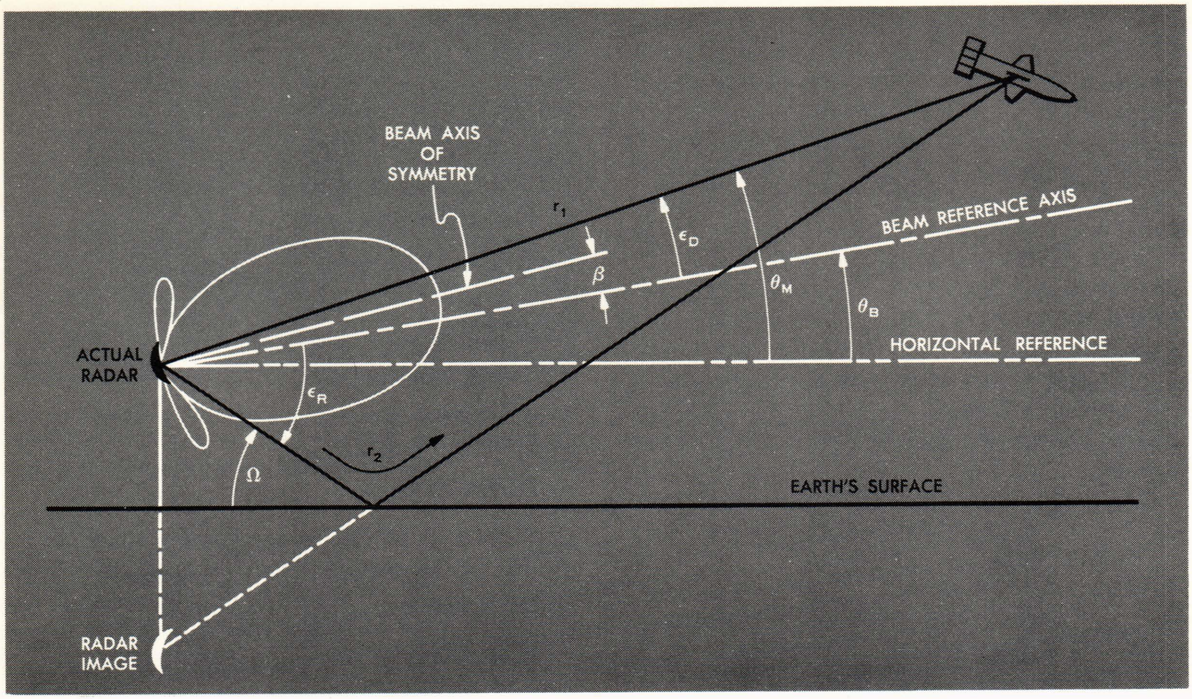
The problem of radar ocean surface reflection is reviewed, emphasizing its effect on beam-riding missile guidance. A vector model of the radar signal, which assumes the total signal to be the vector sum of a noise-free direct signal and a reflected signal, is developed. The over-all effect is readily studied by means of a real-time analog simulation. This permits evaluation of missile performance for any prescribed sea state and target position.

LOW-ANGLE BEAM RIDING
Over the Ocean

W. S. Thompson

In the long-range transmission of electromagnetic radiation, energy reflected from the earth's surface, when combined with that propagated over a direct line-of-sight path, produces an interference pattern in space. If the radiation is composed of microwave signals transmitted by a radar that is used to track a target or guide a missile in flight, the resulting interference pattern induces noise on the tracking or guidance information; its severity depends on the relative amount of reflected energy available. When the microwave transmission takes place over the ocean, the interference pattern becomes dependent upon the physical state of the reflecting surface, where a continuing loss of coherent structure is observed with increasing height of ocean waves.

This discussion is confined to the effects of ocean surface reflections on beam-riding missile guidance at low elevation angles where the effects are most pronounced. These effects have been thoroughly studied by analog simulation techniques. The results are by no means strictly limited to beam-riding guidance but are easily extended to low-angle target tracking, which may be viewed as the reverse of beam riding.



The low-angle beam-riding geometry is presented in two dimensions. The reflected signal appears to originate from an image radar below the ocean surface.

Low-Angle Interference Theory

Consider a missile located at a range r_1 from its guidance radar and at elevation θ_M above a horizontal reference. Considering the microwave signals as electric vectors, the total signal **T** appearing at the missile receiving antenna is the vector sum of a direct signal **D** propagated along the path r_1 and of a reflected signal **R** propagated along the path r_2 but appearing to originate at a radar image directly beneath the actual radar and below the reflecting surface. This vector addition is represented by

$$\mathbf{T} = \mathbf{D} + \mathbf{R} \,. \qquad (1)$$

Assuming no atmospheric attenuation of the signals, the amplitude of **D** is dependent only upon the radar antenna gain distribution. The antenna gain in a given direction is a function of the angle measured from the axis of symmetry of the beam, and on the basis of physical optics can be considered as a Fraunhofer diffraction pattern of a circular aperture. For simplicity, a polar plot of an antenna gain distribution is superimposed on the geometry in the first illustration.

The amplitude of **R** is also dependent on the antenna gain and, in addition, on the amount of attenuation upon reflection from the ocean surface. For convenience **R** is assumed further decomposed into electric vector components **C** and **I** where **C** represents that portion that is coherent with respect to **D**, and **I** is the incoherent portion assumed random in both amplitude and phase.

The vector **I** is actually the vector sum of the reflected signals from a very large number of infinitesimal random scatterers on the fluctuating ocean surface, each adding with its respective amplitude and phase. From statistical theory **I** is considered composed of two independent orthogonal noise vectors, $\mathbf{I}_x$ and $\mathbf{I}_y$, each possessing a Gaussian amplitude distribution with zero mean and standard deviation σ.[1]

The vector summation of all the components yields, for the total signal,

$$\mathbf{T} = \mathbf{D} + \mathbf{C} + \mathbf{I}_x + \mathbf{I}_y, \qquad (2)$$

which can be represented by the vector diagram. The angle ϕ between **C** and **D** represents the instantaneous phase difference between the two signals. In addition, it is the sum of the phase differences arising from the phase distribution

[1] C. I. Beard, I. Katz, and L. M. Spetner, "Phenomenological Vector Model of Microwave Reflection from the Sea," *I. R. E. Transactions* Vol. AP-4, No. 2, April 1956, 162-167.

of the radar antenna pattern, the phase shift of **C** upon reflection, and the path length difference between the direct signal and the reflected signal. If **R** consists only of **C**, i.e., no incoherent reflection, the interference pattern can be considered to result from the superposition of two Fraunhofer diffraction patterns where the phase angle ϕ alone determines the positions of the maxima and minima.

As the missile moves through the interference pattern, the angle ϕ changes continuously. Customarily for this dynamic situation, the change in ϕ is assumed to result from the change in the path-length difference $(r_2 - r_1)$. This proportionality results in an angular rate of change in ϕ of

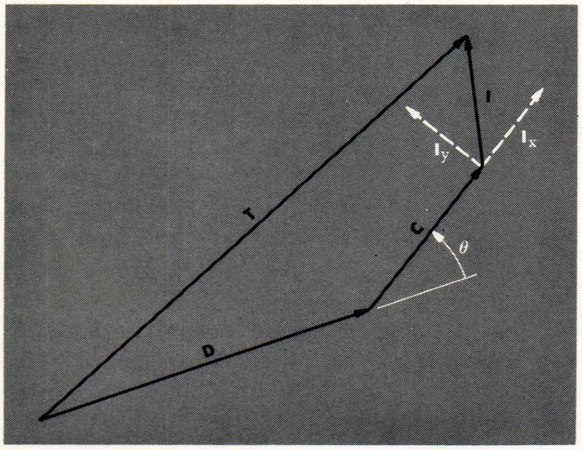
$$\omega_i = \frac{d\phi}{dt} = \frac{2\pi}{\lambda} \frac{d}{dt} (r_2 - r_1), \quad (3)$$

where λ, in the constant of proportionality, is simply the wavelength of the microwave radiation. The angular rate ω_i is expressed in terms of radians per second which, when converted to the lobe cutting or interference frequency f_i in cycles per second, can be expressed to a first approximation, in terms of the geometric quantities, as

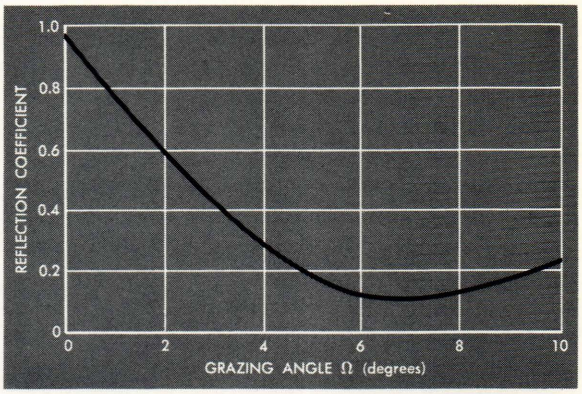
$$f_i = \frac{2d}{\lambda} \cos \theta_M \frac{d\theta_M}{dt}, \quad (4)$$

where d is the height of the radar antenna above the reflecting surface and θ_M is the missile elevation angle.

For a perfectly smooth reflecting surface, the



The total microwave signal is represented vectorially as the vector sum of its direct and reflected components.



A typical variation in the reflection coefficient as a function of the grazing angle is shown for the particular case of a vertically polarized signal with a wavelength of 2 in.

amount of attenuation upon reflection is determined by the surface reflection coefficient. The latter is defined as the amplitude ratio of the reflected signal to the incident signal and, for a given signal wavelength and polarization, is dependent upon the grazing angle Ω defined in the first illustration. A typical plot of the variation of the reflection coefficient with the grazing angle is also shown for a vertically polarized signal with a wavelength of 2 in.

When ocean waves form on the surface, additional signal attenuation results, this being determined by applying correction factors to the coherent and incoherent components. A plot of the correction factor for each component is illustrated as a function of a surface roughness parameter $\frac{h\,\Omega}{\lambda}$ where Ω and λ are the same as previously defined and h is the root-mean-square amplitude of the ocean waves. This plot exhibits a decrease in the amplitude of **C** with increasing ocean surface roughness, while σ, the standard deviation of **I**, builds up rapidly to a constant value. The constancy of σ implies that the total noise power in the incoherent component remains unchanged; however, consideration must also be given to the noise bandwidth. Experimental evidence[2] has shown that the noise bandwidth increases approximately linearly to about 1.5 cycles per second at a value of 500 mils for $\frac{h\,\Omega}{\lambda}$, which indicates that even though the total noise power remains constant with increasing surface rough-

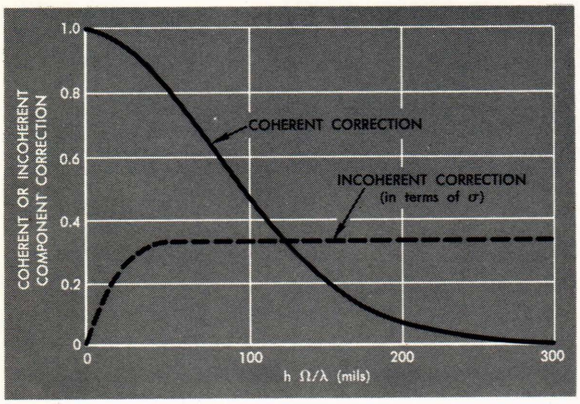
[2] C. I. Beard and I. Katz, "The Dependence of Microwave Radio Signal Spectra on Ocean Roughness and Wave Spectra," *I. R. E. Transactions*, Vol. AP-5, No. 2, April 1957, 183-191.

ness, the power per unit frequency decreases.

The above considerations of amplitude and phase yield a beat effect on the total signal **T** as the missile moves through the interference pattern. This beat effect can be visualized in connection with the vector diagram as a continuous rotation of **C** about the terminus of **D**, while **I** can assume at any instant an arbitrary direction and an amplitude characteristic of its given value of σ.

Low-Angle Beam-Riding Simulation

Beam-riding missile guidance is accomplished by a closed-loop control system employing the principle of negative feedback where the primary aim is to null the difference between the output and an input reference. In this case, the input reference is simply the elevation angle θ_B of the radar reference axis of the first plot. Ideally, the output is the missile elevation angle θ_M which, when fed back and compared with the input, yields the error angle $\theta_B - \theta_M$ denoted by ϵ_D. This forms a single loop feedback system; however, when reflection is present the system assumes a multiple loop character as will be shown.



The relative rough sea corrections for both the coherent and incoherent components of the reflected signals are shown as they vary with a roughness parameter.

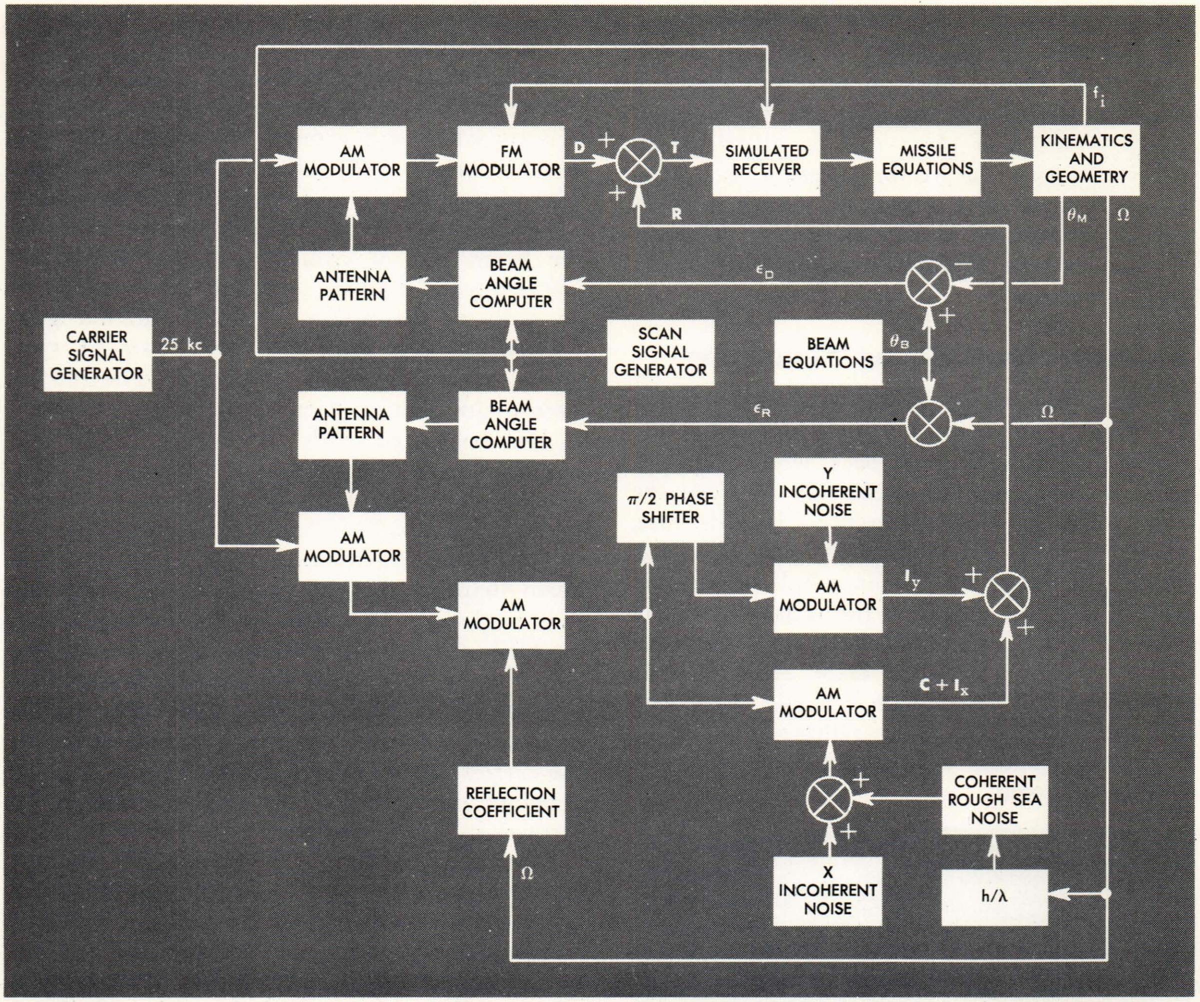
For the missile to maneuver in the proper direction to null the error angle, knowledge of the magnitude and sense of the error angle must be provided to the missile. This is accomplished by employing the microwave radiation from the radar as a carrier signal upon which the error information is impressed as amplitude modulation. The modulation is provided through the process of conical scan action of the radar beam.

Conical scan may be visualized through the following brief explanation. A right circular cone in space, with the vertex at the radar, is defined by revolving the generatrix (the beam axis of symmetry) around the cone axis (the beam reference axis) with a fixed vertex angle. The vertex angle β is defined in the two-dimensional geometry of the first illustration, in which the beam axis of symmetry is assumed to lie instantaneously at the extreme upper point of its path of revolution. As the antenna gain pattern revolves with the beam axis of symmetry, a solid figure is generated. The revolution, maintained at a constant frequency which is low compared with the carrier signal frequency, results in a sinusoidal variation or modulation impressed upon the microwave carrier signal. The missile receiver detects the amount of amplitude modulation which, for an ideal beam shape, is directly proportional to the off-axis angle ϵ_D. The sense of the error angle is established by comparing the phase of the detected modulation with a synchronized reference scan signal transmitted from the radar to the missile by means of a frequency modulation of the carrier signal. The error signal thus obtained provides the necessary steering intelligence information to the missile.

Due to the conical scan action, the reflected carrier signal also contains an amplitude modulation of the same nature but with a different amplitude, depending on the angular position of the reflected signal path in the nutating beam pattern. The addition of the reflected signal to the direct signal results in an amplitude modulation on the total signal that is not indicative of the true off-axis error angle. In general, the resulting error signal contains biases plus random noise where each is dependent upon the physical state of the sea surface, the height of the guidance radar, and the range, elevation, and trajectory of the missile.

Low-angle beam riding is most conveniently studied by a real-time analog simulation. Starting at the left of the diagram shown, a 25-kilocycle sine wave is employed as the analog of the microwave carrier signal. The signal is fed to both an upper channel for development of the direct signal and to a lower channel for development of the reflected signal.

The conical scan action is simulated first in each channel; DC voltages representing the respective off-axis error angles are inputs, along with the scan frequency signal, to beam angle computers. The output is the analog of the



The complete low-angle beam-riding simulation is represented by a simplified closed-loop block diagram.

angle between the beam axis of symmetry and the respective signal path which is varying in a sinusoidal manner about the error angle due to the conical scanning action. A static representation of the radar antenna pattern is contained in a function generator which receives the computed beam angle and generates the proper amplitude level of the simulated carrier along with its modulation component. These signals are then fed to the respective AM (amplitude modulation) modulators which yield the analogs of the direct and reflected signals as they depart from the radar on their respective paths of propagation.

The direct signal is shifted in frequency in the FM (frequency modulation) modulator by an amount equal to the interference frequency which is computed from the kinematic expression given in Eq. 4 above. This shift in frequency will yield the proper interference beat frequency when the direct signal is later summed with the reflected signal.

Meanwhile, the reflected signal is further developed in the sea model simulation. The signal is first modulated by the reflection coefficient of the third illustration. The rough sea correction is then applied by two separate subchannels, one for the inphase signal $\mathbf{C} + \mathbf{I}_x$ and the other for the orthogonal signal $\mathbf{I}_y$.

The coherent rough sea correction as a function of the surface roughness parameter $\frac{h\,\Omega}{\lambda}$ is obtained from the coherent correction curve. This is summed with the inphase incoherent correction noise possessing the standard deviation σ and the bandwidth characteristic of the sea state under investigation. Amplitude modulation of the carrier with the summed signal completes the rough sea correction for the inphase component.

The orthogonal component is developed by first shifting the carrier phase by $\pi/2$ radians, then amplitude-modulating it with the incoherent noise correction from a separate noise source possessing the same statistical characteristics as the inphase noise source.

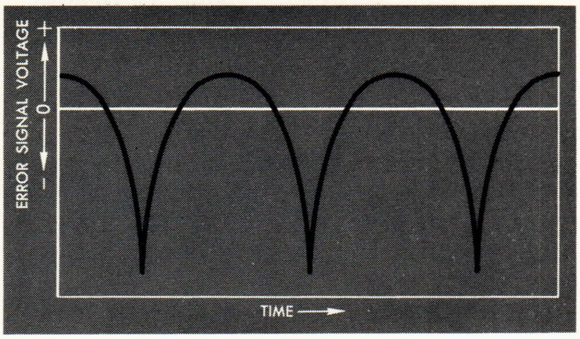
Summation of the two subchannel outputs then results in the analog of the reflected signal **R** which, when summed with the direct signal **D**, yields the total signal **T**.

The simulated missile receiver accepts the total signal and performs the essential operations of automatic gain control (AGC), carrier envelope detection, and phase comparison of the detected signal with the scan reference signal to yield the steering intelligence signal ϵ. If sea reflection were not present, the error angle ϵ_D would be truly represented by the error signal ϵ which, when plotted as a function of time, would appear as a DC voltage with the amplitude varying linearly with ϵ_D. If smooth sea reflection only were present, ϵ would assume a cycloidal shape, as shown in the last illustration, where the frequency of the oscillations is simply the interference or lobe cutting frequency. Increasing roughness of the ocean surface would result in the loss of the coherent structure of the curve which, for an extremely rough ocean surface, would appear only as random noise.

The error signal is the input to the missile equations which simulate the missile computer, the autopilot, the wing servomechanism, and the aerodynamic response. Ideally the missile computer accepts the error signal and computes the magnitude of the missile acceleration required to null the error. The autopilot accepts the computed acceleration command and determines the necessary wing deflection which is effected by the wing servomechanism. For the given wing deflection, the aerodynamic response yields a missile acceleration characteristic of the missile speed, weight, and altitude.

The kinematic and geometric simulator computes the output variables which are used in the feedback loops. The output 0_M—the missile elevation angle—is compared with the beam elevation angle θ_B to generate the error angle ϵ_D for the direct signal. The grazing angle Ω is summed up with θ_B to form the corresponding error angle ϵ_R for the reflected signal. The interference frequency feedback into the FM modulator and the grazing angle feedback into the sea model complete the closed-loop diagram.

The beam equations, which provide the beam elevation angle θ_B continuously, determine the trajectory that the missile takes to the target. The type of trajectory, such as ballistic, line-of-sight, long-range cruise, etc., determines to a great extent the behavior of the missile in the interference region.



A typical beam-riding error signal resulting from smooth sea reflection appears as a cycloidal signal at the characteristic interference or lobe cutting frequency.

An error signal such as the typical one shown permits to some extent a prediction of the behavior of a missile when the specific characteristics of the missile equations are known. Theoretically, the error signal from an ideal receiver would possess a zero mean value over a complete cycle; however, the actual result is that the sharp negative spikes never reach the full theoretical value, yielding an integrated bias that appears as a net "fly-up" error. Additional filtering and nonlinearities in the missile equations yield weird combinations of "fly-up" and "fly-down" net biases that can only be resolved through a series of simulated missile flights.

The missile behavior over a rough sea is not as easily predicted. Analysis of many rough sea simulation runs has shown that the missile closed-loop behavior may be worse than for smooth sea reflection; this, however, is not necessarily so since, in certain situations, actual improvement is obtained.

A prime interest in a study of this nature is to determine the distance by which a missile misses an assigned target. The real-time simulation presented readily permits a determination of the miss distances obtained in the presence of any prescribed low-angle environment. If the established miss distances are found to be greater than the specific tolerances, proposed changes in the system can easily be incorporated into the simulation and evaluated by additional flight simulations.