Assessment of Simultaneous Perturbation Stochastic Approximation Method for Wing Design Optimization

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Introduction

THE need for addressing optimization problems that are characterized by the presence of a large number of design variables, complex constraints, and discrete design parameter values exists in many fields including engineering design. A variety of local and global optimization algorithms have been developed for addressing such problems. Besides deterministic methods, stochastic methods such as genetic algorithm (GA) and simulated annealing (SA) algorithm have recently found applications in many practical engineering design optimization problems. These algorithms are easily implemented in robust computer codes as compared with deterministic methods because they do not depend on direct gradient information, which most deterministic methods do. However, SA and GA methods require a large number of function evaluations and relatively longer computation time than deterministic methods, especially in the case of complex design problems. Although the use of parallel GA and parallel SA as outlined in Wang and Damodaran[1] offers a way to reduce the large computational time, an attractive alternative to SA and GA could be the simultaneous perturbation stochastic approximation (SPSA) method described in Spall.[2] The SPSA method has been applied to numerous difficult multivariate optimization problems in many diverse areas such as statistical parameter estimation, feedback control, simulation-based optimization, signal and image processing, and experimental design. The essential feature of SPSA, which accounts for its power and relative ease of implementation, is the underlying gradient approximation, which requires only two measurements of the objective function regardless of the dimensions of the optimization problem. This feature allows for a significant decrease in the cost of optimization, especially for problems with a large number of variables to be optimized.

The aim of this Note is to compare performance of SPSA with SA and GA and to explore any advantages that SPSA might offer to overcome the large computational efforts of SA and GA when applied to wing-design problems. These methods are briefly outlined following the statement of the wing-design optimization problem, which will form the application problem to assess and compare the performance of SPSA in relation to SA and GA.

Wing-Design Problem

The application concerns the design of wing shape such that the aerodynamic efficiency of the wing or the lift L to drag D ratio reaches a maximum value during cruise with the wing weight acting as a constraint, that is, the goal is to determine the wing geometry by either minimizing D/L or maximizing L/D with the wing weight as a constraint. The D/L ratio can be formulated in detail using the analytic formulas for aerodynamic analysis as defined in Raymer.[3] The lift L is defined as $L = C_L q S$, where $q = \frac{1}{2}\rho V^2$ is the dynamic pressure, ρ is the density of air, V is the flight speed, $C_L = C_{L\alpha}\alpha$ is the the lift coefficient where α is the angle of attack and $C_{L\alpha} = 2\pi A_R/\{2 + \sqrt{[4 + (A_R\beta/\eta)^2(1 + \tan^2\lambda/\beta^2)]}\}$ is the lift curve slope. In the expression for lift curve slope, $A_R (= b^2/S)$ is the wing aspect ratio, where b is the wing span, λ is the wing sweep angle, η (value of which lies in the range 0.95–1.0) is the airfoil efficiency factor, $\beta = 1 - M^2$ is the compressibility factor, and M is the Mach number. The total drag is defined as $D = C_D q S$, where the total drag coefficient is $C_D = C_{\mathrm{Di}} + C_{D0}$, which consists of the induced drag coefficient $C_{\mathrm{Di}} = C_L^2/(\pi A_R e)$ and the zero-lift drag coefficient $C_{D0} = C_f F Q$. In these expressions $e = 4.61(1 - 0.045 A_R^{0.68})(\cos\lambda)^{0.15} - 3.1$ is the wing planform efficiency factor, $C_f = 0.455/[(\log_{10} Re)^{2.58}(1 + 0.144M^2)^{0.65}]$ is the surface skin-friction coefficient, which is a function of the Reynolds number Re, $F = \{1 + [0.6/(x/c)_m](t/c) + 100(t/c)^4\}[1.34M^{0.18}(\cos\lambda)^{0.28}]$ in which t/c is the airfoil thickness-to-chord ratio, $(x/c)_m$ is the chord-wise location of the maximum thickness-to-chord ratio, taken as 0.3 in the present study, and Q is a factor accounting for interference effects on drag taken as 1.0 in the present study. The weight of the wing (in pounds) is $W_{\mathrm{wing}} = 0.0106(W_{\mathrm{dg}} N_z)^{0.5} S^{0.622} A_R^{0.75}(t/c)^{-0.4}(\cos\lambda)^{-1}$, where W_{dg} is the design gross weight in pounds and N_z is the ultimate load factor, which is assumed to be 13.5 for subsonic flow.

The design variables for the wing design optimization, that is, α, b, c, λ, and W_{wing}, represent the angle of attack, wing span, mean aerodynamic chord, sweep angle, and wing weight, respectively. The objective function to be optimized is $F(X) = D/L$ and is defined as follows:

$$\text{Minimize } F(X) \tag{1}$$

subject to six constraints on the design variables defined as follows:

$$\begin{gathered} 1.0\,\mathrm{deg} \le \alpha \le 10.0\,\mathrm{deg}, \qquad 10.0 \le b \le 50.0 \\ 3.5 \le c \le 10.0, \qquad 0.0\,\mathrm{deg} \le \lambda \le 35.0\,\mathrm{deg} \\ 0.5 \le A_R \le 15.0, \qquad W_{\mathrm{wing}} \le 2473\ (\mathrm{lb}) \end{gathered} \tag{2}$$

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Table 1 Comparison of optimal results from SA, GA, and SPSA

Algorithm	α, deg	b, ft	c, ft	λ, deg	W, lb	D/L	NFE
SPSA	3.432	44.994	5.986	18.115	2443.93	0.0319	1,149
GA	3.199	43.861	4.955	19.912	2444.03	0.0324	13,040
SA	3.199	44.987	5.947	18.002	2443.93	0.0319	9,601

An external penalty function method is used to incorporate the constraints so that the composite function to be minimized can be defined as

$$F(X) = \frac{D}{L} + \sum \max(0, g_j)^2 \tag{3}$$

where X is the vector of the six design variables and the design constraints $g_j(X) \leq 0$, which are represented as inequality constraints.

Optimization Algorithms

The optimization algorithms used in this study are stochastic global search methods. SPSA is relatively easy to implement and does not require gradient information. It is a fairly robust method and has the ability to find a global minimum when multiple minima exist. SPSA is an algorithm that is based on a "simultaneous perturbation" gradient approximation. The simultaneous perturbation approximation uses only two function measurements independent of the number of parameters (say, p) being optimized. The SPSA algorithm works by iterating from an initial guess of the optimal vector X_0. First, the counter index k is initialized to a value of 0, an initial guess of the design variable vector X_k is made, and nonnegative empirical coefficients are set. Next a p-dimensional random simultaneous perturbation vector Δ_k is constructed, and two measurements of the objective function, namely, $g(X_k + c_k\Delta_k)$ and $g(X_k - c_k\Delta_k)$, are obtained based on the simultaneous perturbation around the given vector X_k. The parameter $c_k = c_0/(k^m)$, where c_0 is a small positive number taken as 0.01 in this study, k is the loop index, and m is a coefficient taken as $\frac{1}{6}$ in this study. The term Δ_k represents the random perturbation vector generated by Monte–Carlo approaches, and the components of this perturbation are independently generated from a zero-mean probability distribution; a simple distribution that has been used in this study is the Bernoulli ± 1 distribution with probability of $\frac{1}{2}$ for each of the ± 1 outcome. This is followed immediately by the calculation of the gradient approximation based on two measurements of the function based on the simultaneous perturbation around the current value of the design variable vector and the updating of the design vector X_k to a new value X_{k+1} using standard SA form. Finally the algorithm is terminated when insignificant changes in several successive iterations occur or if the maximum allowable number of iterations has been reached. The details of the step-by-step implementation of the SPSA algorithm can be found in Spall.[4,5] The SA method used is described in Deb.[6] For this problem SA is implemented by setting the initial temperature to 5, and the cooling schedule is algebraic of the form $T_{k+1} = \gamma T_k$, where γ takes a value of 0.5. The GA method used in this study is outlined in Deb[6] and essentially follows the method in Goldberg.[7] For this problem GA method is implemented with a population size of 80; a crossover probability of 0.90 and mutation probability of 0.05 have been used to arrive at optimal values.

Results and Discussions

For the wing-design optimization problem the values of the parameters used are $M = 0.7$, $t/c = 0.12$, $Q = 1.0$, $(x/c)_m = 0.3$, and $\eta = 0.95$, and the same termination criterion $|f(x_{k+1}) - f(x_k)| \leq 10^{-6}$ was used to terminate the optimization methods. Table 1 shows the optimum values of the objective function and design variables reached by the two optimization algorithms. In this table NFE refers to the number of function evaluations required to reach the global minima. Figure 1 shows the variation of the objection function with the number of function evaluations required using the SPSA method to reach the optimal value. It also shows the variation of the computed objective function (D/L) with wing weight. Figure 2 shows

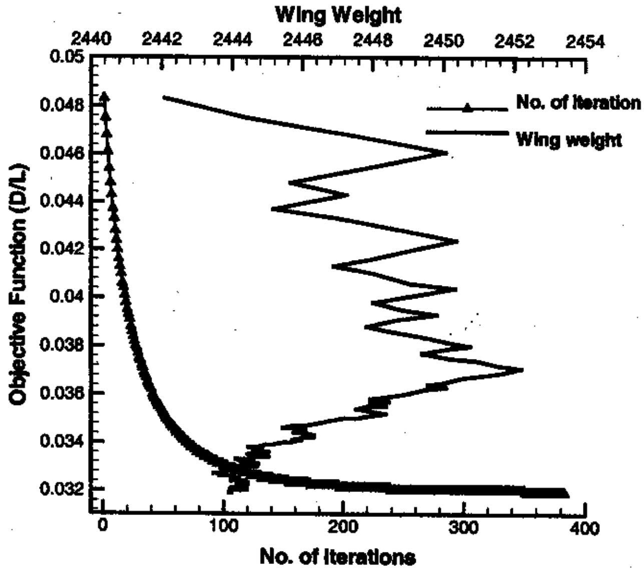

Fig. 1 Convergence of objective function vs design iterations and the variation of objective function vs wing weight toward global optimal values.

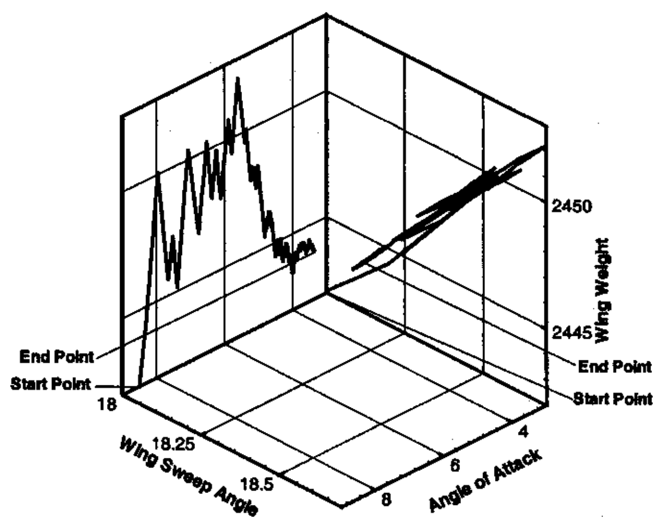

Fig. 2 Projected routes of design variables angle of attack α and sweep angle λ vs wing weight toward the global optimal values.

the variation of angle of attack and wing sweep with wing weight. It can be seen that the optimal values of the objective function and design variables subject to the same constraints from SA and GA are very similar to those attained by SPSA. It can be seen that SPSA attained optimal design values in 383 iteration steps, that is, 1149 measurements of objective function. At the same time GA took more than 13,000 iterations, and SA took more than 9,000 iterations to reach the optimal results. The SPSA method is a significantly faster method than either GA or SA as a global optimization method for this wing-design problem and can serve as a potential cost effective stochastic global optimization design tool than either SA or GA for similar classes of design problems.

Conclusion

A wing-design optimization problem was performed using SPSA, SA, and GA methods in this study. It can be seen that SPSA is more efficient than SA and is also relatively easy to implement. SPSA requires only two measurements of the objective function regardless of the dimensions of the design space corresponding to the optimization problem and the cost of optimization decreases. Although SA and GA can avoid getting trapped in local optima, they require a large number of function evaluations and a long computation time to reach the optima. Future work to assess the performance of SPSA for constrained and unconstrained aerodynamic shape design studies will be carried out in the near future to establish the cost benefits and to investigate the extent to which SPSA offers comparative advantages over GA or SA for aerodynamic design optimization problems.

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