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# Upgrading Complex Systems of Systems: A CAIV Methodology for Warfare Area Requirements Allocation

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APPLICATION AREA: Cost Analysis, Analysis of Alternatives, Decision Analysis

OR METHODOLOGIES: Nonlinear Programming, Cost Analysis, Simulation

## ABSTRACT

"System of systems" terminology is now widely used to describe how the successful, combined operation of many platforms, weapon systems, and communication systems is necessary to achieve an overall warfare objective, especially in joint operations. Although the characteristics and system engineering challenges associated with systems of systems are becoming well understood, effective architecting approaches that enable cost/performance trades are still immature.

A systematic approach to allocating system of systems requirements to component systems has been developed and demonstrated on a naval mine countermeasures (MCM) system of systems of sufficient complexity to demonstrate feasibility of the approach. Treating cost as the independent variable, the process formulates a constrained, nonlinear optimization problem whose objective function is a representation of the top-level measure of effectiveness (MOE), with constraints represented by functionalized Performance Based Cost Models (PBCMs), secondary MOEs, and technology-driven bounds on system measures of performance (MOPs). Both closed-form and simulation-based optimization approaches have been demonstrated and differences quantified, including the sub-optimality of considering just one system at a time. A stochastic simulation of the MCM system of systems was implemented and optimized utilizing a constrained variant of the Simultaneous Perturbation Stochastic Approximation method, in order to demonstrate feasibility on complex systems of systems of national interest.

The process constitutes a *CAIV-based requirements allocation methodology* at the system of systems level. Application of the process is a key enabler to a disciplined, quantitative approach for upgrading a complex system of systems subject to cost and technology constraints. Application during early phases of system acquisition can result in more effective and comprehensive systems acquisition and technology investment strategies. Utilization during O&M phases can be useful in maintaining system performance while examining innovative cost reduction initiatives.

## INTRODUCTION

### Background

Engineering of complex systems of systems has been receiving increased attention recently. System of systems terminology is now widely used to describe how successful, combined operation of many platforms, weapon systems, and communication systems is necessary to achieve an overall warfare objective, especially in joint operations. This increased level of complexity has become a concern at the highest levels of command, as General John Sheehan, former Commander in Chief of U.S. Atlantic Forces, has observed:

> *"Victory will depend on the ability to master the 'system of systems' composed of multiservice hard- and soft-kill capabilities linked by advanced information technologies."* (Sheehan 1996)

These systems of systems have arisen not by design, but in response to the vision of users who recognize the tremendous potential of systems working together towards broad, common objectives, as expressed by Admiral Bill Owens, Vice Chairman of the Joint Chiefs of Staff:

> *"We have cultivated a planning programming and budgeting system that tends to handle programs as discrete entities .... Yet, the interactions and synergisms of these systems constitute something new and very important. What is happening is driven in part by broad conceptual architectures—and in part by serendipity: It is the creation of a new system of systems."* (Owens 1995)

Although the characteristics and system engineering challenges associated with systems of systems are becoming well understood, effective architecting approaches are still immature for systems of system (Manthorpe 1996 and Luman 1996). Until successful methodologies have been demonstrated, there will be little justification for the services to move away from the current acquisition focus on single systems procurements.

This article addresses how best to upgrade a complex system of systems, once the need to do so has been realized. The system of systems is considered as a whole entity and a quantitative methodology for requirements allocation to formulate an optimal upgrade suite under cost and technology constraints is demonstrated. The methodology utilizes a multi-disciplinary approach including operations analysis, cost modeling, nonlinear optimization, and stochastic modeling and simulation. Appropriate sensitivity analyses on technology constraints can yield insights to guide an effective technology investment strategy.

## Systems of Systems Definitions and Concepts

A complex system of systems can be considered to have the following characteristics (Eisner et al. 1991):

- It comprises several independently acquired systems, each under a nominal systems engineering process.
- Time phasing between each system's development is arbitrary and not contractually related.
- System couplings are neither totally dependent nor independent, but rather interdependent.
- Individual systems are generally unifunctional when viewed from the system of systems perspective.
- Optimization of each system does not guarantee overall system of systems optimization.
- Combined operation of the systems constitutes and represents satisfaction of an overall mission or objective.

Although definition of "system of systems" is somewhat arbitrary, it is generally viewed as a coherent entity when there is recognition that overall management control over the autonomously managed systems has become mandatory. Unfortunately, large, complex systems of systems are not developed under a single architecture resulting from a strategic development decision. Component systems are developed one by one, and the full system of systems can evolve over decades as various leaders develop enhanced visions of how systems can be used together to achieve larger objectives evolve. Although each system may have been justified and designed based upon sound system engineering principles to fulfill a perceived functional or performance need, its requirements and design most likely did not develop in response to concerns over the complete system of systems objectives.

A framework for conducting system engineering at the system of systems level has been developed (Eisner et al. 1991) but has not been widely accepted. Figure 1 lists the elements of system of systems engineering and highlights those aspects that require a quantitative analysis of alternatives when upgrading an existing system of systems. The methodology discussed in this article has been developed to support those analyses.

## Recurrent Management Issues

Often a program executive officer will be responsible for a collection of system acquisition programs, each of which belongs to a larger system of systems; however, this collection may not necessarily fully comprise that system of systems. Rather than architecting an entire system of systems, the program executive is often faced with deciding how best to *upgrade* within an existing system of systems. This generally means either beginning a new acquisition program to add a new system to the overall system of systems (additional functionality) or inserting advanced technology into an existing system via the upgrade or modification process (Evans et al. 1995).

Significant constraints are placed upon these executives, including budgets, politics, ill-defined and competing mission objectives, and the technology itself. Many new initiatives are underway under the umbrella of "Acquisition Reform" to encourage acceleration of systems development time, delivery of affordable systems, and risk mitigation through adoption of commercial off-the-shelf (COTS) components or technologies and industrial best practices. These attempts at reducing the usual acquisition cycle include such innovative and complementary measures as Advanced Technology Demonstrations (ATDs) and Advanced Capability Technology Demonstrations (ACTDs), often described, respectively, as "technology pushes" and "military need pulls" (Lynn 1994).

Although these initiatives promote the quick fielding of new, militarily useful technologies, they do not represent a disciplined approach to considering how best to upgrade specific, complex systems of systems under the constraints mentioned above. Development of such an approach is the objective of this research effort.

In summary, management issues are focused on upgrading vs. design of systems of systems because:

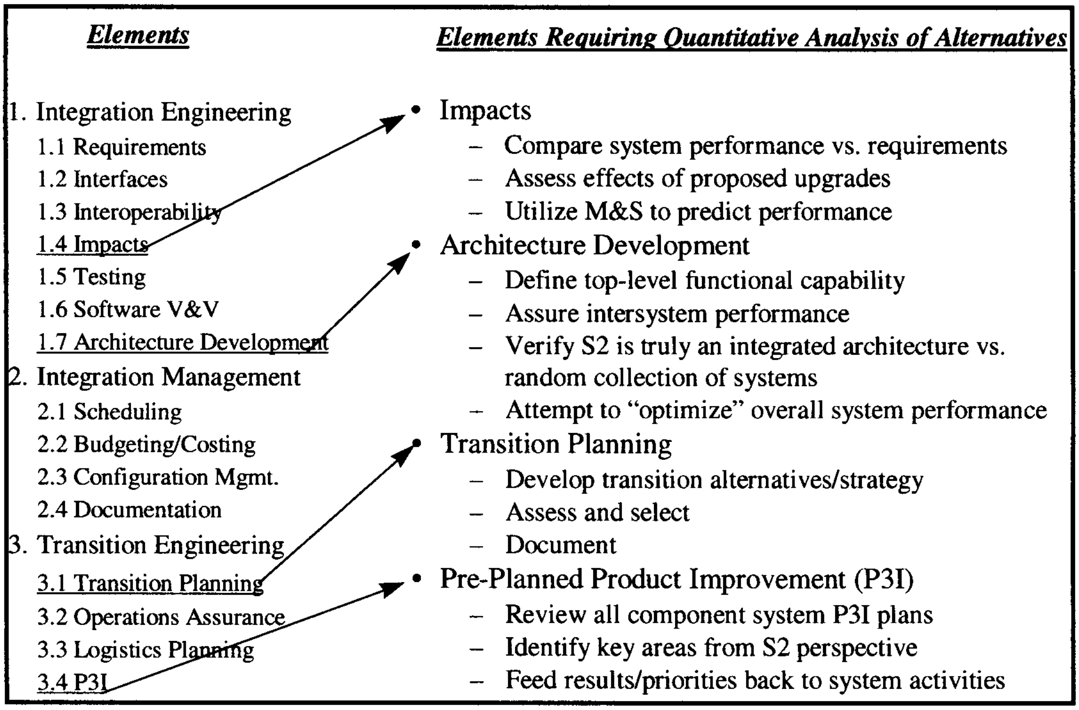


**Figure 1.** System of Systems Engineering Elements

1. All proposed systems/upgrades must fit into an existing system of systems,
2. There is rarely an opportunity to architect a major system of systems from scratch,
3. Requirements usually evolve in consideration of legacy systems' capabilities and management, and
4. We can often take advantage of available models and simulations that can be adapted for decision support.

# OBJECTIVES AND APPROACH

## System of Systems Upgrade Decisions

The decision-maker is generally trying to solve one of two problems: (1) maximize the system of systems' performance subject to a cost constraint or (2) minimize additional cost under performance constraints. While the former is clearly applicable to upgrading or architecting a system of systems, the latter arises in the Operations and Maintenance (O&M) phase of a system life cycle. That is, we may wish to maintain a proven capability while at the same time reduce legacy infrastructure activities.

Although cost-reduction approaches have included "design-to-cost," recent DoD acquisition reform initiatives have softened hard budget allocations in favor of an approach known as Cost as the Independent Variable (CAIV). Application of the CAIV approach requires quantitative understanding of the relationship between cost and performance for major system elements. Representation of a system element's performance as a function of cost will be referred to as a Performance-Based Cost Model (PBCM). While the CAIV terminology has come to represent a specific government approach to acquisition at the individual system level, it is used here simply to indicate that system of systems performance will be displayed and understood as a function of the independent variable, cost.

System of systems upgrade decisions are reviewed annually for all warfare or program areas as part of strategic planning and budgeting processes in DoD. There are four forms of upgrade options depending on which conditions are most pressing:

1. Adding a new type of system (i.e., additional functionality) to the system of systems

2. Procuring additional numbers of existing component systems (enlarging the scope and capability of the system of systems and offering an opportunity to insert advanced technology)
3. Replacing existing component systems due to aging or obsolescence (also offering an opportunity to enhance the system of systems' performance and/or functionality through advanced technology insertion)
4. Upgrading existing component systems due to requirements pressure or availability of advanced technology

## Legacy Decision Support Approach

In assessing whether to go forward with a new system development or major upgrade, DoD usually conducts an analysis of alternatives to determine whether the proposed system is the most cost-effective alternative to meeting a certified military need (Department of Defense 1996). A typical analysis approach is to utilize modeling and simulation (M&S) to estimate the marginal utility of proposed system point designs to a larger warfare or campaign mission objective. The system performance is represented by a set of measures of performance (MOPs) and its contribution to the mission is referred to as a measure of effectiveness (MOE). The simulation is run on a carefully selected set of applicable scenarios with and without the system alternatives to characterize the hypothesized system alternatives' value-added. A multi-objective metric that combines costs and multiple MOEs into a single scalar metric may be used to compare alternatives. This metric may also attempt to reflect expert opinion as to military value of the alternatives that are not captured by quantitative analyses due to limitations of fidelity, scope, or tractability. A primary shortcoming of the analysis of alternatives process from a system of systems perspective is that just one component system is considered at a time, in a "stovepipe" fashion. In a cost-constrained environment, this approach normally will not generate the "best" alternative from the system of systems perspective.

The DoD acquisition community strongly prefers quantitative "engineering analysis" over qualitative "decision support" methods such as the Analytic Hierarchy Process—perhaps because the community is dominated by engineers and scientists who recognize the difficulties in converting opinion and judgments into meaningful metrics, hence the heavy emphasis on modeling and simulation as the basis for decisionmaking. This seems to be a widespread preference throughout the technical and scientific community (Cabral-Cardoso and Payne 1996). The approach of this article attempts to provide objective, quantitative information to decision makers at the system of systems level, thereby minimizing the introduction of subjective judgments at the single system level.

## Proposed Approach

The challenge is to develop a quantitative process or methodology to support system of systems upgrade decisions so to answer the question "From the system of systems perspective, where are the limited upgrade resources best applied?" Equivalently, "Given a system of systems architecture, what is the optimal requirements allocation as a function of overall cost?" "Architecture" here implies that the system of systems functional requirements are well understood and are embodied in definition of the system of systems scope. Whereas the architecture will specify what functions must be accomplished, the CAIV requirements allocation process must address how well each function must be performed by which component system and how many of each system are required.

The process should enable a domain-expert systems architect or engineering team to generate an optimal allocation of design requirements in accordance with a specified MOE for a particular complex system of systems. Here we formulate the general problem and apply it to a real-world, contemporary system of systems in sufficient detail to demonstrate the feasibility of the approach—a practical, proof-of-principle demonstration. The demonstration goes beyond applying closed-form representations of system performance by using simulation to represent the system effectiveness. Substantial investments have been made in system of systems simulations and their use avoids unnecessary simplification of system abstraction resulting from closed-form expressions of typical, complex systems of systems behavior. Model fidelity and execution time must be balanced due to the intense computational burden of advanced M&S. These considerations will drive selection of the system of systems' MOE/MOP and PBCM structure.

There are seven key steps of the proposed system of systems CAIV optimization process, illustrated in Figure 2:

1. Define the overall system of systems, its components and functionality, and its missions or scenarios of interest.
2. Define critical MOPs and MOEs:
   a) One overarching MOE for the full system of systems that characterizes mission success
   b) Secondary MOEs that are of substantial significance from the system of systems perspective
   c) One or more MOPs, or key performance parameters, for each type of component system
3. Specify initial boundary conditions or constraints for the system of systems, as necessary.
4. Formulate Performance-Based Cost Models for each component system by parameterizing each subsystem's cost as a function of one key MOP.
5. If possible, formulate an appropriate closed-form model that will capture the mapping from component system MOPs to system MOEs and eventually the overarching MOE. Alternatively, select an appropriate M&S implementation that evaluates the desired objective function and MOE constraints as a function of component systems' MOPs. (Constructing closed-form expressions that model the system of systems' top-level performance is important for initial problem understanding, but will probably not be sufficient to adequately capture system interactions and performance drivers. It will be necessary to use high fidelity M&S to represent the complexity necessary to provide credible analyses to support decisions regarding complex, high value systems.)
6. Solve the resulting constrained, nonlinear (stochastic) performance optimization problem repeatedly, gradually relaxing the overall cost constraint. A solution to a specific constrained problem formulation yields an optimal set of MOP values that represents one system of systems requirements allocation corresponding to the most effective system of systems design and force structure. The set of solutions will provide insight as to performance and design as a function of CAIV.
7. Effectively communicate results of the process to the decision maker or decision making body. The solution will still require further evaluation to determine design implications for each system. Sensitivity studies should be conducted on secondary MOE constraints and MOP technology con-

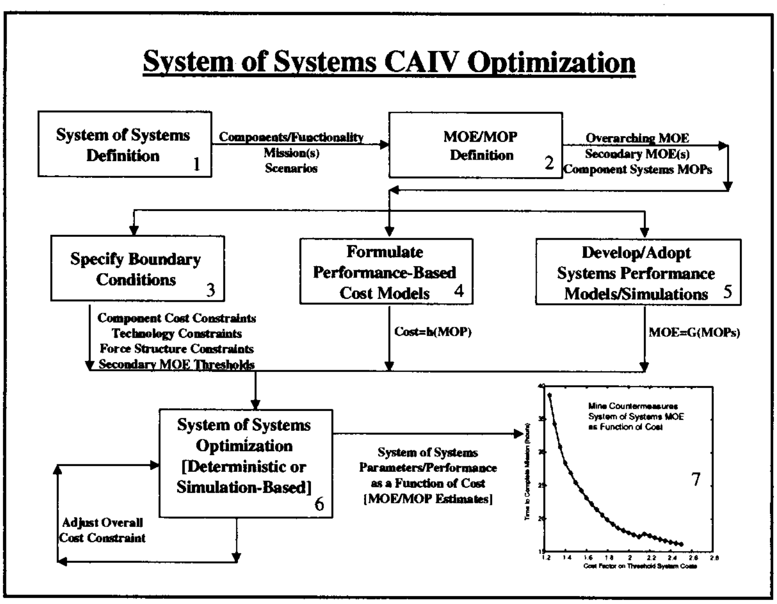


**Figure 2.** System of Systems CAIV Optimization Process

straints to generate operational and technology investment strategy insights, respectively. In this way, the methodology *supports* the decision process rather than makes it.

## GENERAL SYSTEM OF SYSTEMS COST/PERFORMANCE MODEL

Consider $n$ types of systems, $S_i$, that comprise a system of systems, $S$, with the following characteristics and constraints:

- $S = \{S_1, \ldots, S_n\}$
- There are $m_i$ systems of type $i$, and the total number of systems is $m$: $\mathbf{m} = \{m_1, \ldots, m_n\}$ and $m = \Sigma_{i=1}^{n} m_i$. The minimum number of each system type required for the system of systems is designated as $\mathbf{m}^L$.
- Each system type has a set of $r_i$ MOPs:

  $\mathbf{p}_i = \{p_{i,1}, \ldots, p_{i,r_i}\}$. Thus each $\mathbf{p}_i$

  has dimension $r_i$ and $r = \sum_{i=1}^{n} r_i$.
- Each system's MOPs are constrained by low-performance threshold specification values, $\mathbf{p}_i^*$ and realistic technology limitations at the high performance end, resulting in the following upper and lower bound constraints: $\mathbf{p}_i^L \leq \mathbf{p}_i \leq \mathbf{p}_i^U$, or $p_{ij}^L \leq p_{ij} \leq p_{ij}^U$, $\forall j$. Note that for some parameters, such as navigation accuracy, small values are better than large values, hence $\mathbf{p}_i^*$ is not simply the lower bound, $\mathbf{p}_i^L$. In the most general case, these MOP constraints could be functions of program schedule as well, in anticipation of requirements creep and advancing technology.
- Each system's unit cost is a nonlinear function of performance, expressed in terms of its critical MOPs: $c_i = \mathrm{h}_i(\mathbf{p}_i)$, $\mathbf{c} = \{c_1, \ldots, c_n\}$. We denote $c_i^* = \mathrm{h}_i(\mathbf{p}_i^*)$ as the cost associated with the threshold system. This PBCM is generated by considering each critical MOP as a cost driver of a particular subsystem, whose cost can be parameterized on that MOP. Total system of systems' cost is then $C = \mathbf{mc}^T$.
- The system of systems has one overarching measure of effectiveness, $E$, a function of each system's set of MOPs and the number of systems: $E = \mathrm{G}(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n)$.

It is clear from the last assumption that each system type has its own overall MOE, say $E_i$. From the single system perspective, each system's overarching MOE $E_i$ would only be considered as a function of its own MOPs, $\mathbf{p}_i$. But if any $E_i$ depends upon not just $\mathbf{p}_i$ but some elements of $\mathbf{p}_j$ where $i \neq j$ then we say the system of systems is interdependent, and would have to express the individual systems MOEs as $E_i = \mathrm{f}_i(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n)$. Therefore, in general, $E$ will be a complicated function of the full set of component systems' MOPs $E = \mathrm{G}(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n)$, and the single-system MOEs become uninteresting from the system of systems perspective.

When describing a system of systems comprised of relatively simple component systems or using simplified models of complex systems, $E$ could be expressed as a closed-form function of the MOPs. The simplified (but realistic) naval mine countermeasures example developed here has a closed-form, nonlinear expression for $E$, which is intuitive and quite useful. However, MOPs are themselves typically sensitive to scenarios, concept of operations (CONOPS), and environments. So in order to obtain representative, robust, full-fidelity results, it will generally be necessary to use a simulation to evaluate G.

In addition to the constraints on MOPs shown above, several other constraint types can occur and should be considered:

- "Force structure" constraints. There is generally a practical operational or programmatic limitation as to how many systems of each type can comprise the system of systems, known as "force structure" constraints: $\mathbf{m}^L \leq \mathbf{m} \leq \mathbf{m}^U$ or $m_i^L \leq m_i \leq m_i^L$, $\forall i$.
- System effectiveness constraints. Similar to the MOP constraints, a minimum threshold for each system's MOE could exist. This can be generated through a technical performance analysis or (more likely) because the existing component system performs at the threshold level and it is desired to meet or exceed that level. Therefore, the threshold MOE for each system, $S_i$, is $E_i^* = \mathrm{f}_i(\mathbf{m}^L, \mathbf{p}_1^*, \ldots, \mathbf{p}_n^*) \leq E_i$, $\forall i$. When trying to minimize cost subject to performance constraints, there should be a minimum overall system of systems MOE constraint as well: $E^L \leq E$. Without loss of generality, this single system effectiveness constraint will not be addressed further because it would only be applied in practice to ensure that minimal system performance would be achieved to suit a purpose other than being a component of the system of systems under consideration.

- Cost constraints. When applicable, there can be cost constraints on individual systems as well as the full system of systems: $C \leq C^U$ and $\mathbf{c} \leq \mathbf{c}^U$. Implicitly, $\mathbf{c}$ is also bounded below due to the presence of minimum performance thresholds as discussed above. Hence, we have: $\mathbf{c}^* \leq \mathbf{c} \leq \mathbf{c}^U$. Without loss of generality, we will take the system of systems viewpoint, and consider only the cost constraint at the macro level, $C \leq C^U$.
- Secondary MOE constraints. As will be illustrated by the mine countermeasures (MCM) example, there may be one or more secondary MOEs that must be achieved to some minimum level to achieve mission objectives. This can also be necessary in the case where the system of systems effectiveness is not fully expressed by one MOE. Without loss of generality, we will consider just one secondary MOE as a quality constraint: $q(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n) \geq q_T$.

When addressing the system of systems upgrade from the CAIV perspective, we would optimize a sequence of nonlinear programs formed by discretely parameterizing the system of systems cost constraint. This is accomplished by defining a sequence of upper cost bounds, $C_k^U = \text{costfactor}_k \cdot C^*$, where $C^*$ is the cost to produce the threshold system of systems defined by the parameter set, $\{\mathbf{m}^L, \mathbf{p}_1^*, \ldots, \mathbf{p}_n^*\}$. The resulting nonlinear programming problem (with only one MOE constraint) is then to maximize $S = \{S_1, \ldots, S_n\}$ system of systems performance subject to force level, technology, cost, and performance threshold constraints as shown below.

$$\begin{gathered} \text{Max } E = G(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n) \\ \text{subject to:} \\ \mathbf{m}^L \leq \mathbf{m} \leq \mathbf{m}^U \\ \mathbf{p}_i^L \leq \mathbf{p}_i \leq \mathbf{p}_i^U \\ C \leq C_k^U = \text{costfactor}_k \cdot C^* \\ q(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n) \geq q_T \end{gathered}$$

## MINE COUNTERMEASURES' SYSTEM OF SYSTEMS COST/ PERFORMANCE MODEL

A simplified, but realistic model of naval mine countermeasures operations and systems has been developed as a proof-of-principle demonstration. This limited system of systems consists of two systems: a minefield reconnaissance system and a separate, mine neutralization system. The reconnaissance system first conducts a survey of the entire suspected minefield area, attempting to detect, classify, and localize mine-like objects. These contacts are then passed to the neutralization system, which must reacquire the contacts and neutralize each mine-like object, if necessary (that is, if the mine-like object is indeed identified as an actual mine). System descriptions, functionality, measures of effectiveness, measures of performance, and PBCM are provided in sufficient detail to support system of system upgrade decisions and trade-off analyses. Mine countermeasures analysis terminology and notation is described in Appendix I.

The overarching MOE, $E$, for this MCM system of systems, $S$, is the time required to achieve a specified MCM area clearance rate, $\alpha$, with specified confidence level, $\beta$. Knowing the form of $E$ guides our performance model formulation for the component systems, $S_1$ and $S_2$. For the purposes of this analysis, we assume there is only one system of each type, therefore $n = 2$ and $\mathbf{m} = \{1,1\}$.

Following the process described earlier, the mission scenario and minefield that is to be cleared must be specified. The mission is to search a mine danger area of 20 nm$^2$, seeded with 100 mines, corresponding to $S_{\text{minefield}} = 20$ and $M_0 = 100$. The mines are laid out in four rows of 25 each, with a 400-yard separation between mines within each row, and 800 yards between the rows. Hence $d_{\text{mines}} = 600$ yards. Figure 3 illustrates a minefield layout with these characteristics, though this "ground truth" information is unknown to the system of systems.

## $S_1$: MCM Reconnaissance System

This system is used to survey a suspected minefield area, performing the typical MCM mine-hunting functions of *detection, classification*, and *localization*. The CONOPS is that the area is completely covered first with a detection pass followed by a second pass for the purposes of classification. This must be done at a reduced standoff range from each detected object, necessitated by the much higher frequency sensor generally necessary for this more precise function. Localization is done concurrently with detection and classification, and therefore takes no additional time. In consideration of the

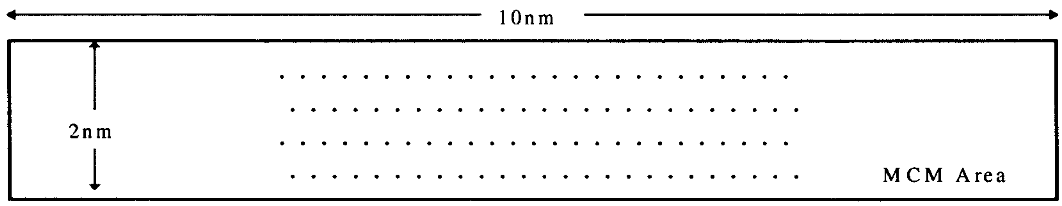


**Figure 3.** Minefield Layout and Area to be Searched/Cleared

overarching system of systems measure of effectiveness, the final form of the MOE for $S_1$ as a function of the MOP vector is the time to complete the reconnaissance function in hours (see Appendix II for details):

$$E_1(\mathbf{p}_1) = S_{\text{minefield}}\left[\frac{24}{p_{1,1}} + \frac{1}{60}[\lambda \cdot p_{1,2}p_{1,4}P_d + (2p_{1,4} - T_{\text{transit}})((1 - p_{1,2})P_d\lambda + p_{1,3} + P_d\lambda_{ft})]\right]$$

where

$$T_{\text{transit}} = \left(\frac{d_{\text{mine}}}{2000 \cdot V_{\text{transit}}}\right).$$

Note that this MOE does not reflect the quality to which the reconnaissance is accomplished, only how long it takes. If we were considering the effectiveness of the stand-alone reconnaissance system, then we would want to have $E_1$ reflect other mission MOEs as well, in order to effect a measure of "minefield characterization." Reconnaissance survey quality will be automatically reflected in $E_2$, via expressions that utilize all the elements of $\mathbf{p}_1$ that affect initialization of the neutralization function provided by $S_2$. Additionally, a minimum threshold quality MOE constraint at the system of systems level will also be imposed.

## $S_2$: MCM Neutralization System

The MCM neutralization system attempts to *relocate*, *identify*, and *neutralize* all mine-like objects detected and classified as such by the reconnaissance system. For the purposes of this analysis, the probabilities of identification and subsequent neutralization are assumed to be one, and we will focus on uncertainty related to re-acquisition of all mine-like objects passed to $S_2$ from $S_1$ as contacts. In consideration of the overarching system of systems measure of effectiveness, the MOE for $S_2$ is the time (hours) to complete neutralization and neutralization attempts on all contacts/objects classified as mine-like by the reconnaissance system, $S_1$. The final form of the MOE for $S_2$ as a function of the MOP vector is (see Appendix II for details):

$$E_2 = \frac{1}{60}[P_L C_m T_n + (1 - P_L)C_m T_{pf} + C_f T_{pf}]$$

$$= \frac{S_{\text{minefield}}}{60}[P_d p_{1,2}p_{2,3}\lambda e^{(-p_{1,5})/(4.481p_{2,1})} + (1 - e^{(-p_{1,5})/(4.481p_{2,1})})P_d p_{1,2}p_{2,2}\lambda + (1 - p_{1,2})(p_{1,3} + P_d\lambda_{ft})p_{2,2}]$$

## $S$: MCM Clearance System of Systems

For the full system of systems, the overarching MOE is then simply the total time to complete clearance operations: $E = G(\mathbf{m}, \mathbf{p}_1, \mathbf{p}_2) = E_1 + E_2$. However, an overall performance or quality constraint must be imposed on the clearance operations, otherwise the optimization will find a very fast yet ineffective system of systems. Specifically, this constraint specifies an MCM area clearance rate, $\alpha$, with an associated confidence level, $\beta$. This should actually be considered as a secondary quality MOE that has a threshold requirement. Recall that $p$ is to be the probability that a particular mine will be cleared, which is the product of the sequential operations' probability of success:

$$p = P_d P_c P_L = P_d p_{1,2} e^{(-p_{1,5})/(4.481p_{2,1})}$$

The expected number of mines successfully cleared is then $pM_0$.

Selection of $\alpha = 0.80$, $M_0 = 100$, and $\beta = 0.90$ will yield a constraint that $p \geq 0.846$ (Luman 1997). Therefore, with 100 mines present, we will be satisfied to be <u>at least</u> 90% confident that <u>at least</u> 80 mines will be cleared. In summary, the performance quality constraint is then:

$$q(\mathbf{p}_1, \mathbf{p}_2) = p = P_d P_c P_L$$
$$= P_d p_{1,2} e^{(-p_{1,5})/(4.481 p_{2,1})}$$
$$\geq 0.846$$

## PBCMs and Parameter Bounds

The reconnaissance system performance ranges and cost modeling are derived from design considerations for an unmanned undersea vehicle (UUV). The neutralization system performance ranges and cost models are based upon a combination of factors, certain operational MCM systems, and COTS information regarding marine navigation systems.

MOPs developed earlier in this article are grouped by the major subsystem for which they act as major cost drivers. The PBCM provides an approximation of subsystem cost as a function of those same primary subsystem MOPs. This synchronization of cost and performance model parameters is crucial, and should become a fundamental feature of the systems engineering process.

Since this type of MCM system would be produced in very small numbers, only developmental costs are considered, neglecting the full system life cycle costs. COTS or nondevelopmental item technologies are also assumed so that developmental costs approximate R&D and production costs combined. It should be noted that since the PBCMs can include nonlinear expressions, a full life cycle model for each PBCM can be accommodated with no change in the approach. The subsystem and associated MOPs are as illustrated in Figure 4.

To illustrate the concept of PBCMs, only area coverage rate will be discussed here; the other seven required models are developed in Luman 1997. There are two sonars in the sensor subsystem: detection and classification sonars. Critical performance parameters affecting area coverage rate are probability of detection, range, and maximum vehicle speed at which the sonars can remain effective in the presence of flow noise. They are of course sensitive to many environmental parameters as well as assumed target characteristics. The approach here is to assume one environment, fix $P_d$ and vehicle speed, and utilize modeled results to derive the following PBCM for the search sonar MOP, area coverage rate, A (nm$^2$/day). (Sensitivity studies are advised to understand dependence upon these assumptions.) The following table represents the data used to generate the PBCM:

| | | | | |
|---|---|---|---|---|
| A (nm$^2$/day) | 10 | 57 | 82 | 94 |
| Cost ($M) | 3 | 4.483 | 7.655 | 11.445 |

A third order polynomial was fit to the data, and together with the upper and lower bounds

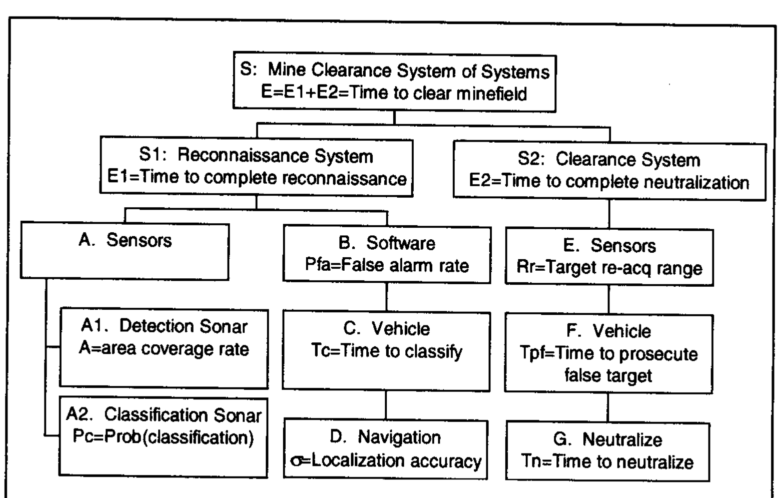


**Figure 4.** Mine Clearance System of Systems MOE/MOP Structure

for A, this constitutes the PBCM. The resulting model is therefore:

$$h_{1,1}(p_{1,1}) = (4.5034E\text{-}5)p_{1,1}^3 - 0.0053861p_{1,1}^2 + 0.21593p_{1,1} + 1.3342$$

with constraints as $p_{1,1}^L = 10$, $p_{1,1}^U = 100$, $p_{1,1}^* = 10$. Figure 5 illustrates the cost/performance relationship for A.

The complete closed-form MCM system of systems model is shown in Figure 6, which incorporates PBCMs for all eight system of system MOPs. The cost constraint indicated in Figure 6 is parameterized by $costfactor_k$ which is a multiplier on the threshold system costs indicating the maximum amount the decision maker is willing to consider spending. In this way, we will consider a series of optimization problems that will provide insight from the CAIV perspective.

## PHASE I RESULTS: CLOSED FORM OBJECTIVE FUNCTION

This section presents the results of optimizing the closed form representation of the MCM system of systems model. For ease of reference, the MCM system of system MOP definitions are repeated below with the correspondence to the optimization vector, $\boldsymbol{x}$.

The system of systems constrained MOE optimization has been solved for an increasing sequence of multipliers ($costfactor_k$) on the cost of the threshold system, denoted by $C^*$, which turns out to be \$28.066M. This provides the decision-maker with information to apply the CAIV approach to system upgrade or initial design. Plots are provided so that one can visualize the top level MOE improvement and corresponding MOP requirements as the system of systems cost upper bound is allowed to increase.

| | |
|---|---|
| $x(1) = p_{1,1} = A =$ | System $S_1$ area coverage rate during detection pass (nm$^2$/day) |
| $x(2) = p_{1,2} = P_c =$ | Probability of correctly classifying a detection as mine-like or nonmine-like, at range $R_c$ |
| $x(3) = p_{1,3} = P_{fa} =$ | Detection false alarm rate (false alarms/nm$^2$) |
| $x(4) = p_{1,4} = T_c =$ | Time required to classify a mine (min) |
| $x(5) = p_{1,5} = \sigma =$ | Standard deviation of minelike object localization error (yards) |
| $x(6) = p_{2,1} = R_r =$ | Contact localization error standoff which yields an 80% probability of reacquisition |
| $x(7) = p_{2,2} = T_{pf} =$ | Time spent prosecuting a non-mine classified as a mine or unsuccessfully attempting to reacquire a correctly classified mine (min) |
| $x(8) = p_{2,3} = T_n =$ | Time spent neutralizing (prosecuting) a classified mine (min) |

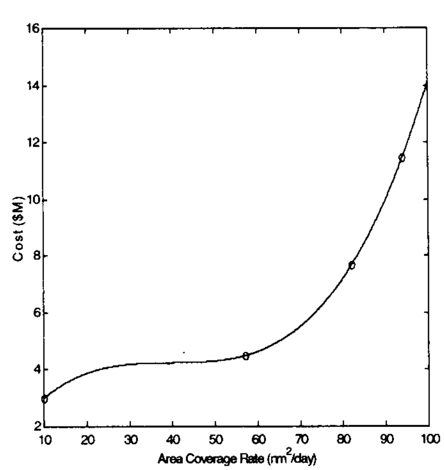


**Figure 5.** PBCM for Area Coverage Rate

The baseline results are obtained through utilization of MATLAB®'s constrained sequential quadratic programming (SQP) algorithm, to solve the fully general nonlinear programming problem in which both objective and constraint functions can be nonlinear (Gill et al. 1981). The particular routine is called CONSTR and is contained in the Optimization Toolbox. Basically, the method formulates a sequence of quadratic programming subproblems based on a quadratic approximation of the Lagrangian function and linearizing the nonlinear constraints about the current iterate. The simpler quadratic programming subproblem (quadratic function with linear constraints) is solved using an active set projection method. The original nonlinear function and constraint sets are then approximated about the new iterate and the

Maximize $S = \{S_1, S_2\}$ system of systems performance (minimize time) subject to technology, cost, and performance threshold constraints:

**Minimize**:

$$E(\mathbf{p}_1,\mathbf{p}_2) = E_1(\mathbf{p}_1) + E_2(\mathbf{p}_1,\mathbf{p}_2)$$

$$= \frac{\mathrm{S}_{\text{minefield}}}{60}\left[\frac{24\cdot 60}{p_{1,1}} + \lambda \cdot p_{1,2}p_{1,4}\mathrm{P_d} + (2p_{1,4} - \mathrm{T}_{\text{transit}})((1-p_{1,2})\mathrm{P_d}\lambda + p_{1,3} + \mathrm{P_d}\lambda_{ft})\right]$$

$$+ \frac{\mathrm{S}_{\text{minefield}}}{60}\left[\mathrm{P_d}p_{1,2}p_{2,3}\lambda e^{\frac{-p_{1,5}}{4.481p_{2,1}}} + \left(1 - e^{\frac{-p_{1,5}}{4.481p_{2,1}}}\right)\mathrm{P_d}p_{1,2}p_{2,2}\lambda + (1-p_{1,2})(p_{1,3} + \mathrm{P_d}\lambda_{ft})p_{2,2}\right]$$

**subject to**:

$$(3, 0.9, 0.25, 3.0, 42)^T \le \mathbf{p}_1 \le (100, 0.98, 2.0, 9.17, 90)^T$$

$$(75, 1.0, 3.0)^T \le \mathbf{p}_2 \le (700, 7.0, 10.0)^T$$

$$C(\mathbf{p}_I,\mathbf{p}_2) \le C_k^U = costfactor_k \cdot C^* \text{ and } \mathrm{q}(\mathbf{p}_1,\mathbf{p}_2) \ge \mathrm{q_T} = 0.846$$

**where**:

$$\begin{aligned}C(\mathbf{p}_1,\mathbf{p}_2) = {} & (4.5034\text{e-}005)\, p_{1,1}^3 - 0.0053861 p_{1,1}^2 + 0.21593\, p_{1,1} + 1.3342 \\ & + 283.4646 p_{1,2}^2 - 507.63784 p_{1,2} + 227.4598 \\ & - 2.0484\, p_{1,3}^3 + 9.9873 p_{1,3}^2 - 17.942\, p_{1,3} + 20.322 \\ & + 0.11597 p_{1,4}^2 - 2.1757\, p_{1,4} + 15.204 \\ & + (2.0618\text{e-}004) p_{1,5}^2 - 0.03776\, p_{1,5} + 1.7778 \\ & + (1.5049\text{e-}007)\, p_{2,1}^3 - (1.5782\text{e-}004) p_{2,1}^2 + 0.055167\, p_{2,1} - 1.8133 \\ & + \text{-}0.28504\, p_{2,2}^3 + 3.8462 p_{2,2}^2 - 17.264\, p_{2,2} + 33.344 \\ & + 0.21024 p_{2,3}^2 - 4.1096\, p_{2,3} + 25.397\end{aligned}$$

$C^*$ = threshold system cost = \$28.066M

$$\mathrm{q}(\mathbf{p}_1,\mathbf{p}_1) = p = \mathrm{P_d P_c P_L} = \mathrm{P_d}\, p_{1,2}\, e^{\frac{-p_{1,5}}{4.481 p_{2,1}}}$$

**Figure 6.** Summary of MCM System of Systems Optimization Problem

sequence is repeated until convergence criteria are satisfied.

Figure 7 summarizes those results, which constitute an allocation of MOE requirements to the lower level MOPs as a function of overall cost. The first two plots (Figures 7a and b) present the top-level MOE ($E$ = time to complete minefield clearance) as a function of increasing cost factor and dollar cost upper bounds. The next two plots (Figures 7c and d) present the corresponding optimal MOPs as a function of increasing cost factor. The MOPs are normalized to their upper and lower bounds, with 0 corresponding to their thresh-

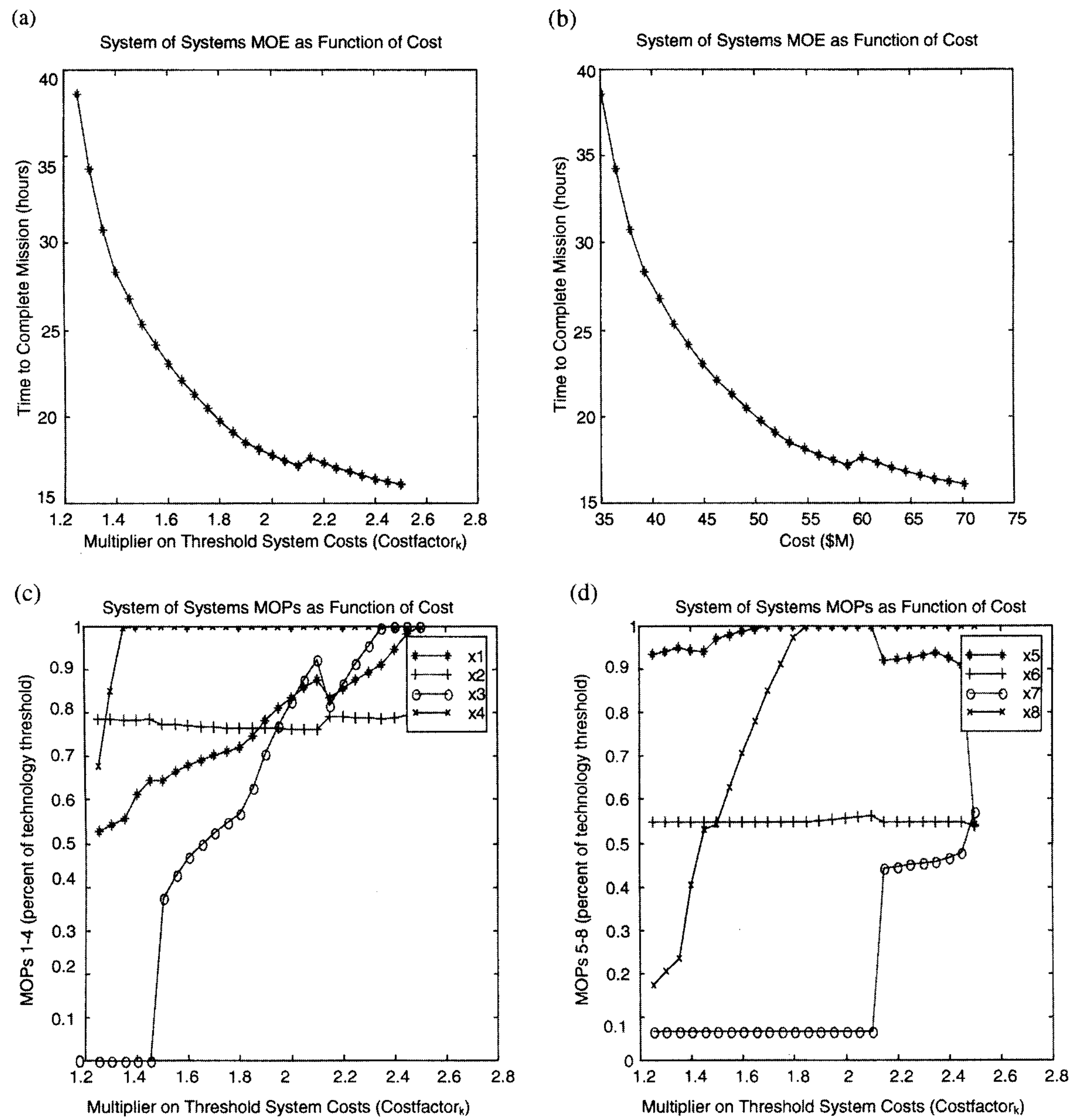


**Figure 7.** Constrained SQP Optimization Results for S

old system values and 1 corresponding to their technology limitations. Several significant insights can be obtained from examination of these plots:

- The system of systems MOE improves steadily to an asymptotic lower bound as the cost limit increases. Due to the imposed technology constraints, after a certain point no amount of money will enhance system performance.
- At the other extreme, if at least $1.25 \cdot C^*$ isn't spent, a feasible solution that satisfies the quality constraint cannot be found. I.e., even a very slow system cannot achieve the clearance rate constraint.
- A subjective "knee of the curve" can be observed to occur somewhere around 1.8–2.0 times the threshold system cost (about $50–56M), after which the rate of MOE improvement significantly decreases.

- The component systems' MOP requirements can be determined from these plots as a function of cost factor. One can see which MOPs become stressed (i.e., move away from their threshold system values) and approach their technology constraint limits as the cost constraint is relaxed. Of course, this behavior is dependent upon the PBCM function developed for each MOP, as well as their significance relative to the objective function and quality constraint. Specifically, initial performance is gained by improving $x_4$ (speed) and $x_5$ (location accuracy). Additional performance gains are most effectively achieved by improving $x_1$ (coverage rate), $x_3$ (false alarm rate), and $x_8$ (neutralization time).
- The hump in the MOE curve near costfactor = 2.15 corresponds to a local objective function minima caused by a prolonged flat area in the PBCM for $x(7)$ (Luman 1997). This effect is common in generating realistic PBCMs wherein there may be only a few discrete technology solutions widely separated in performance and cost. Depending on the situation, a discrete optimization method may be more appropriate.
- These results can be used to design a specific cost-constrained upgrade to the threshold system of systems. For example, if the allowable cost constraint is twice that of the threshold system, then selecting $\mathbf{p}_1$ = [85.3, 0.961, 0.55, 3.0, 42.0] and $\mathbf{p}_2$ = [423.2, 7.0, 3.0] yields $E$ = 17.768, with clearance rate q = 0.846 at a cost of \$56.132M. This is a substantial enhancement to the threshold system represented by $\mathbf{p}_1$ = [10.0, 0.9, 2.0, 9.17, 90.0] and $\mathbf{p}_2$ = [75.0, 6.6, 10.0] that results in an overarching MOE of $E$ = 93.33 hours with clearance rate of only q = 0.620 at a cost of \$28.066M. Since CONSTR would not converge for costfactors less than 1.25, the analysis indicates that a system that satisfies the stringent requirement for 84.6% clearance will cost at least 25% more than a system of systems composed of the threshold component systems, but would take 38.38 hours to complete the clearance mission with a single pass from each system.
- Critical to any cost/performance trade analysis is the concept of sensitivity analyses. Three specific types of sensitivity analyses are especially appropriate for this class of problems:

1. Sensitivity to mission/scenario. This is achieved by varying the parameters that define the threat, environment, mission objective, and systems CONOPS.
2. Sensitivity to secondary MOE constraints. Since these constraints were arbitrarily set, the optimization should be parameterized for excursions about the nominal value to produce families of CAIV curves.
3. Sensitivity to PBCMs. Examination of sensitivities to subsystem cost models and especially technology-driven limitations on MOPs can yield significant insights needed to focus a supporting warfare area technology investment strategy.

Moreover, it can be shown that *optimizing each system separately is suboptimal to optimizing the system of systems as a whole* (Luman 1997). In quantifying the sub-optimality of single-system optimization relative to simultaneously optimizing the entire system of systems, several significant insights were obtained and verified by examining some reasonable assumptions that might be held by component systems' management concerning the concurrent system engineering processes of other systems. For example, if a system engineer assumes that the other component systems are being developed for high performance, he or she will "under-engineer" his or her own system with respect to interfacing parameters and will tend to allocate resources to enhance the single-system MOE. Conversely, if a system engineer assumes that the other systems are not performance-driven, the result is to "over-engineer" his or her system at the interface and since resources are constrained, this forces degradation in the single-system MOE. In both cases, the overall system of systems is suboptimal, because all system engineers are making the same erroneous assumptions.

These effects are accentuated with restrictive cost constraints but become insignificant as overall cost constraints are relaxed to the point where the most advanced technology is affordable for all system components—an intuitive result. In other words, if we are not resource constrained, then the correct course of action is simply to optimize each component system for performance without regard to cost—and it doesn't matter if this is implemented separately or as a system of systems. But as the cost constraint is tightened, it becomes increasingly important to consider the full impact of design decisions on the whole to get the most performance per unit dollar. The results in this regard vividly illustrate the maxim, *"We are short of money, therefore we must think."*

## PHASE II RESULTS: SIMULATION OBJECTIVE FUNCTION

As mentioned previously, obtaining a closed-form, deterministic expression for the system of systems' MOE objective function is not always feasible or would introduce unacceptable simplifying assumptions. A growing number of application areas rely on stochastic modeling and simulation to predict system of systems performance under certain conditions of interest. Therefore, future practical implementations of this approach for warfare area systems of systems will include use of simulation to evaluate the objective function—an extension that will put a premium on minimizing the search algorithm's function evaluations.

Applicable simulations will generally be of the Monte Carlo type, hence, there will be process noise associated with each function evaluation. The simulation produces a stochastic realization of the objective function of the form:

$$y(\mathbf{p}_1, \ldots, \mathbf{p}_n) = G(\mathbf{p}_1, \ldots, \mathbf{p}_n) + \omega,$$ where $\omega$ represents simulation noise.

This stochastic nature of the objective function and quality constraint functions means that we have a stochastic optimization problem—to which classical optimization methods are not directly applicable. Since G will be evaluated by the simulation, the gradient of $y$ will not be available explicitly. Many stochastic optimization methods, like classical methods, require approximations of gradients, but they become extremely costly to compute in this domain since each function evaluation represents a simulation run. Until recently, finite-difference-based gradient search stochastic approximation procedures that are adaptations of deterministic algorithms have been most widely used for this type of optimization. A major drawback of these methods is that the number of function evaluations required at each step is linear in the dimension of the search parameter vector for first order methods and quadratic for second order methods (Glynn 1989). Since we envision eventually using large-scale system of systems simulations with tens of parameters, a much more efficient method is desirable.

The Simultaneous Perturbation Stochastic Approximation (SPSA) Method (Spall 1992) is the most efficient estimator in this domain with respect to function evaluations per iteration, and its first and second order versions have been adapted here for use in solving the MCM system of systems problem. The first order SPSA method is a type of gradient search method that requires only two function evaluations per iteration, independent of the number of parameters to be estimated. The current solution estimate is perturbed in all elements simultaneously in a sort of central difference fashion rather than one component at a time which is generally done in order to estimate the partial derivatives that comprise the gradient vector.

Although SPSA per se is an unconstrained (global) optimization algorithm, a penalty function approach was developed (Luman 1997) to adapt it to the class of nonlinearly constrained problems represented by this system of systems optimization process. However, due to the large range of numerical values of the MCM system parameters, the first order SPSA algorithm exhibited poor convergence. Therefore, a second order version of SPSA (Spall 1997 and Spall 2000) which emulates the convergence acceleration and scaling invariant properties of deterministic Newton-Raphson algorithms was also adapted to the constrained nature of this class of problems. Due to the need to estimate the Hessian matrix, this algorithm (called "2SPSA") requires five function evaluations per iteration, but produced much better results than the first order version (sometimes referred to as "1SPSA"). It is beyond the scope of this paper to discuss the details of the penalty function approach, but they are covered in Luman (1997) and generalized and extended by Wang and Spall (1998).

### Simulation Description

To examine stochastic optimization feasibility for this class of problems, the MCM system of systems model was implemented as a simulation, patterned directly after the parameter dependency diagram shown in Figure 8, with parameters defined explicitly in Appendix I. The simulation was implemented as a MATLAB® function that produces one Monte Carlo realization of $E$ and q with each function call. The simulation randomly generates the speci-

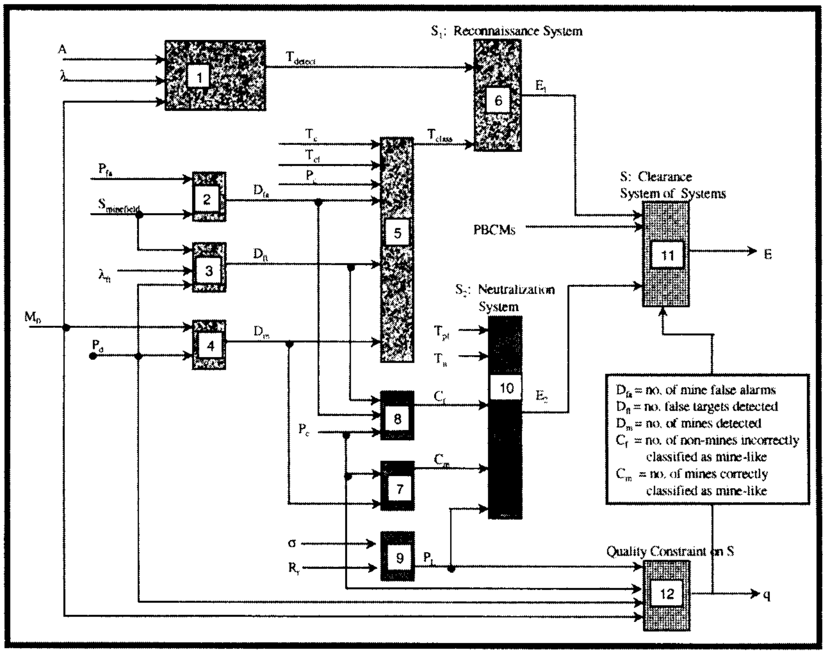


**Figure 8.** MCM Simulation Block Diagram

fied events in accordance with the MOPs. For example, looking at Block 4, if there are 100 mines in the minefield (i.e., $M_0 = 100$) and $P_d = 0.90$, then the number of detected mines ($D_m$) is generated simply as 100 Bernoulli success/failure trials with probability of success equal to 0.90. The randomly generated $D_m$ is then passed to Block 7, which in turn similarly generates the number of correctly classified mines, and so on. Eventually, the MOEs for that realization are produced and the resulting penalty function evaluation is returned by the simulation function MCMSIM after calculating the resultant system cost.

## Second Order Constrained SPSA Optimization Results

Figure 9 displays a comparison of several 2SPSA simulation MOE results compared to the closed form analytic model results. Since the simulations are noisy realizations of the closed form, "expected value" representation, we anticipate the SPSA results to only asymptotically approach the closed form results. Note that 2000 iterations are required to approach the analytic results, which may make this method prohibitive if the only available simulation takes more than a few seconds to generate a realization. Certain post-processing smoothing methods, common in stochastic optimization practical applications, were applied to achieve the final results, generated by interpolating MOP estimates across the CAIV continuum (Luman 1997). These interpolated simulation results are very smooth, approximating the baseline results curve.

However, the interpolated results may not satisfy all constraints, even though the result looks good in the MOE domain. Secondary MOE and cost constraint comparisons are displayed in Figures 10 and 11, and show acceptable levels, considering the variability induced by the simulation. Actually, the interpolated MOP values result in under spending the cost constraint by as much as $4M (about 6%) at the higher levels of the cost constraint, implying that a bit more performance could be extracted.

An interesting aspect of stochastic optimization is that just as the system behavior is stochastic, the optimization process is itself a stochastic process. That is to say, every time an optimization sequence is run, a different solution is obtained, each of which is "close" to the optimum. Therefore, the issue arises as to how to express the "final" answer in both the MOP

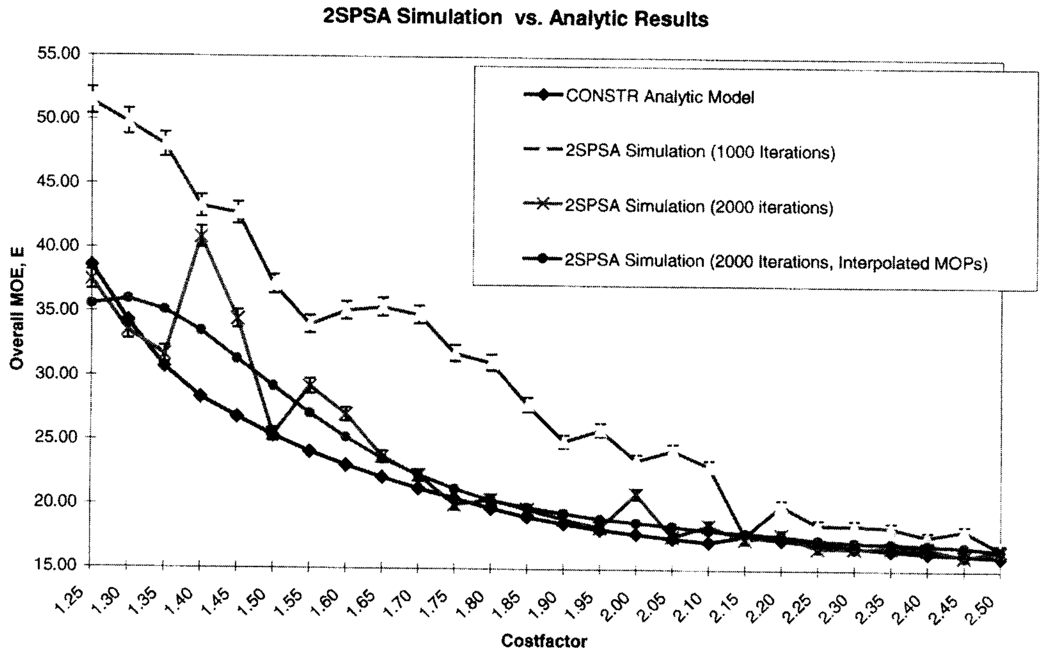


**Figure 9.** 2SPSA Simulation vs. Analytic Model Results

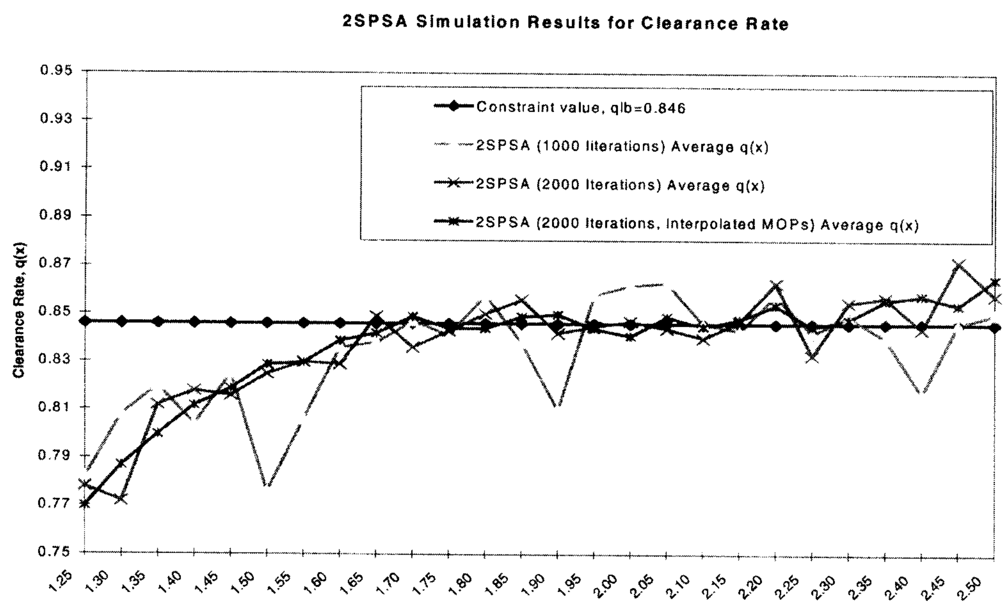


**Figure 10.** 2SPSA Simulation Clearance Rate Results

and MOE domains. For example, what are the values for the MOEs $E$ and q that are associated with the solution vector $x$ (MOPs)? Since MC-MSIM produces a random realization of the objective function, it must be called many times and results averaged to generate expected values for $E$ and q. The results in Figure 9 display the standard deviation bars of 100 such function evaluations about the mean simulation results.

In the baseline analytic results at a representative costfactor constraint value of 2.0,

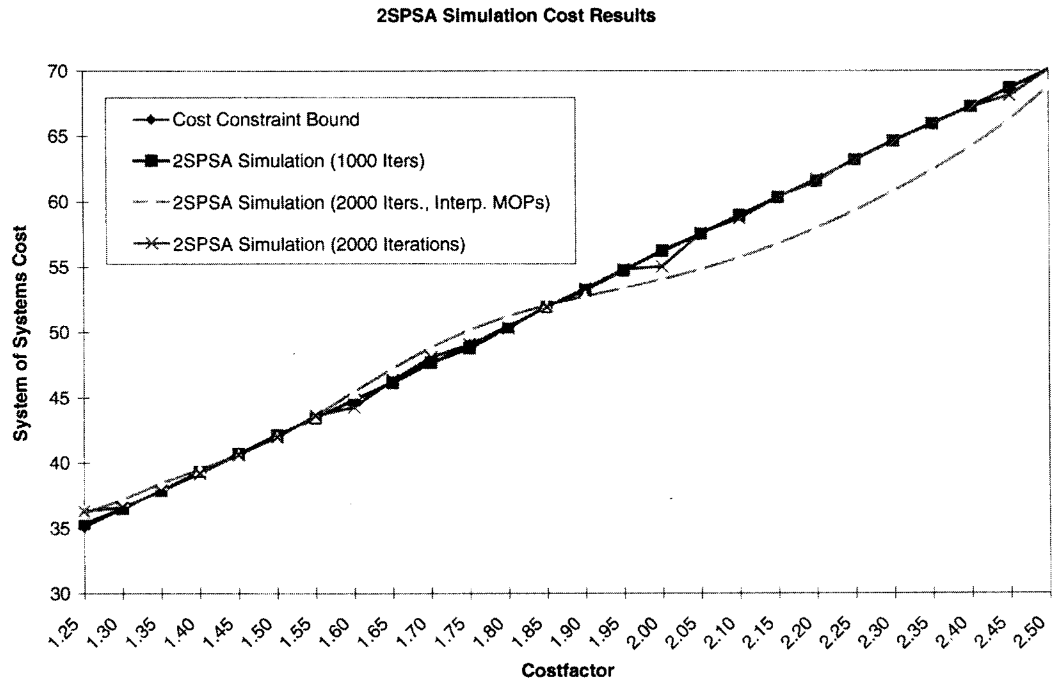


**Figure 11.** 2SPSA Simulation Cost Results

CONSTR produced $\mathbf{p}_1$ = [85.3, 0.961, 0.55, 3.0, 42.0] and $\mathbf{p}_2$ = [423.2, 7.0, 3.0], yielding $E$ = 17.8, with clearance rate q = 0.846 at a cost of $56.1M. The final results of the nonlinear, constrained, stochastic optimization implementation produced $\mathbf{p}_1$ = [74.3, 0.965, 1.1, 3.2, 55.8] and $\mathbf{p}_2$ = [495.6, 4.4, 3.3] yielding $E$ = 18.7, with clearance rate q = 0.841 at a cost of $54.0M. The overall MOE is about 5% worse for $2M less cost and a very slight decrease in clearance rate of 0.005. As expected, this is suboptimal to the analytic formulation due to the complexity introduced by simulation variability or "noise."

This proof-of-principle analysis has highlighted two fundamental difficulties created by the penalty function approach to constrained stochastic optimization: (1) sufficiently large penalty gains to guarantee constraint agreement also makes the transformed objective function very "flat," and (2) the penalty function exaggerates the effect of the simulation noise to the point where it makes convergence very difficult. These factors should motivate further research in more direct methods for constrained stochastic optimization to enhance the likelihood of successful utilization of advanced M&S to support system of systems acquisition decisions.

## SUMMARY

A systematic approach to system of systems requirements allocation has been developed and demonstrated. The process treats cost as the independent variable and seeks to find the "best" point design for upgrading a particular system of systems, subject to cost, operational, and technology constraints, relative to an overarching measure of effectiveness. Sensitivity analysis and utilization of simulation to compute MOEs will explicitly address uncertainties in a quantitative manner. The design requirements so generated represent an improved system of systems that may involve upgrading all component systems simultaneously, not just one at a time. Although final systems requirements decisions must subjectively balance multiple factors, this method objectively integrates cost and performance factors at the initial stage of analysis.

The process has been demonstrated on a naval mine countermeasures system of systems representation of sufficient complexity and detail to demonstrate the feasibility of the approach. This proof of principle demonstration features a constrained, nonlinear optimization algorithm adapted to both (1) closed-form representation of the objective function (i.e., MOEs) and (2) simulation-based objective function. Due to the nature of complex system of systems interactions, the latter approach will be necessary to address full warfare areas or problems of national interest. Their complexity requires simulation to represent mapping of system measures of performance to single system MOEs and on to the overarching system of systems MOE. Various optimization approaches have been demonstrated and differences quantified, including the sub-optimality of considering just one system at a time (Luman 1997). Application of this approach can result in more effective and comprehensive systems acquisition and technology investment strategies, with the secondary benefit that the process can be used as a framework to determine how to utilize campaign-level simulation to support acquisition decisions.

Variants of the process are now being applied to support CAIV analyses for the Navy Theater Wide program, and to focus future science and technology investments for naval mine countermeasures. Such a quantitative approach is not universally applicable, but must be limited to "warfare areas" that can be represented by a comprehensive model or simulation. These applications at the warfare area system of systems level will enable acquisition executives to move from our legacy single-system acquisition approach to a comprehensive warfare area architecting process with a scope spanning (1) new acquisition starts, (2) technology insertion upgrades, (3) force structure, and (4) technology investment strategy in a manner illustrated by Figure 12 (Luman 1998).

## APPENDIX I
## LIST OF SYMBOLS[1]

| | |
|---|---|
| $\alpha$ | Desired MCM area clearance rate |
| A | Reconnaissance system area coverage rate during detection pass ($nm^2$/day) |

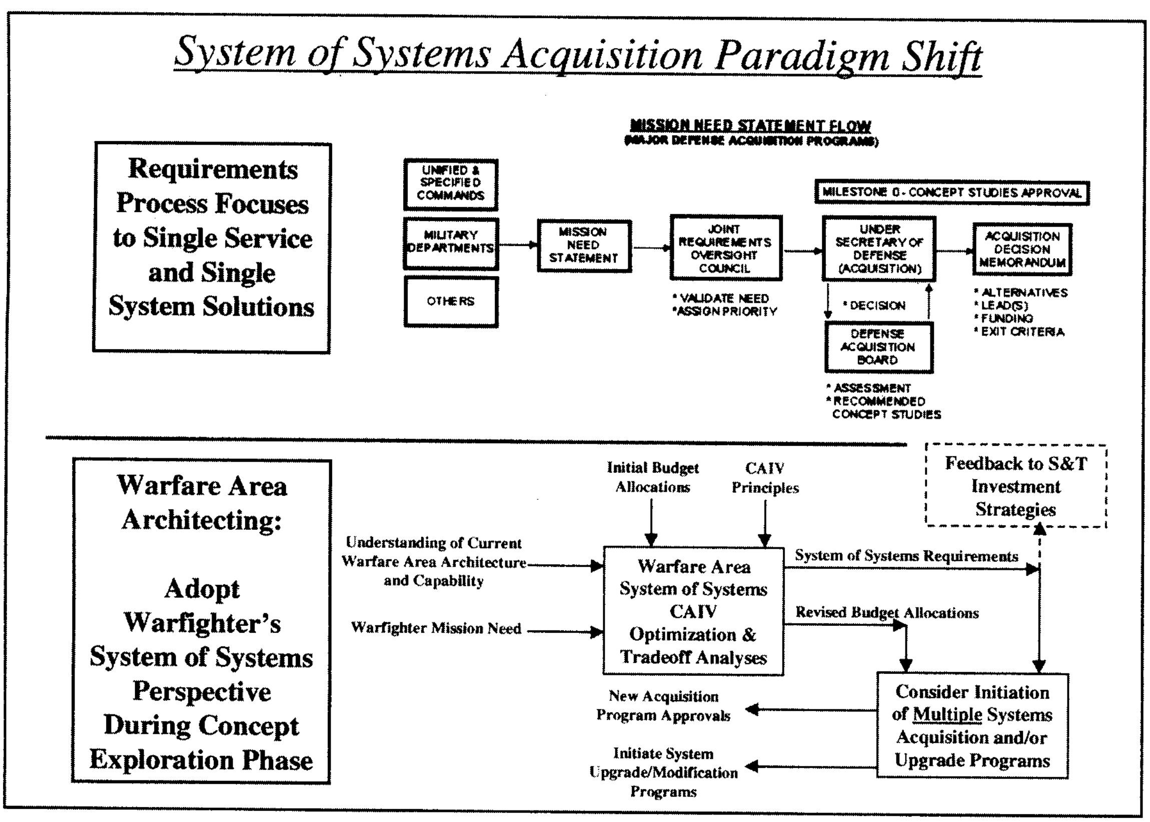


**Figure 12.** System of Systems Architecting Can Support an Acquisition Paradigm Shift

$\beta$ — Confidence level associated with MCM area clearance rate, $\alpha$

$C(\mathbf{p})$ — Total cost for $S$: $C = \mathbf{mc}^T(\mathbf{p})$

$C^*(\mathbf{p}^*)$ — Cost to produce the threshold system

$C_k^U(\mathbf{p})$ — Upper bound for cost constraint: $C_k^U(\mathbf{p}) = \text{costfactor}_k \cdot C^*(\mathbf{p})$

$C_f$ — Number of non-mines falsely classified as mine-like

$C_m$ — Number of mines correctly classified as mine-like

$c_i(\mathbf{p}_i)$ — Cost for system $S_i$: $c_i(\mathbf{p}_i) = \mathrm{h_i}(\mathbf{p}_i) = \Sigma_{j=1}^{r_i} \mathrm{h}_{i,j}(p_{i,j})$

$\mathbf{c}(\mathbf{p})$ — Cost vector: $\mathbf{c}(\mathbf{p}) = \{c_1(\mathbf{p}_1), \ldots, c_n(\mathbf{p}_n)\}$

$\mathbf{c}^L$ — Lower bound for $\mathbf{c}(\mathbf{p})$

$\mathbf{c}^U$ — Upper bound for $\mathbf{c}(\mathbf{p})$

$D_{fa}$ — Number of mine false alarms

$D_{ft}$ — Number of false targets detected

$D_m$ — Number of detected mines

$\mathrm{d_{mines}}$ — Average distance between mines (yards)

$E$ — Overarching MOE for $S$

$E_i$ — MOE for System $S_i$: $E_i = \mathrm{f}_i(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n)$

$\mathrm{F_0}$ — Number of false targets contained in the MCM area, $\mathrm{S_{minefield}}$

G — Overarching MOE objective function, $E = \mathrm{G}(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n)$

$\mathrm{h}_{i,j}(p_{i,j})$ — Performance-Based Cost Model for MOP $p_{i,j}$

$\lambda$ — Minefield density (mines/nm$^2$)

$\lambda_{\mathrm{ft}}$ — False target (non-mine mine-like object) density (objects/nm$^2$);

$m$ — Total number of systems

$m_i$ — Number of systems of type $i$

$\mathbf{m}$ — Force level vector: $\mathbf{m} = \{m_1, \ldots, m_n\}$

$\mathrm{M_0}$ — Number of mines originally laid in the MCM area, $\mathrm{S_{minefield}}$

$\mathbf{m}^L$ — Lower bound for $\mathbf{m}$

$\mathbf{m}^U$ — Upper bound for $\mathbf{m}$

$p$ — Mine clearance probability; i.e., probability that a mine in the MCM area will be cleared

$\mathrm{P}_\alpha$ — Probability that the MCM area will be cleared to the desired minefield clearance rate, $\alpha$

$\mathrm{P_c}$ — Probability of correctly classifying a detection as mine-like or nonmine-like, at range $\mathrm{R_c}$

$\mathrm{P_d}$ — Detection probability at range $\mathrm{R_d}$

$\mathrm{P_{fa}}$ — Detection false alarm rate, (false alarms/nm$^2$)

$\mathbf{p}_i$ — MOP vector for System $S_i$: $\mathbf{p}_i = \{p_{i,1} \ldots p_{i,r_i}\}$

$\mathbf{p}_i^*$ — Low performance threshold specification values for $\mathbf{p}_i$

$\mathbf{p}_i^L$ — Lower bound for $\mathbf{p}_i$

$\mathbf{p}_i^U$ — Upper bound for $\mathbf{p}_i$

$p_{i,j}$ — j-th MOP for System $S_i$

$\mathrm{P_{ID}}$ — Probability of correct mine identification following detection and classification

$\mathrm{P_L}$ — Localization (or re-acquisition) probability

$\mathrm{P_n}$ — Probability of correct mine neutralization, following detection, classification, and identification

$\mathrm{q}(\mathbf{x})$ — Mine clearance rate, $\mathrm{q}(\mathbf{m}, \mathbf{p}_1, \ldots, \mathbf{p}_n)$

$\mathrm{q_T}$ — Threshold value for mine clearance rate (quality threshold)

$r_i$ — Number of MOPs for system $S_i$

$r$ — Total number of MOPs for $S$:

$$r = \sum_{i=1}^{n} r_i$$

$\mathrm{R_c}$ — Mine-like object classification range (yards)

$\mathrm{R_d}$ — Target detection range (yards)

$r_i$ — Dimension of $\mathbf{p}_i$; number of MOPs for $S_i$

$\mathrm{R_r}$ — Range at which $S_2$ has an 80% chance of re-acquiring $S_1$'s detections

$\sigma$ — Standard deviation of mine-like object localization error (yards)

$S$ — System of Systems, comprised of $n$ types of systems: $S = \{S_1, \ldots, S_n\}$

$\mathrm{S_{minefield}}$ — Area to be searched (nm$^2$), referred to as the MCM area

$\mathrm{T_c}$ — Time required to classify a mine (min)

$\mathrm{T_{cf}}$ — Time required to classify a non-mine (min)

$\mathrm{T_{class}}$ — Time required to classify all detections within the search area, $\mathrm{S_{minefield}}$ (hours)

| | |
|---|---|
| $T_{detect}$ | Time required to complete detection pass through search area, $S_{minefield}$ (hours) |
| $T_n$ | Time spent neutralizing (prosecuting) a classified mine (min) |
| $T_{pf}$ | Time spent unsuccessfully attempting to re-acquire a detection (min) |
| $V_{class}$ | Reconnaissance system speed during classification operations (knots) |
| $V_{transit}$ | Reconnaissance system speed during detection and transit (knots) |
| $\omega$ | Simulation-induced process noise on objective function, $G$ |
| $\mathbf{x}$ | $r$-dimensional MOP vector for $S$: $\mathbf{x} = \{\mathbf{p}_1, \ldots, \mathbf{p}_n\}$ |
| $y$ | Noise-corrupted objective function measurement |

# APPENDIX II

## $S_1$: MCM Reconnaissance System

This system is used to survey a suspected minefield area, performing the typical MCM mine-hunting functions of *detection, classification*, and *localization*. The CONOPS is that the area is completely covered first with a detection pass followed by a second pass for the purposes of classification. This must be done at a reduced standoff range from each detected object, necessitated by the much higher frequency sensor generally necessary for this more precise function. Localization is done concurrently with detection and classification, and therefore takes no additional time. In consideration of the overarching system of systems measure of effectiveness, the MOE for $S_1$ is then:

$E_1$ = Time (hours)

to complete reconnaissance of area $S_{minefield}$

given $\lambda$, $\lambda_{ft}$, and $M_0$, where $M_0 = \lambda \cdot S_{minefield}$

and $F_0 = \lambda_{ft} S_{minefield}$.

The time to complete the detection pass over the area (hours) is simply:

$$T_{detect} = \frac{24 \cdot S_{minefield}}{A} = \frac{24 \cdot M_0}{\lambda \cdot A}.$$

Following the detection pass over the MCM area, the reconnaissance system will revisit its localized contacts and attempt to classify each contact as either mine-like or nonmine-like. (Later, the neutralization system will attempt to reacquire and neutralize all declared mine-like objects.) To calculate time to complete classification, we must know how many detections are expected to be made and of what type:

$D_m = P_d \cdot M_0$, Number of detected mines

$D_{fa} = P_{fa} \cdot S_{minefield}$,

Number of mine false alarms

$D_{ft} = P_d \cdot F_0$, Number of false targets detected

To generate expressions for time to classify a real mine as well as false alarms and targets, we must assume a specific classification CONOPS. If we assume that $S_1$ takes the shortest route between contact locations and then executes a semicircle of radius $R_c$ about the contact location, then an approximate expression for the time to classify is:

$$T_c = \frac{60 \cdot d_{mine}}{2000 \cdot V_{transit}} + \frac{60 \cdot \pi R_c}{2000 \cdot V_{class}}$$

What about time spent attempting to classify a target that is, in reality, a false alarm? Let's assume that the CONOPS would be to execute a full circle about the contact location in the event that the first classification pass was unsuccessful during the first half-circle maneuver. The time required to travel to the contact and execute the full circle (minutes) is then:

$$T_{cf} = \frac{60 \cdot d_{mine}}{2000 \cdot V_{transit}} + \frac{60 \cdot 2\pi R_c}{2000 \cdot V_{class}}$$

$$= 2T_c - \frac{60 \cdot d_{mine}}{2000 \cdot V_{transit}}$$

This formulation for $T_{cf}$ keeps it independent of cost drivers for the classification sonar performance, which reduces the number of MOPs necessary in the optimization problem, as the terms $d_{mine}$ and $V_{transit}$ will be considered as fixed for the scenario. The time (hours) required to classify all detections is then:

$$T_{class} = \frac{1}{60}[P_c T_c D_m + (1 - P_c) T_{cf} D_m$$
$$+ T_{cf}(D_{fa} + D_{ft})]$$

$$= \frac{1}{60}\Bigg[P_c T_c P_d M_0 + (1 - P_c) \cdot \left(2T_c - \frac{d_{mine}}{2000 \cdot V_{transit}}\right) P_d M_0 + \left(2T_c - \frac{d_{mine}}{2000 \cdot V_{transit}}\right) \cdot (P_{fa} S_{minefield} + P_d F_0)\Bigg]$$

and we can now formulate the system measure of effectiveness as the sum of $T_{detect}$ and $T_{class}$:

$$E_1 = \frac{24 \cdot M_0}{\lambda \cdot A} + \frac{1}{60}[P_c T_c P_d M_0 + (1 - P_c) T_{cf} P_d M_0 + T_{cf}(P_{fa} S_{minefield} + P_d F_0)].$$

Under the assumptions stated above, we can now list the minimum set of measures of performance that are necessary to formulate an expression for $E_1$ as well as describe performance parameters that will affect the performance of $S_2$. There will be five MOPs, hence $\mathbf{p}_1 = \{p_{1,1}, \ldots, p_{1,5}\}$. Supporting terms and units are defined in the List of Symbols at the end of this paper.

1. Area Coverage Rate: $p_{1,1} = A = (2 \cdot R_d \cdot V_{transit})/2000$. This expression represents a two-sided detection sonar. A typical approximation is that for a particular sonar/target/environment set, $R_d$ is determined by fixing $P_d$ and $V_{transit}$.
2. Probability of Classification: $p_{1,2} = P_c$. For this analysis, the sidescan sonar's $P_c$ is determined at fixed classification range.
3. False Alarm Rate: $p_{1,3} = P_{fa}$
4. Time Required to Classify a Mine: $p_{1,4} = T_c$
5. Mine-like Object Localization Error Standard Deviation: $p_{1,5} = \sigma$. The localization accuracy is a critical parameter for reacquisition, a major function of $S_2$. As a simplification, we have chosen to neglect its effect on $S_1$'s reacquisition during the classification pass, because the reacquisition would be done with the identical sensor suite that performed the initial detections.

After some manipulation (Luman 1997), the final form of the MOE for $S_1$ as a function of the MOP vector is:

$$E_1(\mathbf{p}_1) = S_{minefield}\left[\frac{24}{p_{1,1}} + \frac{1}{60}[\lambda \cdot p_{1,2} p_{1,4} P_d + (2p_{1,4} - T_{transit}) \cdot ((1 - p_{1,2}) P_d \lambda + p_{1,3} + P_d \lambda_{ft})]\right]$$

where

$$T_{transit} = \left(\frac{d_{mine}}{2000 \cdot V_{transit}}\right).$$

Note that this MOE does not reflect the quality to which the reconnaissance is accomplished, only how long it takes. If we were considering the effectiveness of the stand-alone reconnaissance system, then we would want to have $E_1$ reflect other mission MOEs as well, in order to effect a measure of "minefield characterization." Reconnaissance survey quality will be automatically reflected in $E_2$, via expressions that utilize all the elements of $\mathbf{p}_1$ that affect initialization of the neutralization function provided by $S_2$. Additionally, a minimum threshold quality MOE constraint at the system of systems level will also be imposed.

# APPENDIX III

## $S_2$: MCM Neutralization System

The MCM neutralization system attempts to *relocate*, *identify*, and *neutralize* all mine-like objects detected and classified as such by the reconnaissance system. For the purposes of this analysis, the probabilities of identification and subsequent neutralization are assumed to be one, and we will focus on uncertainty related to re-acquisition of all mine-like objects passed to $S_2$ from $S_1$ as contacts. In consideration of the overarching system of systems measure of effectiveness, the MOE for $S_2$ is:

$E_2$ = Time (hours) to complete neutralization and neutralization attempts on all contacts/objects classified as mine-like by the reconnaissance system, $S_1$.

Clearly, $E_2$ will depend upon how many objects of what type are detected and subsequently classified as mine-like objects by $S_1$. Since the neutralization system will attempt to neutralize all declared mine-like objects, it is important to know how many such objects are expected. Expressions for the number of mines correctly classified as mine-like, $C_m$, and the number of non-mines incorrectly classified as mine-like, $C_f$, are as follows:

$$C_m = D_m \cdot \mathrm{P_c} = \mathrm{P_d \cdot P_c \cdot M_0}$$

$$C_f = (D_{fa} + D_{ft}) \cdot (1 - \mathrm{P_c})$$
$$= (\mathrm{P_{fa} \cdot S_{minefield}} + \mathrm{P_d \cdot F_0}) \cdot (1 - \mathrm{P_c})$$

$E_2$ can now be formulated using three MOPs, $\mathbf{p}_2 = \{p_{2,1}, p_{2,2}, p_{2,3}\}$, whose units and supporting terms are defined in the List of Symbols:

1. Contact Reacquisition Range: $p_{2,1} = \mathrm{R_r}$. This is the standoff distance from the localized target which yields an 80% probability of reacquisition.
2. Failed Reacquisition Time: $p_{2,2} = \mathrm{T_{pf}}$. The average time spent in a failed attempt to reacquire a target handed off from $S_1$.
3. Neutralization Time: $p_{2,3} = \mathrm{T_n}$. The average time required to neutralize a correctly classified mine.

The contact reacquisition range is used to calculate the probability of reacquisition, or localization as $\mathrm{P_L} = e^{(-\sigma)/(4.481\mathrm{R_r})} = e^{(-p_{1,5})/(4.481p_{2,1})}$. This yields $\mathrm{P_L} = 0.80$ when $\mathrm{R_r} = \sigma$. This model assumes an exponential decay depending upon localization accuracy, and reacquisition capability of the neutralization system, $S_1$ and $S_2$ MOPs, respectively. Dependence of $\mathrm{P_L}$ on $\mathrm{R_r}$ and $\sigma$ is illustrated in Figure 13. It is this localization error quantity from $S_1$ that has the most direct affect on the performance of $S_2$.

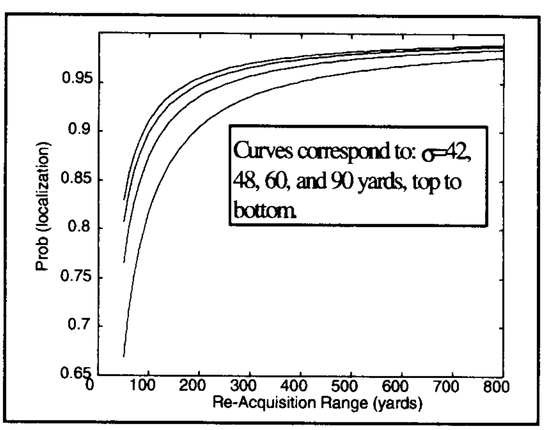


**Figure 13.** Probability of Localization as a Function of Reacquisition Range

Therefore, the MOE for $S_2$ can be expressed as the sum of (1) time to successfully reacquire and neutralize mine-like objects, (2) time spent in unsuccessful attempts to reacquire min-like objects, and (3) time spent prosecuting non-mine-like objects classified incorrectly. After some manipulation (Luman 1997), the final form of the MOE for $S_2$ as a function of the MOP vector is:

$$E_2 = \frac{1}{60}[\mathrm{P_L}C_m\mathrm{T_n} + (1 - \mathrm{P_L})C_m\mathrm{T_{pf}} + C_f\mathrm{T_{pf}}]$$
$$= \frac{\mathrm{S_{minefield}}}{60}[\mathrm{P_d}p_{1,2}p_{2,3}\lambda e^{(-p_{1,5})/(4.481p_{2,1})} + (1$$
$$- e^{(-p_{1,5})/(4.481p_{2,1})})\mathrm{P_d}p_{1,2}p_{2,2}\lambda + (1 - p_{1,2})$$
$$\cdot (p_{1,3} + \mathrm{P_d}\lambda_{ft})p_{2,2}]$$

Therefore, we see that the MOE for $S_2$ depends on three MOPs from $S_1$, making the system of systems "interdependent."

## ENDNOTES

[1] Note: All vectors are row vectors and superscript $^{\mathrm{T}}$ denotes vector transpose. E.g., $\mathbf{x}^{\mathrm{T}}$ is a column vector.

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