

# Fifty Years of Orbit Determination: Development of Modern Astrodynamics Methods

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*Precision orbit determination (OD) methodologies have evolved over the past 50 years through research by astrodynamics specialists from industry, university, and government organizations. Refinements have included improvements in modeling techniques from analysis of satellite tracking data over a wide range of orbits. Methods have been developed to evaluate force models and the enhancement of model fidelity using a variety of geodetic-quality satellites placed into orbit since the early days of the space program and continuing today. This article provides an overview of OD methodologies and their evolution as well as a brief description of modern OD and estimation methods that are being used routinely in the 21st century by the astrodynamics community. The subject matter should also be useful reading for the nonspecialist.*

## INTRODUCTION

Satellite orbit determination (OD) can be described as the method of determining the position and velocity (i.e., the state vector, state, or ephemeris) of an orbiting object such as an interplanetary spacecraft or an Earth-orbiting satellite. The object's motion is approximated by a set of equations of motion with the state adjusted in response to a set of discrete observations and subject to both random and systematic errors. In the context of this article, the OD problem is generally described by the computational process (generally solved by applying statistical estimation techniques) of determining the

state of a satellite as a function of time using the set of measurements collected onboard the satellite and/or by ground-based tracking stations.

The satellite is usually assumed to be influenced by a variety of external forces, including gravity, atmospheric drag, solar radiation pressure, third-body perturbations, Earth tidal effects, and general relativity in addition to satellite propulsive maneuvers. The complex description of these forces results in a highly nonlinear set of dynamical equations of motion. Furthermore, the lack of detailed knowledge of the physics of the environment

through which the satellite travels limits the accuracy with which the state of the satellite can be determined at any given time. Similarly, observational data are inherently nonlinear with respect to the state of the satellite. Since the OD equations are also highly nonlinear, linearization is normally performed so that linear estimation techniques can be used to resolve the OD problem. The solution can be obtained over a short orbit arc of less than 1 h or a long orbit arc approaching many days or longer. Different techniques have also been applied to obtain an accurate solution.

These ideas can be applied to a wide variety of OD problems, ranging from near-Earth satellite orbits to lunar and interplanetary transfer orbits. This article focuses on definitive OD and the statistical OD techniques associated with it that are used by DoD, NASA, and the global astrodynamics community. Associated topics, such as initial OD (IOD) and specific orbit propagators used by NASA and DoD for space object tracking and catalog maintenance are touched upon briefly but are not addressed with any rigor. However, the principles described are broadly applicable to near-Earth as well as deep space (interplanetary) missions.

Because of the sophistication and complexity encompassing OD, the article does not describe the issue in depth. There are, however, a number of excellent textbooks that cover the technical details on the theory and applications of OD.<sup>1-5</sup> An excellent review of atmospheric drag studies and research on satellites conducted in the United Kingdom before the launch of Sputnik I in the late 1950s through the 1980s is provided by King-Hele.<sup>6</sup>

As noted above, the state vector of an orbiting satellite is composed of a set of position and velocity components that are usually defined in a Cartesian reference frame, normally referenced to the Earth's center of mass. The

term “state vector” is sometimes used interchangeably with the word “state” to describe the satellite’s location in 3-D space. The term “definitive OD” is referenced in the astrodynamics literature as precision OD (POD). The objective of POD is to obtain an accurate orbit that accounts for the dynamical environment in which the motion occurs, including all relevant forces affecting the satellite’s motion. Through this process, a preliminary orbit is estimated using a minimum number of observations. This estimate provides the initial conditions for numerical integration of the nonlinear differential equations of motion to obtain a reference orbit. A differential correction procedure is then used to iteratively correct the reference orbit and refine the final orbit solution. An improved orbit is thus obtained by using many observations or observational data sets along with an accurate physics-based model describing the dynamical environment. POD orbits are those that best satisfy all available observations and require the ultimate in observational accuracy.

A context diagram for a near-Earth satellite OD solution is illustrated in Fig. 1. This map shows that there are at least three requirements for an OD system to work effectively: the completeness of the underlying physics and mathematical rigor of the theory, the observational (tracking) data or measurements to satisfy the observability features in space and time, and the computational techniques employed, which may be influenced by the computer hardware and software used. The computational process includes both orbit propagation and statistical estimation techniques.

### HISTORICAL BACKGROUND OF OD

OD for celestial bodies is a topic that has attracted the interest of some of the best astronomers and

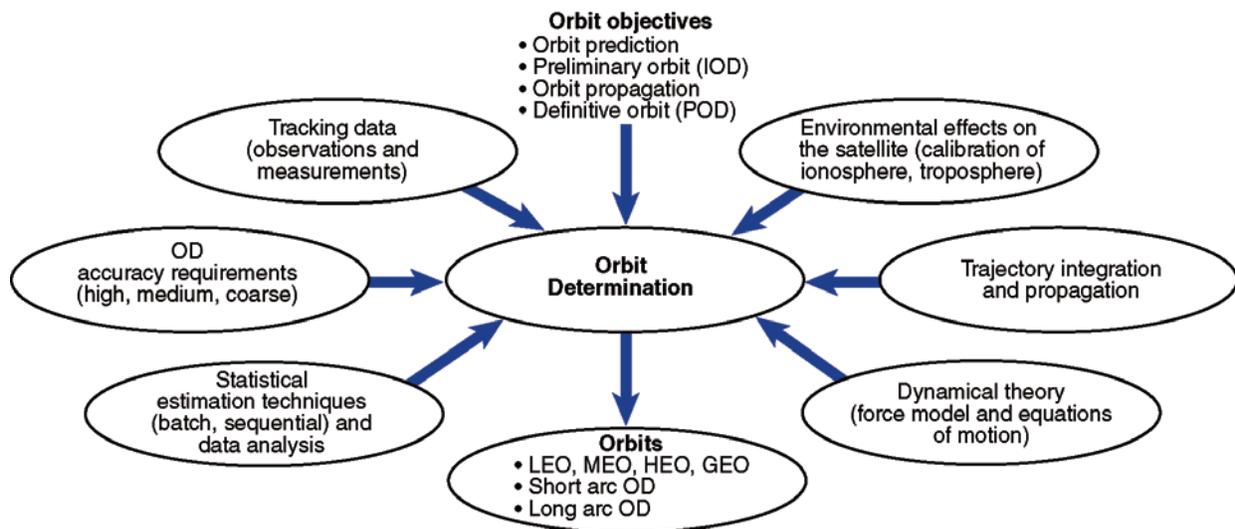


Figure 1. Context diagram for standard OD.

mathematicians for centuries. The method of IOD was solved by K. Gauss (1801) for the orbit of the asteroid Ceres using three angles and the method of least squares (LS), which he developed. The need for OD of Earth satellite orbits arose after the launch of Sputnik in 1957. APL's G. Weiffenbach and W. Guier<sup>7,8</sup> used data from Doppler tracking of the Sputnik I satellite in 1957, which formed the basis of the Transit system. In the early days

of satellite OD, only a handful of satellites and instrumentation systems were available for use in orbital analysis. Table 1 lists the ground instruments that were used in satellite tracking in the very early days of the satellite era. For example, initially Sputnik was tracked mostly by visual observations that were accurate to only a few kilometers. Consequently, with accuracies such as these, the orbits obtained from short data arcs fitted over the

**Table 1. Tracking and orbital accuracies of ground instruments used in the early satellite tracking era.**

Ground instrument	Tracking accuracy	Orbit accuracy at 1000-km altitude (3-D, root mean square)	System description
DOVAP (Doppler Velocity and Position) (1945)	10 m	NA	Doppler velocity and position system for missile tracking (Army, White Sands Missile Range); radio beacon in vehicle; UHF, 36 MHz (uplink) and 77 MHz (downlink); phase comparison at three ground receivers
Schmidt cameras (1950)	1–3 arc-sec, 1 ms	10 m	Air Force/Smithsonian Astrophysical Observatory (SAO); great resolution but required clear weather and darkness
UDOP (UHF Doppler) (1950)	10 m	10 m	UHF Doppler at 440 MHz; similar to DOVAP with higher resolution and less loss of accuracy from ionospheric effects; Pershing missile tracking and satellites
Hewitt cameras (1954)	1 arc-sec, 1 ms	5 m	UK; best-resolution camera; nontracking
Baker-Nunn cameras (1956)	1–3 arc-sec, 1 ms	10 m	Air Force/SAO tracking camera evolved from Schmidt cameras
MiniTrack I (1956)	3 arc-sec, 1 ms	200 m	Navy; three system angles only; radio interferometer by phase comparison; 137 MHz (VHF)
Azusa (1958)	5–10 m	10 m	Army; radio interferometer; continuous wave-based system at C-band
Micro-Lock (1958)	10 m	10–20 m	Air Force/NASA; radio interferometer
MiniTrack II (1959)	3 arc-sec; 1 ms	200 m	Navy; two system angles only; radio interferometer by phase comparison; 137 MHz (VHF)
MISTRAM (MISsile TRAjectory Measurement) (1960)	1 m	5 m	Range, range-rate, and angle measurements at five Air Force sites; radio continuous-wave interferometer; MM 1 and satellites
Transit (1959–1996)	0.15 m/s	50 m	Dual-frequency Doppler at 150 MHz (uplink) and 400 MHz (downlink)
Jodrell Bank Observatory (JB) (UK)	1°, 1 s	200 m	Range and angle data from passive big dishes (JB, LL); pulse radar trackers (LL, JB); data at the time too sparse and inconsistent for definitive satellite applications
Millstone Hill (MIT Lincoln Lab [LL])	5 km		
Goldstone Station (NASA) (1960)	30–50 km		
SECOR (Sequential Correlation of Range) (1961)	5 m	10 m	Army; continuous-wave four-frequency ranging system at 421 MHz (UHF)
DOPLOC (Doppler Phase Lock) (1963)	5–10 m	10 m	Army; Doppler phase-locked narrowband tracking filter by radio reflection at 108 MHz; single-pass orbit solution

observation span were of modest accuracy, ranging from several kilometers to several hundreds of meters.

In the 1960s, camera tracking and radio Doppler tracking systems used in missile tracking tests by the Army were developed and employed for satellite tracking. Many of these early systems were used for missile guidance system analysis only and not directly for satellite tracking, e.g., DOVAP (Doppler Velocity and Position). Other tracking systems such as the GLOTRAC (Global Tracking System) and ODAP (Offset UHF Doppler), which were spin-offs of these early systems, provided comparable accuracies. After the launch of the early satellites, these systems were moved to the Air Force range in Florida and evolved into radio systems commonly used at the time such as MiniTrack, Azusa, MISTRAM (MISsile TRAjectory Measurement), and DOPLOC (Doppler Phase Lock), etc.

The observations used in the solution process for the orbit consisted of a mix of optical data obtained passively and collected by cinetheodolites and Baker Nunn, Schmidt, and Hewitt cameras. In many cases, visual sightings obtained using binoculars, Doppler data collected from Transit satellites that transmitted an RF signal in space, and MiniTrack radio interferometer measurements that yielded purely angular measurements were also used in the solution. The accuracy of these measurements was not particularly good, ranging from  $0.030^\circ$  (cinetheodolites) to  $0.003^\circ$  (Hewitt camera for tracking satellites, 1957–1965). For a 1200-km orbit this represented a worst-case positional uncertainty of about 0.5 km. With improvements in the performance of the photographic and Doppler techniques, orbit position accuracy improved to about 10–20 m. With the development of laser ranging systems in the mid to late 1960s, the precision of the observations approached 5–10 m. Since the 1970s, advances in laser technology, radio tracking techniques, and force modeling have improved orbit accuracies to better than 5 cm in orbit altitude. The Topex/Poseidon mission is one such case.<sup>9</sup> Today, orbit precision 3-D accuracies are routinely in the 2- to 5-cm range.

The techniques used in OD today are the product of an evolutionary process from the early 1960s through the late 1980s and incorporate refinements made in modeling techniques and improvements in ground-based tracking systems. The early geodetic satellites that started with Transit and expanded to the GEOS (Geodetic Earth Orbiting Satellite) suites of satellites (1965–1980) provided a wealth of high-quality tracking data that allowed a definitive assessment of the gravity field, evolving into the high-fidelity force models

used today. A recent paper by Thomas Thompson<sup>10</sup> on the development of the Transit navigation program provides an interesting insight into the technological challenges encountered during the early experimental and prototype phases of this program. Table 2 (on the following page) shows the evolution of the many specially designed satellite systems used in developing force models. Geodetic satellites are still evolving today (e.g., Topex, GRACE [Gravity Recovery and Climate Experiment], CHAMP [Challenging Mini-satellite Payload], and GOCE [Gravity Field and Steady-State Ocean Circulation Explorer]) and now include onboard instruments such as three-axis accelerometers and gyro inertial instruments as well as 3-D gravity gradiometers.

The principal applications of OD are for Earth-orbiting satellites. However, POD can also apply to spacecraft motion beyond the Earth's gravitational attraction, including vehicles orbiting the Moon and planets as well as spacecraft in interplanetary space (heliocentric orbits). Very accurate angular measurements in the form of very long baseline interferometry (VLBI) observations are used in interplanetary navigation and OD. These consist of differenced one-way range and Doppler measurements from two Deep Space Network sites on different continents that track the spacecraft simultaneously using a precise timing source along with an extragalactic source such as a quasar whose location is precisely known. In this manner, the planetary ephemeride is tied to a celestial reference source that provides a much tighter fit for the interplanetary orbit.

Beyond the Earth's sphere of influence, the effects of third-body perturbations become the more dominant force model effect on the spacecraft. However, a satellite in Earth orbit is affected by a large number of different perturbations depending on orbital altitude. Figure 2 illustrates the effect of different force model perturbations on three different orbit regimes from low-Earth orbit (LEO) to geosynchronous Earth orbit (GEO). Generally LEOs are at 1,000-km altitudes (400–1,600 nm), medium-Earth orbits (MEO) at 10,000-km altitudes (1,500–6,500 nm), and GEO at 35,000-km altitudes. Typical design lifetimes for satellites in LEO are

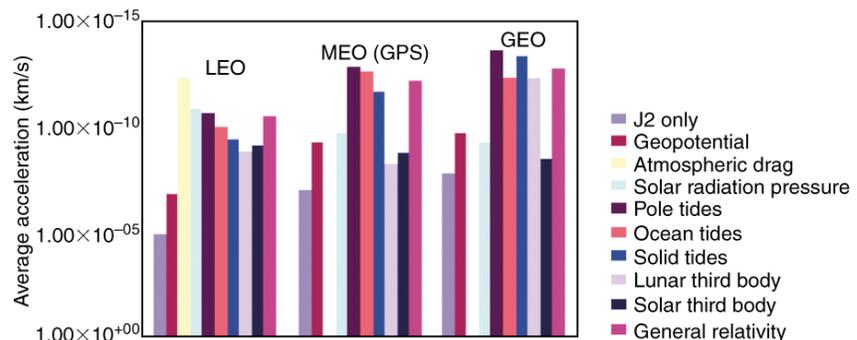


Figure 2. Effect of force model errors on satellite orbit altitudes.

**Table 2. Geodetic satellite missions used in the development of force models.**

Satellite	Date	Application
Transit/Oscar	1959–1967	Two-frequency Doppler system (150 and 400 MHz)
Echo	1960	Passive balloon for photography against a star background; built by Bell Telephone Laboratories for first communications experiments
ANNA 1B (Army/Navy/NASA/Air Force)	1962	Flashing light satellite; ultrastable oscillators
SECOR	1962	Army map service range-only system at four ground sites
GEOS (Geodetic Earth Orbiting Satellite) 1	1965–1968	APL-built optical beacons, laser reflectors, radio ranging transponder (SECOR), Doppler beacons, and range and range-rate transponder
PAGEOS (Passive GEO satellites)	1966	Passive GEOS (aluminum-coated plastic balloon) used as a passive geodetic satellite
GEOS 2	1968–1974	APL-built instrument; same as GEOS-1 except for two C-band beacons and a passive radar reflector
Triad/Transit Improvement Program (TIP)	1972	APL-built TIP; NavPak, drag compensation system, time and frequency
Starlette	1975	Passive laser satellite tracked by NASA/Single Laser Tracker (SLR) sites (French)
GEOS (Geodynamic Experimental Ocean Science) 3	1975–1978	APL-built instruments; same as GEOS-2 except for radar altimeter; S-band transponder and satellite-satellite tracking (SST) using the ATS geostationary satellite ranging with an LEO satellite
LAGEOS (Laser Geodynamics Satellite) I	1976	Passive laser with corner reflectors tracked by NASA/Goddard Space Flight Center (GSFC) SLR sites
SEASAT	1978	Radar altimeter
GPS	1978–present	Blocks I, II, II R-M, IIF, III, GPS satellites (DoD/Air Force)
LAGEOS II	1980	Passive laser with corner reflectors tracked by NASA/SLR sites
Nova	1981–1988	Advanced Oscar
GEOSAT (Geodetic/Geophysical Satellite)	1985–1989	APL-built radar altimeter
Topex/Poseidon	1992–2006	APL-built radar altimeter plus GPS receiver
Stella	1993	Passive laser satellite tracked by NASA/SLR sites (French)
CHAMP (Challenging Mini-satellite Payload)	2000	SST (Hi-Lo) + three-axis accelerometer; GPS dual-frequency Blackjack (BJ) P-code receiver (NASA/GeoForschungsZentrum Potsdam [GFZ])
GRACE (Gravity Recovery and Climate Experiment)	2001	SST (formation flying in same orbit); accelerometer; BJ GPS receiver (GFZ)
Jason	2001	Radar altimeter plus GPS BJ receiver; Topex/Poseidon follow-on (European Space Agency [ESA])
GOCE (Gravity Field and Steady-State Ocean Circulation Explorer)	2008 (est.)	SST (Hi-Lo) + three-axis gravity gradiometer (GG) (ESA); Ku/C-band altimeter; GPS 12-channel codeless receiver; accelerometer and gyro inertial measurement unit (ESA)

less than 5 years, in MEO close to 10 years, and in GEO 10–15 years based on battery and electronic components in the space environment and fuel/propellant capabilities. However, theoretical (natural) orbit lifetimes in LEO depend on satellite area-to-mass ratios and solar conditions but are generally less than 100 years,<sup>11</sup> whereas MEO and GEO orbit lifetimes are

greater than 1,000 years. Recent studies of GPS disposal orbits have shown that significant long-term resonance effects may cause the orbit to evolve into a highly elliptical orbit (HEO) after 200 years and reenter the atmosphere.<sup>12</sup> The Starlette/Stella retroreflector satellites (noninstrumented) have a predicted lifetime of 2,000 years.

## EVOLUTION OF ANALYTICAL THEORIES AND DEVELOPMENT OF MODERN OD

In the terminology of astrodynamics, analytical procedures are categorized as general perturbation (GP) methods and numerical integration procedures are referred to as special perturbation (SP) methods. Figure 3 shows an evolutionary growth tree from analytical theories to modern POD astrodynamics codes used today. The development of analytical orbit theory began in the late 1950s and early 1960s with the work of Brouwer<sup>13</sup> and Kozai.<sup>14</sup> Brouwer adapted the Hill-Brown lunar theory in 1946 to the low-Earth satellite problem using rectangular coordinates. The theory was developed to second-order terms using mean orbital elements and included inclination and eccentricity as power series; it was, however, precise only for nearly circular orbits

or near-equatorial orbits. The theory involved canonical transformations of Hamiltonian mechanics using Delaunay variables to simplify the theory and incorporated only low-order zonal terms of the Earth's potential. Brouwer's method was later modified by Lydanne<sup>15</sup> to handle the singularities of eccentricity and inclination and by A. Deprit et al.<sup>16</sup> for critical inclination.

Although Kozai's method was a first-order theory and was easier to understand using Lagrange's planetary equations, Brouwer's analytical theory was selected as the basis of the Navy's Position and Partial Model (PPT3) and the Air Force's Simplified General Perturbation Model (SGP4) used by the Navy and Air Force Space Commands, respectively. Both PPT3 and SGP4 produce the two-line element sets for maintaining the space catalog and are employed by most satellite users today but are accurate to only a 1- to 10-km

level. Kozai's theory was the basis of the SAO (Smithsonian Institution Astrophysical Observatory) Differential Orbit Improvement (DOI) program that was used in the early to mid 1960s to analyze very accurate Baker-Nunn camera observations. It formed the basis of the standard Earth gravity models such as the  $8 \times 8$  gravity field in 1963 and  $16 \times 16$  field description in 1966. This was later replaced by NASA's GEODYN program that is used today for POD and precision geophysics applications. Later, Kaula<sup>17</sup> developed an orbital theory in Keplerian orbital element space using osculating or instantaneous orbital elements. This allowed, for example, third-body, resonance (see below), and solid and ocean tidal perturbation effects (in terms of Love numbers obtained from terrestrial observations or numerical Earth models for the amplitude and phase) to be handled more easily. It was also incorporated into the orbit element space, did not suffer from singularities, and handled more general cases. (Resonances are orbital disturbances caused by repeating ground tracks over the same features on Earth. In fact, this effect is due to the impressed frequency of some high-order harmonics [11th–29th] becoming equal to the natural frequency of the satellite's motion. If not

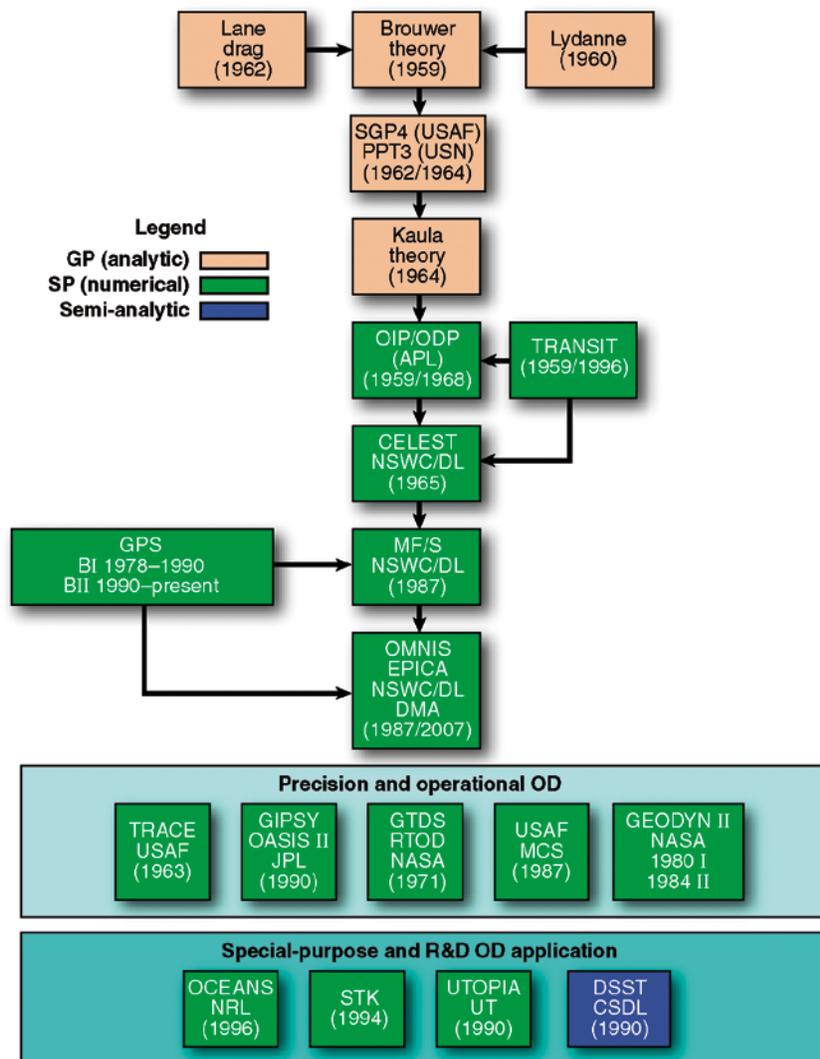


Figure 3. Evolutionary growth of analytic theory and modern astrodynamics codes.

accounted for, the accuracy goal of high-accuracy missions [e.g., Topex/Poseidon] could not have been met if resonant terms in the Earth's geopotential were not incorporated.)

### CONCEPTS OF OD

The general procedure for all definitive or precision ODs is to set up a dynamical model of the orbit that uses observations from all sources available to improve the parameters of the orbit by the process of differential corrections. Applications include

- Orbit propagation, which uses a blend of SP (e.g., Cowell numerical integration or semi-analytic methods) and GP analytic methods (e.g., SGP4, PPT3, Brouwer-Lyddane theory) to propagate an orbit in time and space
- Orbit prediction, which uses a fairly accurate orbit model with perturbation terms and a prediction algorithm such as extended Kalman filtering (EFK) to predict the future state of the satellite beyond the data arc used in POD
- Definitive OD (POD), which uses a set of observations from tracking measurements to estimate the orbit solution with a statistical level of confidence using weighted LS or sequential estimation methods

The specific orbits may range from LEO to GEO. The model can either be a set of differential equations representing the satellite's motion or a set of functions in time that represent changes to the fundamental parameters of the system.

In a "batch method" approach, the measurements are used to determine the state of the satellite at some epoch, which is then mapped forward in time using the dynamical models (a "fully dynamic" approach). However, the state may also be determined sequentially at each desired time with little dependence on dynamics

if continuous tracking data are available (a "fully kinematic" approach, such as with GPS). "Reduced dynamic" OD, done mostly with GPS orbits, is a hybrid approach<sup>18</sup> whereby the dynamic model parameters are held fixed once a converged solution is obtained, with additional accelerations estimated using the observation geometry alone. Figure 4 depicts all basic areas used in practice for the solution of routine OD problems.

The process of differential correction cited earlier in this article is used in computing the residuals or differences between the observations of the satellite being tracked and the predicted position from the estimator. The residuals are then used to calculate a set of corrections to the starting state vector to minimize the residuals, and the solution is iterated in this fashion until convergence is achieved. All orbit propagators, including the SGP4 and PPT3 analytical orbit propagators, and all numerical SP programs operate nearly identically, regardless of whether they are a batch processor or a sequential processor. The dynamical models describing the forces acting on the satellite include both conservative forces, such as gravitational attraction of the Earth, Moon, and planets, as well as nonconservative forces, such as atmospheric drag and solar radiation pressure. Some of the models can be described in terms of accurate analytical or semi-analytical formulas or by numerical techniques.

The source and type of tracking data collected may consist of ground-based and/or onboard measurements that have varying spatial and temporal distributions. In some computational approaches, biases in station location, ground or satellite clocks, onboard oscillators, etc., can be included as part of the state to be estimated.

The computational approaches for processing the measurement data are called statistical estimation techniques, of which classical LS (batch estimate) and state-space KF (sequential estimate) have been used most commonly. Most POD is done on the ground in a postflight mode, but many recent satellite missions—particularly

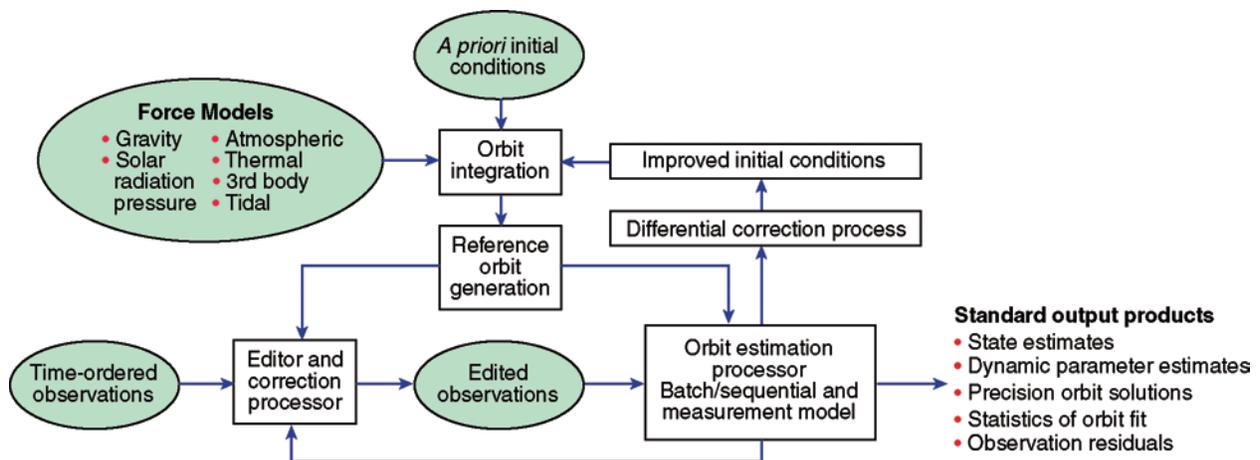


Figure 4. General model of the OD process.

NASA satellite missions that incorporated GPS receivers beginning in the early 1990s and continuing today—have used autonomous navigation. For example, TDRSS (Tracking and Data Relay Satellite System) demonstrated OD in 1994; GPS Enhanced OnBoard Navigation System (GEONS) demonstrated OD in 1996 and is currently achieving accuracies of 20 m and 3 cm/s in real time; Topex/Poseidon, launched in 1992, demonstrated real-time POD in 1998; and the APL-built TIMED (Thermosphere, Ionosphere, Mesosphere Energetics and Dynamics) satellite, launched in late 2001 with an onboard GPS receiver, demonstrated OD in 2002.

## THE OD PROBLEM

For a near-Earth satellite, the OD problem can be described mathematically by representing the propagation of the orbit by equations of motion with respect to a geocentric inertial reference frame expressed simply as

$$\ddot{\vec{r}} = -\frac{GM}{r^3}\vec{r} + f_G + f_{NG},$$

where  $\vec{r}$  is the state vector of the satellite. The term  $f_{NG}$  is the sum of all nongravitational accelerations acting on the satellite and includes atmospheric drag and possible lift, solar radiation pressure, Earth radiation pressure due to changes in Earth albedo and emissivity, satellite thermal radiation due to temperature variations over the satellite, charged particle-induced drag, and empirically derived forces. The term  $f_G$  is the sum of all noncentral gravitational accelerations (static Earth field and Sun, Moon, and planetary effects) and includes temporal gravity effects from solid-Earth tides, atmospheric and oceanic tides, and general relativity. The state vector can be generalized to include any quantities directly affecting the motion of the satellite dynamically such as the Earth's gravity field (described by tesseral harmonic coefficients) or drag ( $C_D$  coefficients), or the observation-state relationship kinematically (e.g., tracking station biases). In general, the estimate of the state will differ from the true state because of a combination of effects, including

- Mathematical formulation and parameter errors embedded in the equations of motion
- Mathematical formulation and parameter errors in the observation-state relationship
- Random or systematic errors in the observations
- Numerical errors in the computational procedures used in the estimation process

## OD BATCH METHODS

All early statistical estimation algorithms employed for OD consisted of batch methods whereby all measurements are used in a single estimation solution using

Bayesian LS techniques. OD batch processes range from solutions based on single-pass short arcs of tracking data (an hour or less) to very long arcs ranging from 30 days to several years. The types and duration of the fit spans are very important ingredients in defining the orbit characteristics and the accuracy of the solution obtained. Between these two extremes, medium-arc fit solutions cover time spans on the order of 1 week or less. For short data arcs, the dominant error sources are usually observation errors. As data arc lengths increase, dynamic model errors become more dominant in the batch solution. In general, the longer the period during which the dynamic models are applied and the complexity of the solution require sophisticated numerical processing techniques to solve common arc parameters and describe a set of global parameters that are applicable over all arcs. Batch methods can be used for POD for many applications, including LEO to GEO orbits, lunar and planetary transfer, deep space (heliocentric) missions, and lunar and planetary OD. Likewise, the OD solution can be performed autonomously in-orbit or on the ground.

## OD ESTIMATION TECHNIQUES

While the batch method has been the hallmark approach for some time, it is slowly being supplanted and replaced with sequential estimation in which new estimates of the state are derived with the addition of each new measurement. Typically, KF formulation is used, although the standard formulation is often recast into forms that take advantage of the numerical power embedded in mathematical algorithms for solving large dimensional matrices. These newer forms may cast the problem in terms of a factorized upper-diagonal (UD) or square-root information filter/smoothing (SRIF/SRIS) formulations.<sup>19</sup> Smoothers can be thought of as Kalman filters that run backward in time. In addition, hybrid techniques have been used that employ a mini-batch approach that blends both batch and sequential methodology. The consider-state<sup>18</sup> approach, whereby the statistical effects of poorly modeled or unknown states can be modeled in the covariance matrix but not estimated in the state, can also be used. This method recognizes the presence of these states and is less sensitive to modeling errors than a filter that tries to estimate these states. Kalman formulation has been used effectively in some autonomous on-orbit OD solutions with good success. The UD and SRIF methods have been used when a large number of measurements are used, such as with planetary missions involving optical and radio-based measurements. These methods have been extensively employed for GPS orbital fits as well (e.g., the Jet Propulsion Laboratory [JPL] GIPSY program uses both SRIF and UD techniques for OD). Although the sequential forms have their uses, most precision geodetic

and geophysical studies that require POD solutions use a standard batch LS approach (e.g., NASA’s GEODYN II). Real-time and postflight programs such as NASA’s GTDS use a combination of batch-weighted LS KF (both linear and extended versions) and mini-batch options, depending on the mission. Newer and special filter types have been used over the last several years, including particle, unscented, and sigma-point filters, and apply specific mathematical techniques to avoid filter divergence caused by highly nonlinear problems where the EKF approach tends to underestimate the covariance of the state. However, none of these types of filters has found particularly wide application to POD solutions over the standard approaches.

**COMMON DATA TYPES FOR OD**

Several types of observations are routinely used today in OD. Table 3 lists the various observation types and sources.

Figure 5 illustrates a comparison of GPS orbit differences between the Naval Surface Warfare Center (NSWC) OMNIS precision-derived ephemerides used as the reference orbit and the International Geodynamics Service (IGS) and ESA derived precision ephemerides over a fit span of approximately 10 h. IGS uses the GIPSY/OASIS II software for POD propagations (see

Table 5). Transformation rotations from the J2000 to the ITRF92/ECEF frame were made to make the comparison. Both IGS and ESA precision ephemerides compared to better than 2 m, with a standard deviation of 50 cm (3-D) over the fit span. Comparisons today have agreed to the 50-cm level with a one-sigma of 5 cm.

**OD SOFTWARE**

A wide range of software exists today within the scientific community (university, government, and industry) for the analysis of precise observational data and the generation of high-accuracy orbits. In contrast, data from the early days of OD lacked accurate tracking data, software, or satellites over various inclinations and eccentricities. In those days, analytic satellite theories (Brouwer and Kozai) were the predominant means used to solve for short tracking arcs of data by fitting an LS solution to a set of data. Longer arcs were not very accurate because of limited knowledge in the gravity field, the unknown effects of drag, and other inaccuracies in force field modeling. Modern software packages have been designed for specific applications, such as GPS, some for a variety of satellites (independent of eccentricity, orbital inclination, or satellite altitudes) and others for simultaneous estimation and prediction of a constellation of satellites.

Early in the 1960s, almost all of these codes were run on mainframe computers such as IBM 7094 and 360 machines. For example, the APL OIP program used for Transit OD was developed in 1959 in assembly language and run initially on a UNIVAC 1103 and then on an IBM 7094; a later version, ODP, was developed in PL1 and run on the IBM 360. The NSWC CELEST program was used as the operational post-processing Doppler software at the Navy Astronautics Group and was checked quarterly with APL orbit solutions throughout the life of the Transit program. Today, many of the POD software codes are run on workstations. Some of these are completely flexible and can be used in multiple applications by manipulating a list of operating parameters. These new packages achieve high accuracy by using high-fidelity models and high-order numerical integration codes between epochs. Typically, numerical integration errors are not a large error source; rather, errors in the force field modeling

**Table 3. Common data types used in orbit determination.**

Content	Source
Azimuth and elevation angles and slant range	Passive or active radars
Right ascension and declination	Baker-Nunn cameras, telescopes, binoculars, visual sightings, big dish radar telescopes, cine-theodolites
Azimuth	Direction finders
Time of closest approach	Radars, radio receivers (for transmitting [Doppler] satellites)
Range, angles, and range-rate	Special Doppler radars
Two- and three-way Doppler	NASA/Goddard range and range-rate
Space-based observations	Onboard instruments (magnetometers, star trackers, gravity gradiometers, GPS receivers, accelerometers)
GPS	Pseudo-range and carrier phase; single, double and triple differences of the basic measurement types
Direction cosines	Interferometer systems (MiniTrack, MISTRAM), Air Force radar interferometer sensor network
VLBI and DVLBI, delta DOR	Very-long baseline interferometry (VLBI) and differential VLBI (DLBI) measurements; delta-differenced range and range-rate (DOR) measurements of artificial (planetary spacecraft) or natural radiating (e.g., quasars) sources

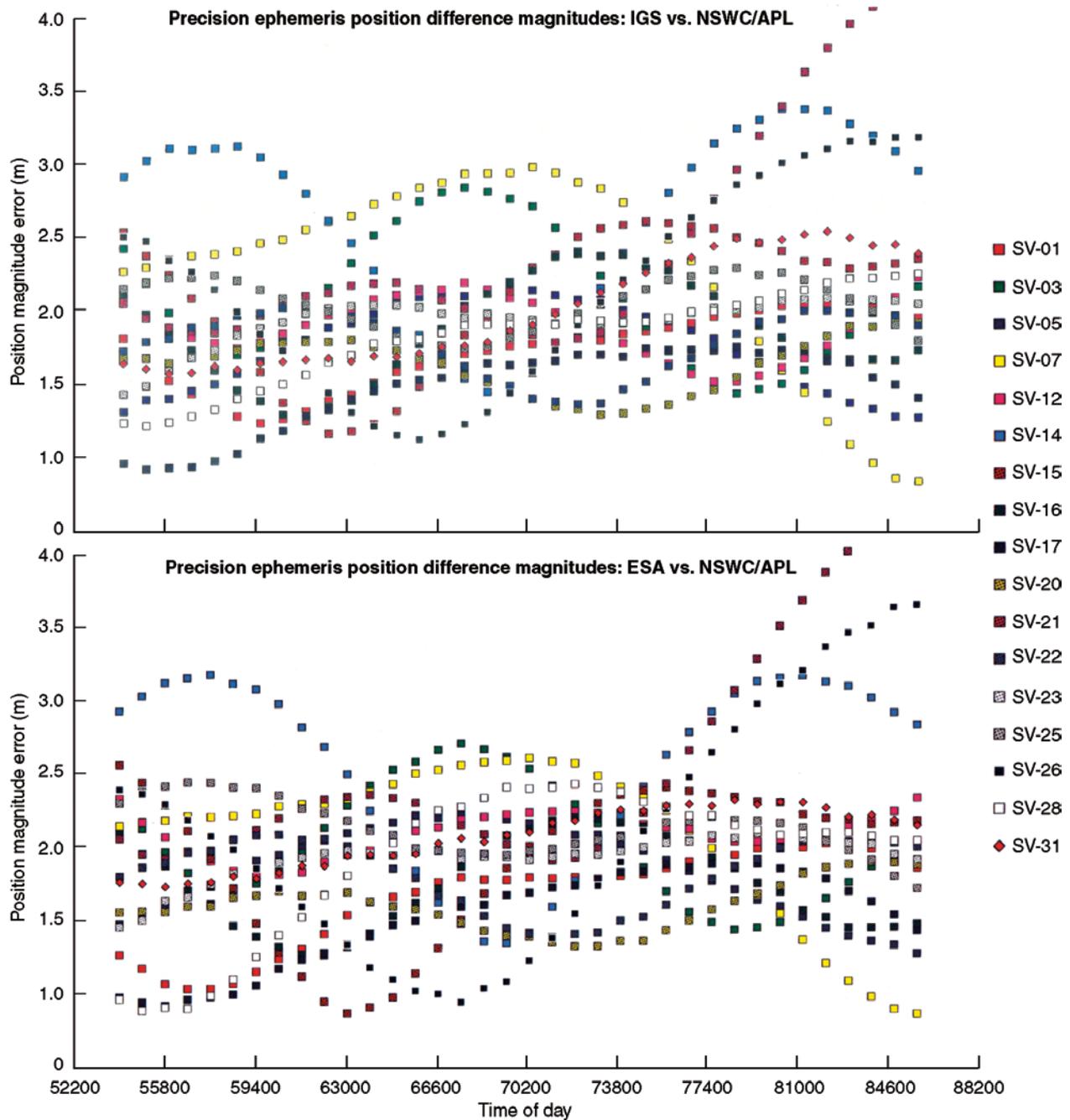


Figure 5. Examples of GPS satellite comparisons derived from precision orbits (SV = satellite vehicle).

tend to be the dominant error sources, assuming that a properly tuned integration and step size are used. GEODYN II, used for POD and geophysical research processing, is run on CRAY and CYBER machines. Some special-purpose variants of OD software from the late 1990s have been adapted to vector processes on supercomputers, and some applications, such as geodetic and geophysical solutions, are even being adapted to parallel-processing architectures. Finally, some of the codes used in special-purpose applications and research activities (e.g., GTDS) have been ported to

PCs whereby semi-analytic methods have been applied for OD and estimation studies.

### THE OD USER COMMUNITY

The OD community in the United States consists of universities, government laboratories, and commercial organizations. The organizations that are routinely involved in OD are NASA/GSFC, NASA/JPL, The Aerospace Corporation, the Naval Research Laboratory (NRL), APL, the University of Texas at Austin,

NSWC/DL (Dahlgren Laboratory), the National Geospatial-Intelligence Agency (NGA), AFSPACECOM, the University of Colorado, MIT/CSDL (Charles Stark Draper Laboratory), MIT/LL (Lincoln Laboratory), and NAVSPACECOM. The European community has a comparable suite of users. Table 4 provides a brief summary of the relevant software programs commonly

used today and their specific application areas. Table 5 provides a list of some of the major OD programs currently employed with a description of primary applications areas, data types, and program capabilities. Most of these programs undergo continual refinements and improvements to adapt to model improvements and mission requirements.

**Table 4. Organization-specific orbit propagator and determination programs and applications.**

Organization	Software program	Primary application
Aerospace Corporation/USAF	TRACE	Operational OD evaluation and covariance analysis <a href="http://www.aero.org/publications/crosslink/summer2002/04.html">www.aero.org/publications/crosslink/summer2002/04.html</a>
Analytical Graphics Inc.	STK/HPOP	Integrated graphics and numerical processing <a href="http://www.agi.com/products/desktopApp/odtk">www.agi.com/products/desktopApp/odtk</a>
Charles Stark Draper Laboratory	DSST	Precision semianalytical OD technique <a href="http://www.csdl.org">www.csdl.org</a>
	DGTDS	POD
APL	OIP/ODP	Transit Doppler post-processing OD used in the 1960s through the 1980s
MICROCOSM	MICROCOSM	Commercial software OD package of the NASA GEODYN program <a href="http://www.vmsi_microcosm.com">www.vmsi_microcosm.com</a>
MIT/LL	DYNAMO	POD, specifically for HEO and GEO satellites <a href="http://www.ll.mit.edu">www.ll.mit.edu</a>
NASA/GSFC	GTDS	Operational OD for LEO, MEO, and GEO orbits (TDRSS) and lunar and interplanetary orbits <a href="http://fdab.gsfc.nasa.gov/live/Home/Tools_Nav_GTDS.html">fdab.gsfc.nasa.gov/live/Home/Tools_Nav_GTDS.html</a>
	RTOD	Precision real-time OD for onboard spacecraft using Kalman filtering <a href="http://nctn.oact.hg.nasa.gov/ft-tech-GEONS.html">nctn.oact.hg.nasa.gov/ft-tech-GEONS.html</a>
NASA/GSFC	GEODYN II	POD for geodesy and geophysics <a href="http://bowie.gsfc.nasa.gov/697/POD/POD.html">bowie.gsfc.nasa.gov/697/POD/POD.html</a>
NASA/JPL	MIRAGE	Multiple satellite OD using GPS
NASA/JPL	DPTRAJ	Interplanetary OD
NASA/JPL	GIPSY/OASIS II (GOA)	POD of satellites using GPS, SLR, and DORIS observations <a href="http://gipsy.jpl.nasa.gov/orms/goa">gipsy.jpl.nasa.gov/orms/goa</a>
Navy/NSWC	OMNIS/EPICA	GPS precision orbits <a href="http://earth-info.nga.mil/GanG/sathtml/gpsdoc2006_11a.html">earth-info.nga.mil/GanG/sathtml/gpsdoc2006_11a.html</a>
Navy/NSWC	PPT3 <sup>a</sup>	Surveillance and space debris tracking and propagation
Navy/NSWC	Special-K	Operational numerical OD program
Navy/NRL	OCEANS	Orbit studies, covariance analyses, and GPS orbits <a href="http://www.nrl.navy.mil">www.nrl.navy.mil</a>
SAO	DOI	Used in the early 1960s for OD of Baker-Nunn camera data and development of standard Earth gravity models
USAF/SPACECOM	MCS	GPS operational orbits
USAF/SPACECOM	SGP4 <sup>a</sup>	Surveillance and space debris tracking and propagation
USAF/SPACECOM	SPADOC/ SPECTR	Operational numerical OD program used by Shreiver and Kirkland AFBs
USAF/SPACECOM	ASW	Workstation numerical OD program
University of Texas	UTOPIA, MSODP	Precision orbits using GPS, SLR, and DORIS observations; TRANET, OPNET, altimetry <a href="http://www.csr.utexas.edu">www.csr.utexas.edu</a>

<sup>a</sup>Not used for OD.

**Table 5. Major OD software programs in community use.**

Program	Organization (sponsor)	Data types useable	Filter type	Models handled and integrator	Primary application	PC- or mainframe (MF)-based	Program capabilities
CELEST	NSWC/DL (Navy) (1965)	Doppler	Batch LS		Transit, GPS	MF	Multi-arc, multi-satellite
GEODYN II	NASA/GSFC (1984)	All data types	Batch LS	11th-order Cowell predictor-corrector	All satellite types for POD and geophysics	Cyber205 Fortran-based	Multi-arc, multi-satellite
GIPSY/OASIS II Real-Time GIPSY	NASA/JPL (1990)	GPS, SLR, DORIS	SRIF/SRIS	High-order Adams predictor-corrector	High-precision orbit types with GPS receivers	UNIX WKS	Multi-arc, multi-satellite
GTDS	NASA/GSFC (1975)	All data types	Batch LS	4th-order Runga Kutta, Cowell Adams predictor-corrector	NASA operational satellites, analytic and research support	MF/Fortran, R&D (PC/WIN), VAX, Sun IBM MF, SiG WKS	Multi-arc, multi-satellite (50) solve for parameters
MCS	USAF/SPACECOM (1987)	GPS pseudo-range (PR) or carrier phase	Partitioned six-state LS filter only	High-order predictor-corrector	GPS	MF or PC	Fixed-state partition
MicroCosm	VMS, Inc. (1990)	All data types	Batch LS	Cowell predictor-corrector	All satellite types	UNIX, VAX, or PC	No multi-arc capability
OCEANS	NRL (1996)	Laser PR Carrier-phase R A E (range, azimuth, elevation)	Batch KF (GPS)	Cowell 4th-order Runga Kutta, 9th-order predictor-corrector	Covariance studies, research applications	PC	Multi-arc, multi-satellite
OIP/ODP	APL (Navy) (1960)	Doppler	Batch LS	4th-order Runga Kutta	Transit	MF	Multi-arc, single satellite
OMNIS (GPS)	NGA (National Imagery and Mapping Agency [NIMA]; Defense Mapping Agency [DMA]) (1987)	GPS PR or carrier phase	SRIF/SRIS MiniBatch	Cowell predictor-corrector	GPS or satellite vehicle (SV) with GPS receiver (GPSr)	RISC6000 and SuperMini	Multi-arc, epoch state
OMNIS (GPS)	NSWC/DL (Navy) (1987)	PR or carrier phase	SRIF/SRIS, Mini-batch	Cowell predictor-corrector	GPS or SV with GPSr	UNIX and RISC6000	Multi-arc, epoch state
STK Version 5	Analytical Graphics (2007)	All data types	Optimal Kalman filter and fixed-interval smoother	Runga Kutta, Gauss-Jackson	All satellite types	UNIX and PC	Multi-satellite
TRACE	Aerospace Corp. (Air Force) (1960)	R,A,E GPS PR, Doppler, range rate, optical data	SRIF/SRIS, Sequential batch LS	10th-order Gauss-Jackson w/ regularized time option	General analysis of operational systems and evaluation of prototype systems	UNIX, WKS, and PC	Multi-satellite (60), 1000 estimated parameters, 200 tracking stations
UTOPIA, MSODP	University of Texas at Austin, Center for Space Research (1990)	Laser, altimeter, range-rate (one- and two-way), GPS, Doppler	SRIF/SRIS	Fixed-step, fixed-order integrator	POD	Cray, HP, UNIX workstation	UTOPIA for single satellite, MSODP for multi-satellite

Special-purpose OD programs
  Operational OD programs for all data types
  GPS operational programs

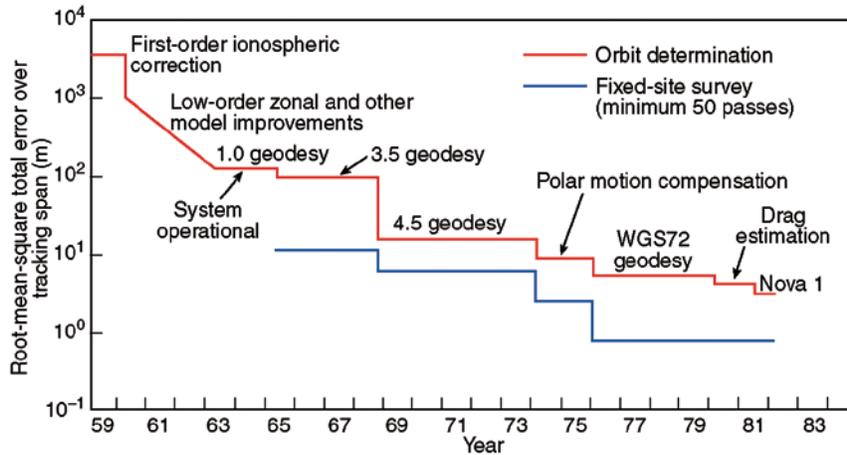


Figure 6. History of Transit orbit accuracies (1959–1983).

**ACCURACY**

In the early days of the satellite tracking era, orbits were generally tracked from a single station using optical instruments at high altitudes (>1000 km) to avoid the effects of drag. The orbital accuracies obtained in this early period were good to 10- to 100-m levels over short arcs. As additional instrumentation systems came along, such as Transit and the Laser Tracking System, the accuracy of the solutions improved from a single-station short-arc solution to a multi-site, multi-arc solution, which took from 1 to 2 weeks. The history of the Transit satellite orbital accuracies from 1959–1983 is shown in Fig. 6. By 1972, it was found that Transit needed an OD every 1 to 2 days as a result of the unpredictable effects of drag. Orbital fits in the 1960s were at the 100- to 200-m level, which evolved to 10 m by the mid-1980s, largely because of system improvements and the development of a high-fidelity gravity field. Transit was removed from service for navigational purposes at the end of 1996, is now used for ionospheric studies, and has been renamed NIMS (Navy Ionospheric Monitoring System).

By the end of the 1960s, it was felt that with the determination of a full  $16 \times 16$  gravity model, the orbit fits might be good to better than 10–20 m for LEO orbits until the effects of resonance were discovered and evaluated more thoroughly. Including high-order resonance terms (11th–30th order) in the

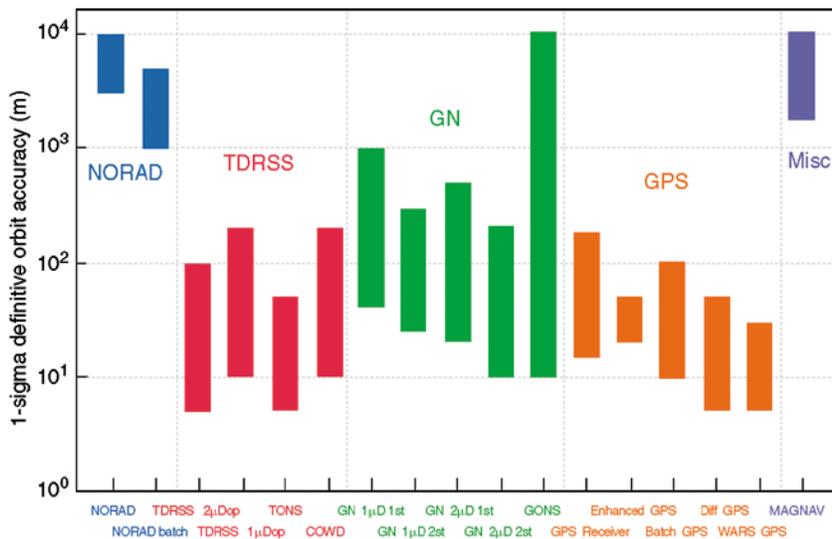


Figure 7. Typical orbital accuracies from GTDS OD.

Earth’s potential led to more confidence on the orbit accuracies by the late 1970s. By then, enough satellites were in orbit and in a variety of inclinations and eccentricities to be able to separate out the effects of many of the tesseral harmonic coefficients in the Earth’s potential. Typical examples of the orbital accuracies that were obtainable as of the end of CY2000 are provided in Fig. 7 from GTDS operational orbit processing from the North American Aerospace Defense Command (NORAD), TDRSS, general navigation (GN) satellites, and GPS (pre-selective availability) and range between 5–1000 m for LEO orbits. GTDS is considered by many to be the benchmark OD system.

It should also be mentioned that several methods are typically used in assessing the accuracy of orbit fits. One method is determined by the covariance of the statistical estimation process itself, which depends inherently on the accuracy of the physics modeling. A second method is to use the residuals from the fitting process to the observational data. A third program is to use overlap methods that calculate orbits over a short several-day period, with 1 day used as the overlap interval, and then record the maximum and minimum position and velocity differences in this overlap region. These differences then provide a measure of the uncertainty of the orbit that reflects the use of different data sets and tracking stations employed in the solutions, tracking geometry, and system errors. All three methods should provide consistent results for orbital accuracy evaluation.

**SUMMARY**

OD and satellite analysis over the last 50 years has evolved through a rigorous scientific discipline, starting from the most basic observations from the best optical camera measurements available at the time, through the Transit Doppler

program and radar altimetry and GPS, to the rich variety of satellite instruments carried into space today. The period from 1957 to 1970 concentrated on orbit improvements obtained primarily from optical and Transit Doppler measurements; from 1970 to 1980, improvements came with the NASA laser upgrade program and use of geodetic quality satellites; and finally in the period from 1980 to 1990, improvements in Earth's rotation rate and polar motion and time and frequency were dominant. Throughout this period, the accuracies of all instruments, including laser and Doppler tracking, altimeters, and onboard clocks, continually improved along the way as well. This entire evolution was an iterative process that required certain steps to be undertaken, with confidence gained along the path as new knowledge of the force models and gravity field in particular were acquired. All of these refinements contributed to these results. Today's satellites carry autonomous navigation capabilities that allow high-precision orbits to be calculated onboard the vehicle and are enabling fairly high accuracies to be attained. With the new LEO satellites using SST techniques and onboard gravity gradiometers, the years 2005 to 2015 are being called the decade of high space resolution gravity. At the beginning of the 21st century, more exciting discoveries await the scientific and astrodynamics community as the critical challenges ahead are waiting to be revealed.

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