

Photonic Analog-to-Digital Conversion

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The analog-to-digital converter (ADC) performs the crucial transformation of physical electromagnetic signals into ones and zeros that can be processed by computers. As such, it is of great importance to a vast number of electronic systems and is often a limiting component for the performance of such systems. Photonic technology is capable of overcoming many of the limitations in traditional electronic ADCs and is therefore the subject of much experimental investigation. In this article we provide a brief overview of ADCs within the context of receiver systems and of the advantages that photonic technology has to offer, as well as a discussion of recent developments in research performed at APL that promise to extend the functionality of photonic ADCs. Finally, we present the results of experiments implementing nonuniform photonic sampling techniques to unambiguously identify signals separated by many traditional Nyquist zones.

INTRODUCTION

Accurate collection and processing of electromagnetic information is critical to a wide variety of applications. Present day RF and microwave sensor systems must cover many frequency bands, detect and identify a large range of signal powers, and analyze the signal information on an ever-decreasing time scale. Simultaneously achieving these attributes typically requires multiple hardware systems, stressing even the most accommodating platforms and requiring the elimination of functionality on some. The power of digital signal processing to deliver increased functionality and improved system performance has long been recognized.

The resulting preference for digital representation of signals as the format for receiver system outputs has elevated the importance of the analog-to-digital converter (ADC), which serves as the interface between the received analog signals and the digital domain. With this move to the digital domain, the ADC is and will continue to be a major bottleneck for many systems.¹ The aim of the authors' current research is to leverage the unique benefits of photonic technology to build ADC systems capable of far greater speed, bandwidth, and accuracy. In this article we describe novel sampling techniques developed and implemented at APL that

enable the accurate capture and processing of incoming signals without requiring high-speed electronics.

ELECTRONIC ADCs

ADC Fundamentals

An ADC performs two basic functions: sampling and quantization of an incoming continuous-time signal. These consist of the discretization of a signal in time and amplitude, respectively, as illustrated in Fig. 1. The number of discrete amplitude levels for the ADC is most often written in terms of the number of bits required to express the levels in binary form. For example, an ADC with a resolution of 8 bits would have $2^8 = 256$ different amplitude levels to approximate the continuous signal. The quantization function is therefore completely specified by two system parameters, the number of bits and the full-scale voltage of the ADC.

Since most ADCs take a measurement of the signal amplitude once every μ seconds, the timing resolution is described in terms of this uniform sample rate $1/\mu$. The sampling function is specified by the frequency of sampling instants ($1/\mu$), the temporal precision of sampling instants, and the duration of the sampling window. Timing jitter of the uniform clock that defines the sampling instants will lead to errors in the digital representation of the sampled signal. In general, this effect can be ignored if the timing jitter δt is small compared with the error that is introduced by quantization. This condi-

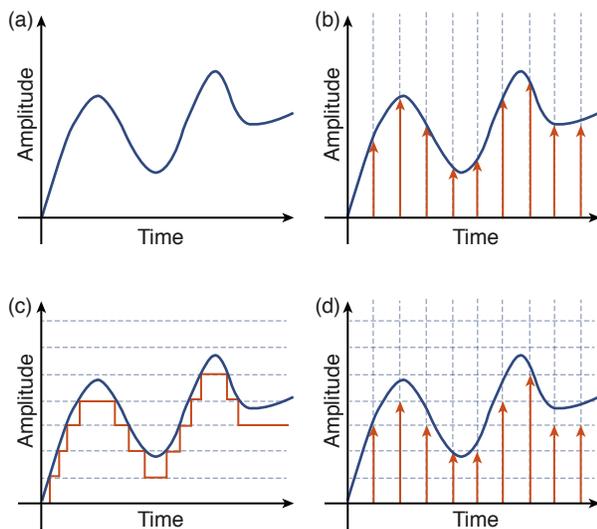


Figure 1. Illustration of sampling and quantization functions. The quantization function is performed by rounding down to the nearest discrete level. (a) A continuous-time, continuous-amplitude signal (analog). (b) A discrete-time, continuous-amplitude signal (sampled). (c) A continuous-time, discrete-amplitude signal (quantized). (d) A discrete-time, discrete-amplitude signal (digital). The original analog signal is shown in blue for comparison.

tion can be expressed mathematically as $\delta t < 1/(2^q \pi f_0)$, where f_0 is the highest frequency of the input signal and q is the number of bits of the ADC. If this condition is not met, the timing jitter of the sampling clock will degrade the effective resolution of the ADC, hence the need for high-precision clock sources. For the case of excessive jitter, the equation can be inverted to find the effective number of bits.

Another error that can be caused by the sampling process is signal aliasing, which is a fundamental limitation on the ability of uniform sampling to accurately represent the original continuous-time signal. The sampling theorem states that a signal of finite bandwidth can be completely reconstructed by an interpolation formula from its uniformly sampled values, so long as the sample rate is at least twice the highest frequency component of the original signal.² Another way of stating this theorem is to say that for a given sample rate $f_s = 1/\mu$, unambiguous identification of the input signal frequency is only possible for frequencies at less than half the sample rate $f_s/2$, known as the *Nyquist frequency*. After interpolation, sampled signals of higher frequency will erroneously appear to have a frequency between 0 and $f_s/2$, known as the *first Nyquist zone*. For example, a signal with frequency $f_0 = f_s/2 + \alpha$ (where $0 < \alpha < f_s/2$) will appear to have the frequency $f_r = f_s/2 - \alpha$ after reconstruction. In general, this *alias* will have the frequency $f_r = f_s \min[\text{mod}[f_0, f_s], 1 - \text{mod}[f_0, f_s]]$, where $\text{mod}[x, y]$ is the remainder function. There is therefore a tradeoff between Nyquist frequency and resolution that must be balanced for uniformly clocked ADCs: sampling higher-frequency signals makes the timing jitter requirements increasingly difficult to satisfy, resulting in a reduced effective number of bits.

Traditional Electronic Receiver Architectures

As an alternative to building an ADC with high-rate digitization hardware, one approach that is often used is to interleave several lower-sample-rate digitizers, which will achieve a high sample rate in aggregate. The channels are staggered in time, as shown in Fig. 2a, collecting samples at a low rate which are then interleaved using digital signal processing. This allows for the use of high-resolution, low-sample-rate digitizers, but it does not entirely avoid the need for high-precision timing: any timing or gain errors between channels will lead to spurious signals in the processed spectrum, known as *interleaving spurs*, which will distort the reconstructed signal. Another approach, shown in Fig. 2b, is to interleave in the frequency domain, using filters and mixers to downconvert a region of the spectrum to within the range of low-rate ADCs. In this architecture, each channel downconverts a different region of frequency space, using filters to eliminate aliasing. This approach is especially well suited to applications in which the signals of interest consist of narrow-bandwidth information encoded on high-frequency carriers. However, although this system

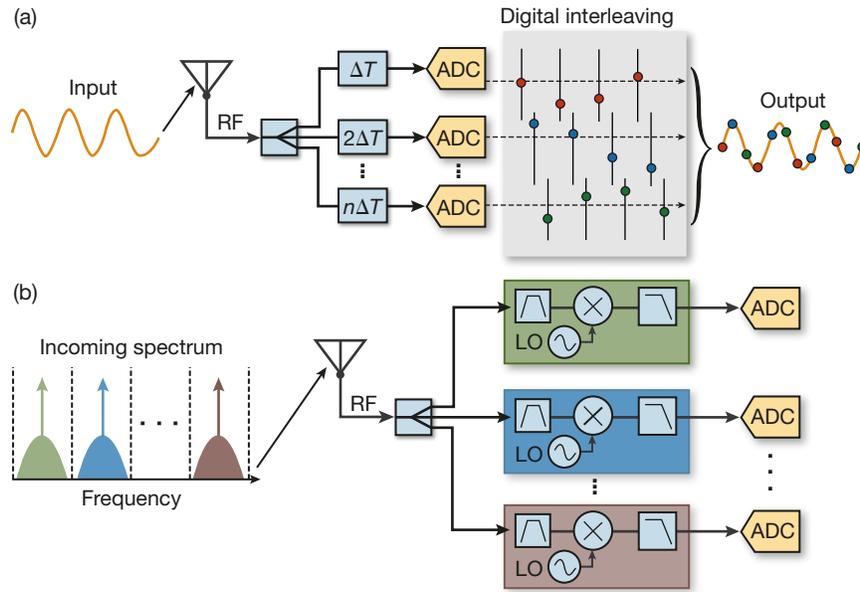


Figure 2. Electronic receiver architectures for a time-interleaved system (a) and a channelized downconversion system (b). ΔT , time delay; LO, local oscillator.

successfully reduces the timing jitter requirements on the ADCs and avoids the signal aliasing problem, downconversion introduces added noise, harmonic distortion, and loss via the mixing process, not to mention the fact that such a system is very hardware intensive. The size, weight, and power requirements of a downconversion system can be prohibitive for many applications in which a wide operational bandwidth is needed.

PHOTONIC SOLUTIONS

As has been investigated at length in Ref. 3, the use of photonic technology in analog-to-digital conversion can be organized into four broad categories: photonics-assisted ADCs, in which photonic components augment the capabilities of an electronic ADC that is responsible for both the sampling and quantization functions; photonic sampling with electronic quantization; electronic sampling with photonic quantization; and photonic sampling and quantization. This article will focus on photonic sampling with electronic quantization. It has long been recognized that the ultrashort pulses generated by mode-locked lasers can be of great utility to the sampling function within the analog-

to-digital conversion process.⁴ Mode-locked lasers capable of generating subpicosecond pulse widths with extraordinary pulse-to-pulse timing stability have developed to the point that they are now commercially available from several vendors. Coupled with high-speed electro-optic modulators, these pulse sources can provide direct sampling capability over 50 GHz of bandwidth or wider, with jitter levels that enable greater than 10-bit resolution systems.

The basic design for a photonic sampled ADC is shown in Fig. 3a. A stable mode-locked laser provides a stream of ultrashort optical pulses, which define the sampling instants for the signal to be digitized. The signal of interest is sampled by an electro-

optic modulator, which encodes the RF input onto the pulse train as an amplitude variation. The encoded optical signal is then converted to the electrical domain using a high-speed photodiode, the output of which is quantized by an electronic ADC. The main benefit of this system is that all of the timing characteristics are controlled by the low-noise optical clock. The sampling time is set by the pulse width and the bandwidth of the electro-optic modulator, the sampling rate is set by the pulse repetition rate, and the timing jitter is set by the jitter of the laser. Although the photonic sampling

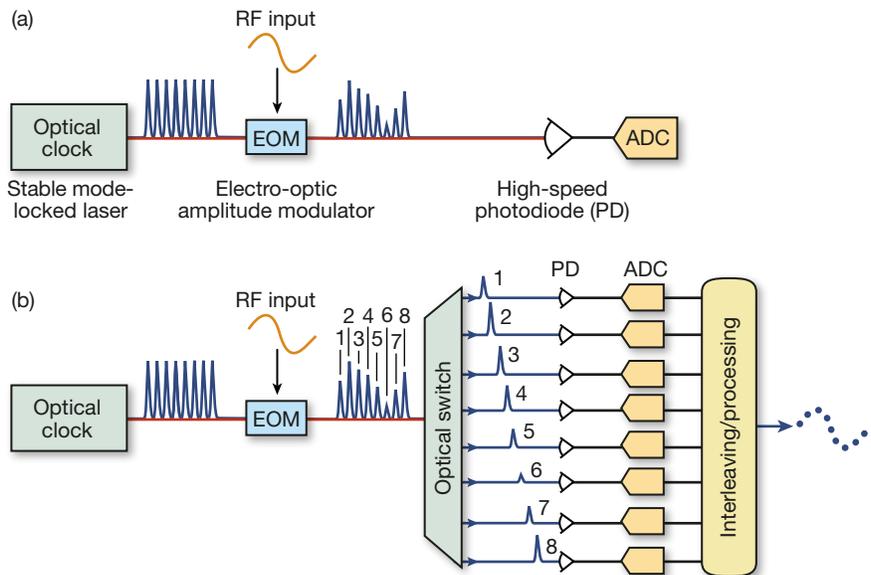


Figure 3. Traditional photonic-sampled ADC architectures. (a) Basic photonic link. (b) Time-demultiplexed system.

improves the timing noise of the system, the architecture of Fig. 3a does not reduce the rate at which the quantization function must take place. Because this simple architecture relies on a single electronic ADC to quantize the photodiode output, the ADC must be clocked at the sampling rate. For continuous broadband signals, such as high-data-rate communications, the aliasing problem once again necessitates a high sample rate and thus high-speed electronics. If the information bandwidth is small compared with the carrier frequency, a technique known as photonic downsampling can be used. This is most useful for systems in which the carrier frequency is known to be in some fixed band and the information on the carrier occupies a bandwidth less than half the sample rate. In this case the mode-locked laser and the modulator accurately sample the incoming signal below its Nyquist rate, allowing the operational bandwidth of the photodiode and electronic ADC to be significantly reduced. An RF filter before the electro-optic modulator can be used to prevent signals outside the frequency band of interest from aliasing into the bandwidth of the ADC. This technique allows for information encoded on high-frequency carriers to be captured with high precision without the use of high-speed electronics or RF downconversion hardware, but it is inherently limited to applications that require only a relatively narrow operational bandwidth.

As with the all-electronic ADC, time demultiplexing into several interleaved channels can reduce the rate requirement for individual ADCs, as well as the operational bandwidth needed for the photodiode. An example of such a system is shown in Fig. 3b, where the demultiplexing is accomplished using an optical switch.^{5,6} Another method that has been used for an interleaved photonic ADC relies instead on a sequence of pulses at different wavelengths such that the optical switch is replaced by a wavelength demultiplexer.⁷ Both of these architectures suffer from the limitations common to all interleaved systems, in particular path-mismatch of time and amplitude between channels, which can lead to unwanted spurs in the reconstructed spectrum. What is needed is a method for sampling below the Nyquist rate that has a wide operational bandwidth with-

out aliasing. Uniform sampling is simply not capable of achieving this goal with a single ADC channel.

THEORY: NONUNIFORM SAMPLING

Randomized Sampling Techniques

Nonuniform sampling techniques can defeat aliasing of high-frequency carriers while sampling well below their Nyquist rate, allowing the sample rate requirement of the system to be dictated by the bandwidth of the encoded data on the incoming signal rather than by its carrier frequency. The basic concept behind nonuniform sampling is illustrated in Fig. 4. Figure 4a shows an example of uniform sampling in which several signals of different frequencies could all equally have produced the same set of samples; there is no way to determine the frequency of the original signal from these samples alone. This is another formulation of the signal aliasing problem described in Electronic ADCs. However, if we allow the sampling instances to deviate from the uniform times (denoted by $\{t_i\}$ in the figure), it is clear that only one of the candidate frequencies could have produced this new set of samples. This result is shown in Fig. 4b, where the nonuniform sample times are denoted in red by $\{t'_i\}$. Nonuniform sampling, coupled with the

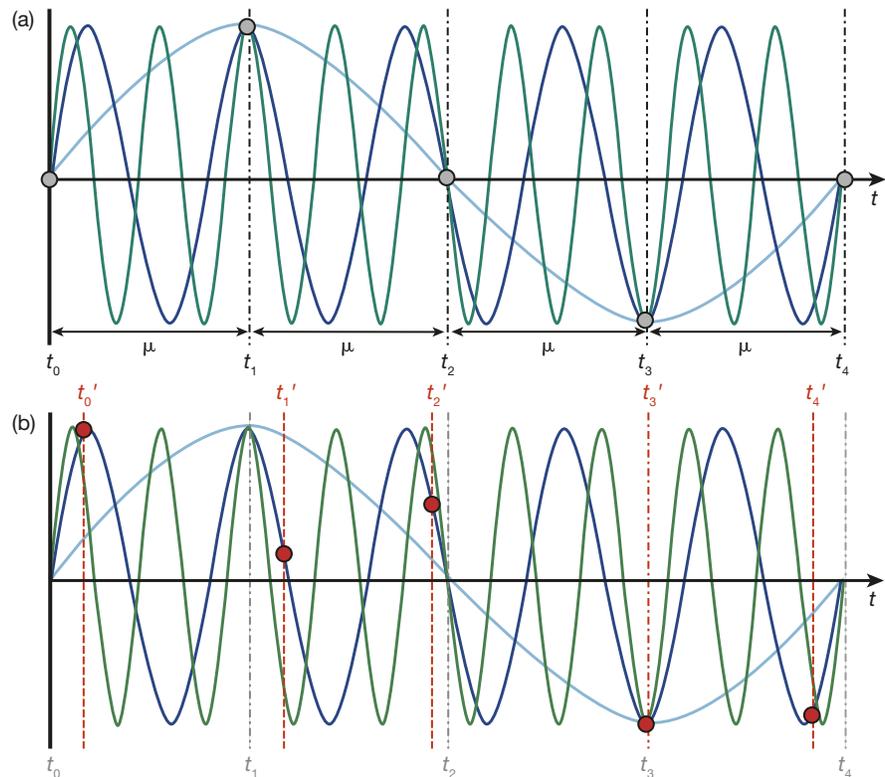


Figure 4. Illustration of nonuniform sampling. Shown as gray dots in panel a are the samples obtained for a uniform sampling process at the rate $1/\mu$, whose sampling instants are shown on the time axis as $\{t_i\}$. Red dots in panel b denote the samples obtained for a nonuniform sampling process, whose sampling instants are shown in red as $\{t'_i\}$.

appropriate reconstruction algorithms, can therefore determine the original carrier frequency of a sampled signal without sampling at twice the frequency of interest. Although there are several possible ways to sample nonuniformly, optimal reconstruction of a signal with an arbitrary carrier frequency will only be achieved for sampling processes that meet certain statistical requirements. A more complete theoretical investigation of the statistical merits of various nonuniform sampling techniques can be found in Ref. 8 and in much more mathematical detail in numerous papers on compressive sampling.^{9,10}

Signal Reconstruction

An intuitive method for signal reconstruction involves the Fourier decomposition of the nonuniformly sampled signal by projecting onto a function space and performing a least-squares analysis. This is simply a “best fit” of the data to a set of frequencies. The basis functions are constructed by using the vector of nonuniform sampling instants and a set of frequencies $\{f_i\}$ chosen to be analyzed. Choosing sines and cosines as the basis functions, a matrix Φ of these transforms can be formed as follows:

$$\Phi = \begin{bmatrix} \cos(2\pi f_1 t_1) & \cos(2\pi f_1 t_2) & \cdots & \cos(2\pi f_1 t_N) \\ \sin(2\pi f_1 t_1) & \sin(2\pi f_1 t_2) & \cdots & \sin(2\pi f_1 t_N) \\ \vdots & \vdots & \ddots & \vdots \\ \cos(2\pi f_M t_1) & \cos(2\pi f_M t_2) & \cdots & \cos(2\pi f_M t_N) \\ \sin(2\pi f_M t_1) & \sin(2\pi f_M t_2) & \cdots & \sin(2\pi f_M t_N) \end{bmatrix},$$

where $\{t_k\}$ are the known sampling instants. Note that the number of frequencies M in the analyzed set is directly related to the bandwidth and frequency resolution considered in the reconstruction algorithm. In order for reconstruction to be possible, M must be less than half the number of samples N , $2M < N$. Letting \bar{c} denote the vector of Fourier coefficients and \bar{y} the vector of sampled data, the optimal estimate of the unknown coefficients can be obtained using the pseudoinverse of the matrix Φ ,

$$\bar{c} = ((\Phi\Phi^T)^{-1}\Phi)\bar{y},$$

which can then be used to approximate the frequency spectrum of the original signal. It is important to note that signal reconstruction requires knowledge of the sampling times, which has consequences for the hardware implementation of a nonuniform sampling system. Aliasing will manifest itself as degeneracy in the rows of Φ , that is to say that basis functions of aliased frequen-

cies will yield identical values when evaluated at the sampling times $\{t_k\}$, rendering the matrix singular and making reconstruction impossible. Therefore there still exist restrictions on $\{f_i\}$ in order to ensure successful reconstruction, but with careful choice of the sampling times, the alias-free bandwidth can be extended to a bandwidth that is many times greater than half the mean sample rate. The key advantage for nonuniform sampling is that information about the carrier frequency can be obtained without having to sample at twice the carrier frequency. To accurately capture the encoded data, all that is required is a mean sample rate scaling with the information bandwidth, a much less stringent requirement than scaling with the carrier frequency.

NONUNIFORM PHOTONIC SAMPLING SYSTEM

Experimental System Design

Figure 5 diagrams the general experimental setup of our nonuniformly sampled photonic ADC.^{11–13} We used an actively mode-locked laser with a pulse repetition rate of 10 GHz to act as a grid of sampling times from which to choose. Individual pulses were then selected pseudorandomly using an amplitude modulator configured to act as an on–off switch. A programmable pulse-pattern generator (PPG) controlled the transmission of pulses, with the pattern designed such that the mean time interval between two successive pulses was $\mu = 10$ ns. The PPG was clocked with the same 10-GHz reference oscillator driving the laser, thereby ensuring that the PPG output would be synchronous with the optical pulses entering the modulator. The length of the pattern and therefore the total measurement time was set to be $\approx 20 \mu\text{s}$. After generating the nonuniformly spaced train of sampling pulses, the incoming RF signal was captured using a second amplitude modulator with a 3-dB bandwidth of 20 GHz. The encoded pulse train was then converted to the electrical domain by a photodiode and passed to an electrical track-and-hold (T/H) circuit. This created

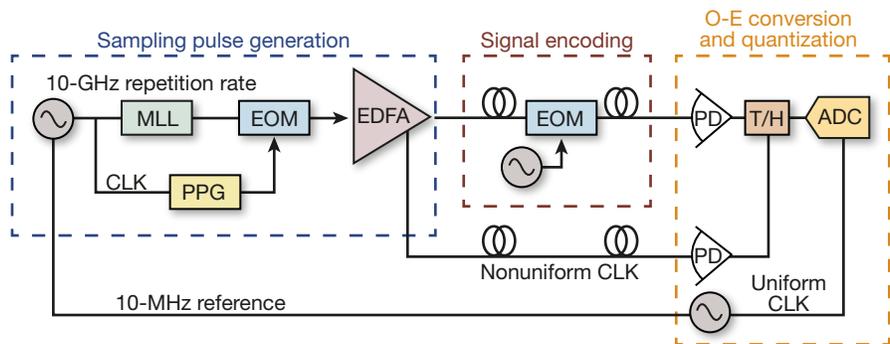


Figure 5. Diagram of the nonuniform photonic sampling system. CLK, clock; EDFA, erbium-doped fiber amplifier; EOM, electro-optic modulator; MLL, mode-locked laser; O-E, optical to electrical.

a series of held voltages for the digitizer to analyze. Note that the input bandwidth of the system is still determined by the photonic sampling operation and can be greater than 50 GHz with commercially available technology. The purpose of the T/H circuit was to allow the ADC to be uniformly clocked, thus converting the nonuniformly sampled signal to a uniformly sampled signal. Quantization is easier to implement with an electronic ADC when the clock is periodic, and the nonuniform-to-uniform conversion facilitates the use of commercially available high-resolution ADCs to perform this function. Crucial to these experiments was the synchronous nonuniform clocking of the T/H circuit with an electronic replica of the sampling pulse train, which ensured that none of the encoded pulses (and thus signal samples) were missed by the electronic digitizer. Finally, the T/H output was digitized by a 14-bit ADC, which was clocked uniformly at the rate of 100 MHz. The timing of the uniform clock, which was synchronous with the master laser oscillator, was adjusted with respect to the T/H output such that the uniform sample times always fell approximately in the middle of the held voltages and not on an edge.

Experimental Results and Discussion

To demonstrate the alias-defeating capability of this technique, two-tone X-band signals from a frequency synthesizer were sampled by the system and reconstructed using the algorithm described in *Signal Reconstruction*. The results are shown in Fig. 6 for several different frequency separations of the two-tone signal. The nonuniform sampling techniques implemented by the system yielded an alias-free bandwidth of 5 GHz, even though the mean sample rate was 100 MHz. As can be seen from the spectra, the system is able to unambiguously identify signals that are separated by many traditional Nyquist zones (a Nyquist zone for this case would span 50 MHz). In Fig. 6c, the two tones are separated by the equivalent of 74 traditional Nyquist zones. The signal-to-noise-floor ratio for this system is limited primarily by the poor amplitude noise performance of the actively mode-locked laser used as the master optical clock. Significant improvement in noise can be achieved by using a passively mode-locked laser, although these sources typically have lower pulse repetition rates. A nonuniform sampling system using this type of laser as the optical clock

would therefore require some method of pulse multiplication in order to be practical. This is an area of research that will be investigated in the future. We are also currently developing a high-repetition-rate, low-noise laser that promises to provide a dramatic increase in the achievable signal-to-noise performance.

We have developed an architecture that capitalizes on the very wide bandwidth of photonic sampling and the data-management efficiency of digital alias-free signal processing, enabling a single hardware system to simultaneously observe many signals across multiple frequency bands without scanning and without parallel filtered receivers. There will be no instantaneous blindness to signals at any frequency, as would occur when scanning one frequency region while a signal resides in another, nor will there be static frequency blind spots, as would occur in parallel filtered receivers. This work represents the first and only demonstration, to our knowledge, of a nonuniformly sampled photonic ADC and is presently being evaluated as a very wide input bandwidth (covering from 1 GHz up to 100 GHz) electronic receiver to support applications with a very wide processing bandwidth (up to 50 GHz in a single output). This approach will improve the receiver probability of detection by eliminating frequency blindness and significantly improve the size, weight, and power requirements as compared with parallel receiver systems, for which a 50-GHz processing bandwidth would be inconceivable for all but the largest of platforms.

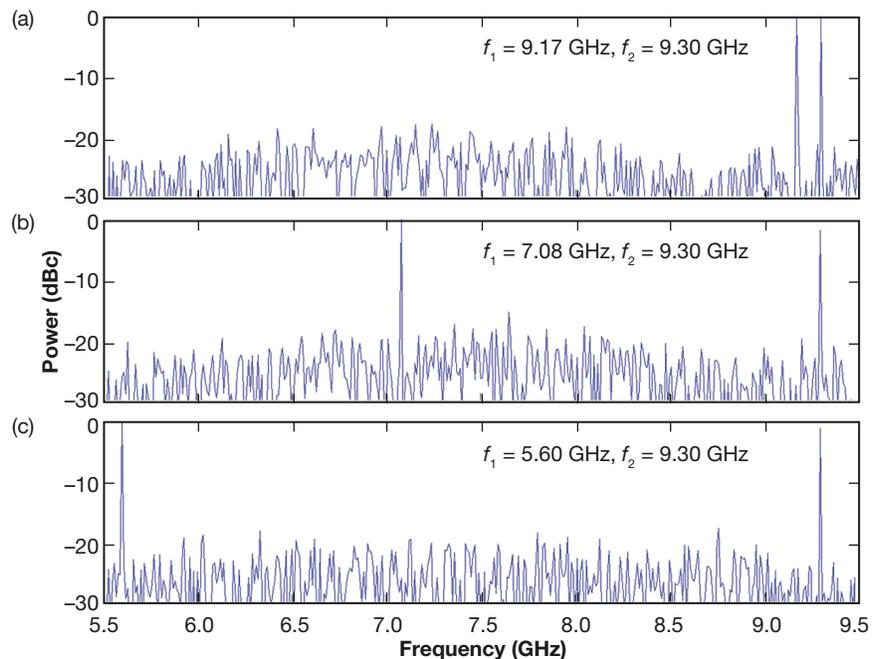


Figure 6. Reconstructed RF spectra for two-tone input signals. Input signal frequencies are indicated as f_1 and f_2 . Mean sample rate was 100 MHz, and the equivalent Nyquist-limited bandwidth is therefore 50 MHz. The input frequencies are accurately identified over an operating bandwidth of up to 80 Nyquist zones. dBc, decibels relative to the carrier.

FUTURE WORK

Photonic analog-to-digital conversion architectures have long been recognized as having the potential to provide significant advantages in bandwidth, timing precision, and timing stability. In our work we have focused on designing a system architecture that will maximally realize the advantages of photonics while retaining the ability to utilize the high performance and maturity of electronic digitization and processing. Experimental demonstration of a photonic sampling system based on nonuniform sampling techniques has shown the capability of such methods to defeat traditional signal aliasing. This suggests that optimization of the signal reconstruction algorithms could further extend the alias-free bandwidth of nonuniform sampling systems. The vast literature available in the related field of compressed sensing presents a fertile ground for further inquiry. Future work will also include optimization of the hardware implementation to better align the functionalities offered by photonic technology with the signal-processing techniques.

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