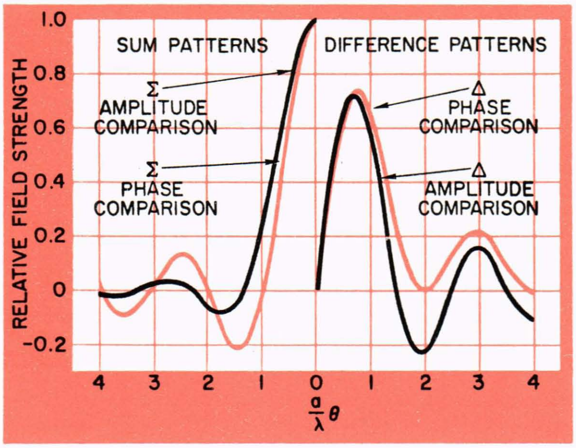

Fig. 9—Amplitude and phase comparison patterns.

the substitution $a/\lambda = \dfrac{\frac{8}{9}}{BW}$. This gives

$$S = 140\ \tan(\phi/2) \quad \%\ \text{per}\ BW, \qquad (21)$$

$$S' = 24\ \tan(\phi/2) \quad \text{db per}\ BW, \qquad (22)$$

The loss in forward gain is obtained from Eq. (12),

$$G = -20 \log \cos(\phi/2) \quad \text{db}. \qquad (23)$$

Figure 5 shows the modulation sensitivity as a function of loss in forward gain and compares the results with those obtained with amplitude comparison. For a two-dimensional conical-scan system, the required peak-to-peak value of sinusoidally-varying phase shift is $\sqrt{2}$ times the value of ϕ shown. Experimental verification within 5% has been obtained.

RADIATION PATTERN IMPROVEMENTS—Overall pattern improvements may be achieved by providing coupling between the adjacent regions of the four separate apertures of Fig. 7, particularly near the center. This coupling has no effect on the sum pattern where all apertures are excited in phase. For the difference pattern, the apertures are excited in anti-phase. The aperture distribution for the difference pattern is therefore reduced in amplitude for those regions where the coupling is effective. Coupling can thus give a suitable difference pattern distribution, with the amplitude smoothly reduced to zero at the center and, at the same time, permit a suitable sum pattern distribution with maximum amplitude at the center.

In one antenna, mutual coupling was used to reduce the measured sidelobes in the difference pattern from about −9 db without coupling to

[4] J. F. Ramsay, "Fourier Transforms in Aerial Theory," *Marconi Review*, Nos. 83-89, 1946.

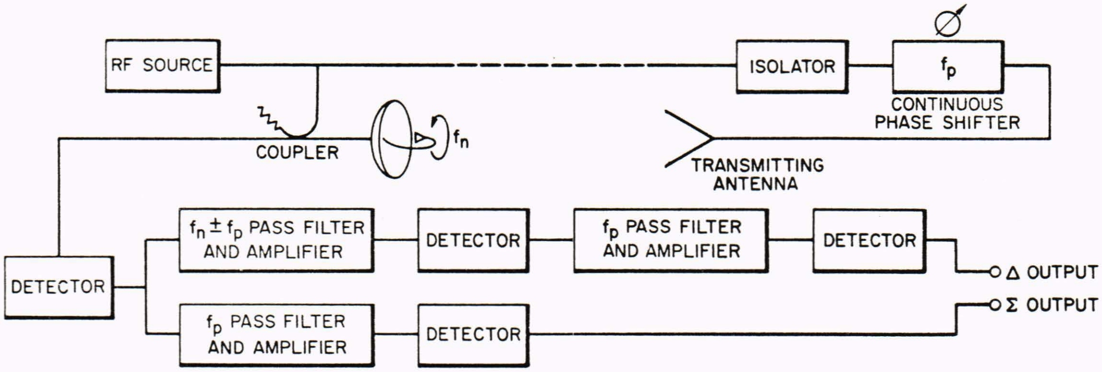

Fig. 10—Measurement of sum and difference patterns.

about -18 db with coupling, without affecting the sum pattern.

Antenna Pattern Measurements

Special techniques are required to measure the sum and difference patterns of conically-scanned beams where the amplitude-comparison method is used. These patterns are the vector sum and difference of the static patterns. Even with the beam nutating, they cannot be measured directly since the RF phase would have to be "remembered" from the time the beam points in one direction to the time when it points in the opposite direction. Figure 10 shows a simple method of measuring the sum and difference patterns.

The antenna under test is used as receiver. The beam is nutated in the normal operating mode at frequency f_n. Some of the transmitter power is tapped off and added to the received signal so that the received signal power is always small in comparison to the power added. The crystal detector then acts as a linear mixer where the IF frequency is zero. With the correct phasing of the two signals, the DC variations in the output would be a measure of the sum pattern, and the output at the nutation frequency, a measure of the difference pattern.

To avoid point-by-point phase adjustments and DC measurements, the transmitted power is single-side-band-modulated at a low frequency, f_p, ($f_p \ll f_n$) by a continuously-variable phase shifter such as, for example, described by Fox.[5] The mixer output may then be continuously analyzed in two channels: first, in a channel which passes only the frequency f_p and whose output measures the sum pattern; second, in a channel which passes $f_n \pm f_p$. Detection then yields f_p, which measures the difference pattern.

[5] A. G. Fox, "An Adjustable Waveguide Phase Changer," *Proc. I.R.E.* 35, Dec. 1947.

With conically-scanned antennas where the phase-comparison system is used, sum and difference patterns may be measured directly by adjusting the phase shifters to $+90°$ and $-90°$, respectively, for the difference pattern and to $0°$ for the sum pattern. Alternatively, a conventional monopulse-type feed system may be connected to the apertures with sum and difference channel outputs.

Conclusions

It has been shown that, in contrast to generally accepted theory, the radiation pattern characteristics which are of interest for a conically-scanned tracking antenna are identical with the equivalent monopulse sum and difference patterns.

For an amplitude-comparison system, it has been shown that the sum and difference patterns are those derived from the given amplitude distribution multiplied by a cosine or sine function, respectively. The modulation sensitivity has been calculated and plotted as a function of beamwidth and beam separation, or loss in forward gain.

For a phase-comparison system, it was found that the sum and difference patterns were invariant functions of the antenna, entirely independent of the amount of modulation. The static patterns, however, vary with modulation from the sum pattern to the difference pattern. The sum pattern may be optimized by a suitable amplitude distribution and, concurrently and independently, the difference pattern may be optimized by the use of coupling between the separate apertures. The modulation sensitivity of a phase-comparison antenna made up of identical apertures has been shown to be a function only of the spacing of the antennas and the amount of modulation, but independent of the amplitude distribution.