

RUSSIAN ACADEMY OF SCIENCE

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**RANDOMIZED
ALGORITHMS
OF ESTIMATION
AND OPTIMIZATION
UNDER
ALMOST ARBITRARY
NOISE**

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The book is devoted to the theory of effective recursive algorithms of multidimensional optimization and estimation which achieve a good performance without standard assumptions on observation noise as zero-mean and independence. The new approach is based on randomized simultaneous perturbations. The offered algorithms are widely used in an identification, recognition, control theory and economics.

This book is useful for practical engineers in developing and programming the devices of telecommunication, measurement or control. It is also interesting for a student and post graduate student which specialize in cybernetics.

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