

*Experiments are being carried out to explore the interactions between a plasma and an electron beam. The plasma can be excited into several modes of oscillation, some of which grow in amplitude at the expense of the beam energy. The theory is developed in terms of the dispersion relationship, the dependence of wavelength on frequency. Calculations elucidate the coupling between the various simple modes. Such studies are helpful in understanding similar processes occurring in nature and in man-made devices.*

# BEAM-PLASMA

**I**n recent years there has been a great renewal of interest in the properties and the potential uses of a fourth state of matter—the plasma state. The distinguishing feature of this state is that a significant number of the constituent atoms or molecules are ionized. This results in some unusual properties because it means that the dominant forces in a plasma are the long-range electric and magnetic forces between the charged particles rather than the short-range collisional forces between the neutral atoms or molecules in the three usual states (solid, liquid, and gas) of matter. One very prominent characteristic of a plasma is the variety of cooperative motions it undergoes in response to applied electric or magnetic disturbances. Cooperative motions are those involving coherent movement of many particles. These may take on a large number of forms, whose study constitutes one of the major subjects of plasma physics.

Around 1950 several groups of scientists and engineers in various countries started a head-on attack on the problem of producing useful power from the fusion of the nuclei of the heavy isotopes of hydrogen—deuterium and tritium. The problem to be solved was that of heating the nuclei to so high a temperature that their thermal motion would be sufficient to overcome their electrostatic repulsion, and then to confine them long enough and at sufficient densities to make the fusion reaction probable. When these requirements are translated into numbers, they are awesome enough

to give all but the most stout-hearted (or foolhardy) pause: in the most favorable case (using tritium), the “gas” has to be maintained at a temperature of about  $10^8$  °K for about one second at a concentration of about  $10^{14}$  atoms per cc (corresponding to a pressure of about one atmosphere). Under these conditions the “gas” is completely ionized and is therefore a plasma. Only the realization that strong magnetic fields could perhaps keep this plasma confined and away from material walls gave any hope of success to the undertaking.

Almost 15 years have elapsed, but the goal of controlled thermonuclear fusion seems only a trifle nearer now than it was then. All frontal assaults on the problem have foundered, in essence because, given enough time (and one second is plenty of time on the atomic scale), the plasma has always found a way to organize the myriad of collective modes of motion of which it is capable in such a way as to cancel out the confining field in some vulnerable spot and to escape through the breach. After a frustrating and often embarrassing series of failures, the frontal assault has been abandoned. Instead, hopes for success are now pinned on, first, patiently learning to unravel these plasma modes one by one, and then finding ways of preventing the plasma from utilizing them to escape.

An extensive basic research program has therefore come into being, having as its objective the

# INTERACTIONS

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study of the possible collective modes of plasma motion—how they can be excited and suppressed, whether they grow or decay once excited, and how rapidly. One such program has been under way in the Plasma Dynamics Research Group at APL for some two years and is now starting to produce interesting results. The experiment to be described here is only one of several in progress. It has been singled out for this report because it is concerned more directly than the others with stimulating, measuring, and exploring the collective motion of plasma. The first few steps we have taken along that road, together with some background material, form the subject of this discussion.

The experiment consists of sending a thin, medium-to-high-velocity beam of electrons into a slowly counter-streaming plasma confined by a magnetic field, and observing the excitation and subsequent growth or decay of collective oscillations or waves in the plasma. The primary purpose of the experiment is to understand the mechanism for transfer of streaming energy from the electron beam to wave energy in the plasma. A long-range goal is to attempt to heat the plasma ions directly by causing the electron beam to excite a growing plasma wave involving the ions. Such a direct coupling to the ions would constitute a significant step toward overcoming one of the major stumbling blocks in the way of controlled thermonuclear fusion: heating the plasma ions in a time that is short compared to the relaxation time for

achieving collisional equilibrium with hot electrons (which are easily produced in a variety of ways).

## Theory

**A MODEL FOR THE PLASMA**—The subject of waves in plasma is extremely complicated. At least four monographs have appeared in the past two years alone on that important special topic, and more are in print. The complication arises in part from the multitude of different experimental regimes that exist, each with its own particular dominant plasma mode or interaction. For example, the oscillation can be so fast that the (massive) ions are unable to follow it, and only the electrons take part; or it can involve principally the ions. The electrons and ions can move in-phase or out-of-phase. The wave can be carried by the electrons or ions gyrating about the magnetic lines of force (cyclotron waves); or they can be propagated by density fluctuations in the electrons or ions, with the coulomb force dominant (plasma waves). Interaction with electromagnetic waves can occur. Furthermore, when two plasmas interact, as in our experiment, resonances between different types of waves in the two plasmas can give rise to a whole class of new phenomena.

In order to render the present discussion more understandable, therefore, we shall make two simplifications. First, the ions will be considered infinitely heavy, their only purpose being to pro-

vide a sort of smeared-out neutralization for the electronic charge. This is a good approximation in most experimental situations; when ion motion becomes important, however, it can easily be relaxed. Second, the magnetic field will be considered infinitely strong, thus restricting the electrons to move along field lines (longitudinal motion). This is not a good approximation in most experimental situations, and has been adopted here mostly for simplicity of presentation. A more correct treatment is being undertaken to help in the interpretation of the experiments, and takes into account electron motion transverse to the field lines. Phenomena that depend on a finite magnetic field—such as resonance of the cyclotron frequency with the frequency of some driving field, or the coupling of transverse with longitudinal waves—then make their appearance, and may, under some circumstances, even be dominant. The major justification for using a model which cannot describe these finite magnetic-field effects lies in the separation it affords between phenomena involving longitudinal electron oscillations and those depending on magnetic field.

**PLASMA OSCILLATIONS, THE DISPERSION RELATION, AND LANDAU DAMPING**—If the uniform equilibrium concentration of electrons in the plasma described above is disturbed by the presence of a small density fluctuation, the electrostatic restoring forces acting against the inertia of the electrons will cause them to undergo oscillations. In the absence of thermal electron motion, it can easily be shown that the angular frequency of oscillations (called the plasma electron frequency,  $\omega_p$ ) is given by

$$\omega_p = \left[ \frac{4\pi n_0 e^2}{m} \right]^{1/2},$$

where  $e$ ,  $m$  and  $n_0$  are, respectively, the electron charge, mass, and average concentration. This oscillation is not a traveling wave; the electron concentration merely oscillates about its equilibrium value. It is the simplest and most fundamental cooperative motion of which a plasma is capable, and it was discovered in 1929 by Langmuir and Tonks during their extensive studies of space charge in electron tubes.<sup>1</sup>

When the electrons have a finite temperature, a more sophisticated treatment must be adopted. It is now no longer possible to treat the motion of a "typical" individual electron. Instead, we must deal with the electron-velocity distribution function  $f(v, x, t)$ , and ask how it will be affected by a density fluctuation. With the electron motion re-

stricted to one dimension (say  $x$ ) by the infinite magnetic field, and in the absence of collisions (which are unimportant during the short times relevant to the present experiment),  $f$  must satisfy the so-called collisionless Boltzmann, or Vlasov, equation,

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} - \frac{eE}{m} \frac{\partial f}{\partial v} = 0,$$

with the electric field  $E$  related to  $f$  by the Poisson equation,

$$\frac{\partial E}{\partial x} = 4\pi e \left[ n_0 - \int_{-\infty}^{\infty} f(v, x, t) dv \right].$$

If we now assume that  $f$  is composed of a large, uniform, stationary part  $f_0(v)$ , and a small fluctuating part  $f_1(v, x, t)$ , and that the only electric field is that arising from charge-density fluctuations, we can then linearize the Vlasov and Poisson equations by consistently neglecting products of small quantities, obtaining

$$\frac{\partial f_1}{\partial t} + v \frac{\partial f_1}{\partial x} - \frac{eE}{m} \frac{\partial f_0}{\partial v} = 0$$

and

$$\frac{\partial E}{\partial x} = -4\pi e \int_{-\infty}^{\infty} f_1 dv.$$

We shall now look for plane-wave solutions for  $f_1$  and  $E$  by assuming that their time- and space-dependence is given by  $\exp i(kx - \omega t)$ . This assumption will not limit us to any very special class of solutions, since any reasonable function can be represented as a suitable superposition of such plane waves by means of the Fourier theorem. Of course, it may turn out that plane wave solutions exist only for complex  $k$ : ( $k = k_r + ik_i$ ), or  $\omega$ : ( $\omega = \omega_r + i\omega_i$ ), i.e., that the plasma will support only waves which grow or decay in space or time (as  $e^{-k_i x} e^{i(k_r x - \omega t)}$  for  $\omega$  real, or as  $e^{\omega_i t} e^{i(kx - \omega_r t)}$  for  $k$  real) rather than purely sinusoidal waves. But this is precisely the type of information we need for understanding what happens when our electron beam interacts with the plasma, so we shall continue.

If we substitute the above plane-wave space- and time-dependence for  $f_1$  and  $E$  into the linearized Vlasov and Poisson equations, we find that a solution can exist only if  $\omega$  and  $k$  are related as follows:

$$D(k, \omega/k) \equiv \frac{k^2}{\omega_p^2} - \int_{-\infty}^{\infty} \frac{G(v)}{v - \omega/k} dv = 0,$$

where

$$G(v) \equiv \frac{1}{n_0} \frac{\partial f_0}{\partial v}.$$

This equation is called the dispersion relation because its solution,  $k(\omega)$ , determines the frequency

<sup>1</sup> L. Tonks and I. Langmuir, "Oscillations in Ionized Gases," *Phys. Rev.*, **33**, Feb. 1929, 195-210.

dependence of the index of refraction,

$$n(\omega) = \frac{ck(\omega)}{\omega},$$

where  $c$  is the velocity of light, and of the phase velocity

$$v_{ph}(\omega) = \frac{\omega}{k(\omega)}.$$

As it stands, however, the dispersion relation is ambiguous because it does not specify how to evaluate the integral when both  $k$  and  $\omega$  are real, since there is then a pole on the path of integration. The additional information needed to formulate an unambiguous prescription can be obtained only from consideration of the physical consequences of various choices. For this purpose the above formulation of the problem is still not sufficiently precise.

One must ask a question that has a unique solution. One which naturally suggests itself is the initial-value problem: suppose that  $f_1$  has a given initial functional form; how do  $f_1$  and  $E$  evolve in time? This problem can be solved using Fourier and Laplace transforms, and involves multiple integrations over  $v$ ,  $k$  and  $\omega$ . The most significant fact that emerges is that the asymptotic behavior of  $f_1$  and  $E$ , as  $t \rightarrow \infty$ , depends only in an unimportant way on the initial-value function, and is mainly given by the zeros of  $D(k, \omega/k)$ , which appears in the denominator of the integrand. The location of the zeros above or below the real  $\omega$ -axis determines whether  $E$  and  $f_1$  grow or decay asymptotically. The path of integration in the complex  $\omega$ -plane lies above all singularities of the integrand, thereby avoiding the troublesome pole previously appearing on the integration path of the dispersion relation  $D(k, \omega/k)$ . Physically, the meaning is that the plasma has certain preferred modes of oscillation which persist for long times regardless of the form of the initial disturbance; these modes are determined from the zeros of the dispersion relation.

While this formulation has eliminated our major difficulty, one fault still remains. In the derivation it was assumed that  $D(k, \omega/k)$  is an analytic function of  $\omega$ . The expression for  $D(k, \omega/k)$ , as given above, however, is not analytic in  $\omega$ , for if the integral in the defining equation is evaluated by contour integration, the result depends on the sign of  $Im(\omega/k)$ , changing discontinuously as this quantity passes through zero. This difficulty can be cured by redefining  $D(k, \omega/k)$  as

$$D(k, \omega/k) = \frac{k^2}{\omega_p^2} - \int_C \frac{G(v)}{v - \omega/k} dv,$$

where the contour  $C$  is the real axis of the complex

velocity plane if  $Im(\omega/k) > 0$ , and is modified as shown in Fig. 1 if  $Im(\omega/k) < 0$ . This expression is manifestly analytic in  $\omega$ ; the final obstacle has now been overcome.

We are at last in a position to study the asymptotic behavior of  $E$  and  $f_1$  by examining the location of the roots of the dispersion relation. The analysis is particularly simple if the roots are near the real  $\omega$ -axis (corresponding to a small decrement or increment of the oscillation). It can then be shown that

$$Im \omega \cong \frac{\pi\omega}{2} \left( \frac{\omega_p}{k} \right)^2 G \left( \frac{\omega}{k} \right) \left( 1 - \frac{k}{\omega} \frac{d\omega}{dk} \right),$$

where  $\omega$  on the right-hand side denotes the real part of  $\omega$ . In the long wavelength limit, and using a Maxwellian distribution for  $f_0$ , this becomes

$$Im \omega \cong - \sqrt{\frac{\pi}{8}} \omega_p \left( \frac{\omega}{kc_t} \right)^3 e^{-\omega^2/2k^2c_t^2},$$

where  $c_t$  is the rms thermal velocity. The negative imaginary part for  $\omega$  means a *damped* plasma oscillation. This noncollisional damping mechanism has been called Landau damping for its discoverer.<sup>2</sup>

A simple physical explanation can be given for this damping mechanism. Consider a mode with angular frequency  $\omega$  and wave number  $k$ , i.e. a wave with phase velocity  $\omega/k$ . This wave can be regarded as a periodic potential well moving with the phase velocity. Electrons moving at exactly that speed neither gain energy from the wave nor lose their energy to it. Electrons moving slightly slower, however, tend to be accelerated and drain energy from the wave, while those moving slightly faster are decelerated and give up some of their energy to the wave. Thus, the wave will grow or decay depending on whether there are more fast or slow electrons. In a Maxwellian distribution, of course, there are always more slow-moving than fast-moving electrons, so that all such waves are damped. If, however, there is a "bump" in the tail of the Maxwell distribution, such as would be produced by a beam of fast electrons penetrating a

<sup>2</sup> L. D. Landau, "On The Vibrations of the Electronic Plasma," *J. Phys. (U.S.S.R.)*, **10**, 1946, 25-34.

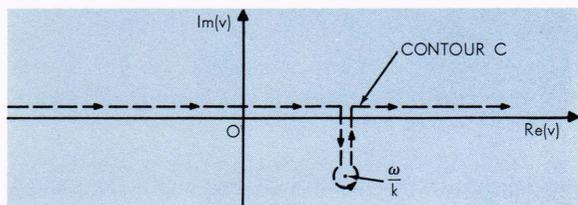


Fig. 1—Path of integration in the complex velocity plane to be used for evaluating  $D(k, \omega/k)$ .

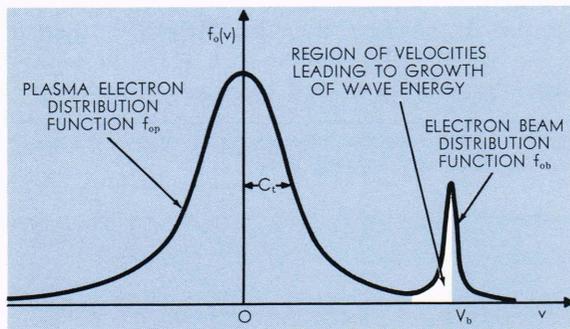
stationary plasma, then growing waves become possible since, in a limited region of  $v$ , there are more fast than slow electrons. Such a distribution function is shown in Fig. 2. It was the possibility of generating growing waves by this mechanism, called inverse Landau damping, that provided the initial motivation for the present experiment.

**TWO-STREAM INSTABILITY**—There is another aspect to the beam-plasma interaction that can lead to growing waves—the two-stream instability. While the inverse Landau damping discussed above depends on the trapping of beam electrons in the moving potential well of the plasma wave (and hence on a thermal spread of velocities), the two-stream interaction arises chiefly from charge bunching or clumping in a fashion familiar from klystron theory. In contrast to Landau damping or inverse damping, it does not depend on a thermal distribution of electrons but can be considered the limiting case of this mechanism as both beam and plasma temperatures tend to zero. This is equivalent to the plasma and beam equilibrium distribution functions  $f_{op}(v)$  and  $f_{ob}(v)$ , tending to delta functions  $n_p\delta(v)$  and  $n_b\delta(v - V_b)$  where  $V_b$  is beam velocity. A simple calculation with the dispersion function  $D(k, \omega/k)$  then gives for the beam-plasma dispersion relation the equation

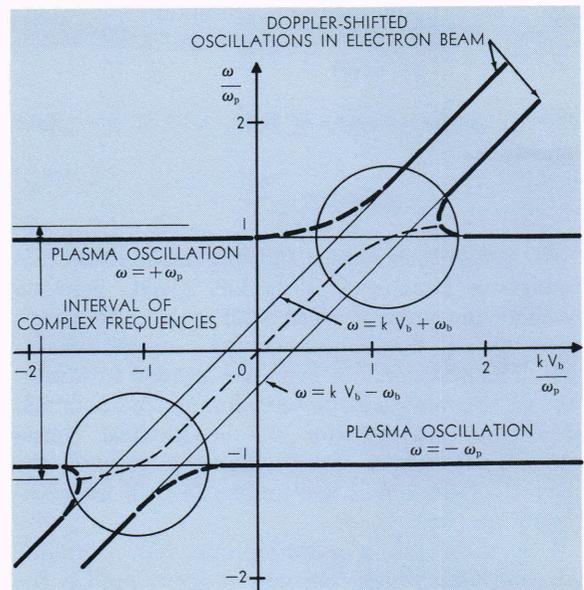
$$\frac{\omega_p^2}{\omega^2} + \frac{\omega_b^2}{(\omega - kV_b)^2} = 1,$$

where  $\omega_b$  is the plasma frequency of the beam electrons,  $\omega_b \equiv \frac{4\pi n_b e^2}{m}$ . The solutions to this equation describe the modes of the two-stream instability.

A very simple physical explanation of this instability can be given in terms of the plasma oscillations of the stationary plasma and of the beam, each at its own plasma frequency. First note that



**Fig. 2**—Form of the equilibrium velocity distribution function for a stationary plasma and a streaming electron beam. The rms thermal velocity is  $c_t$  and the streaming velocity is  $V_b$ . This distribution leads to growing waves.



**Fig. 3**—Mode coupling diagram for a cold plasma and a cold beam. The dispersion equations for plasma oscillations in each component are shown. Interactions occur, and the mode structure is drastically altered near the circled regions. Over the interval of about  $2\omega_p$ , complex frequencies arise, the real parts of which are shown as dotted lines.

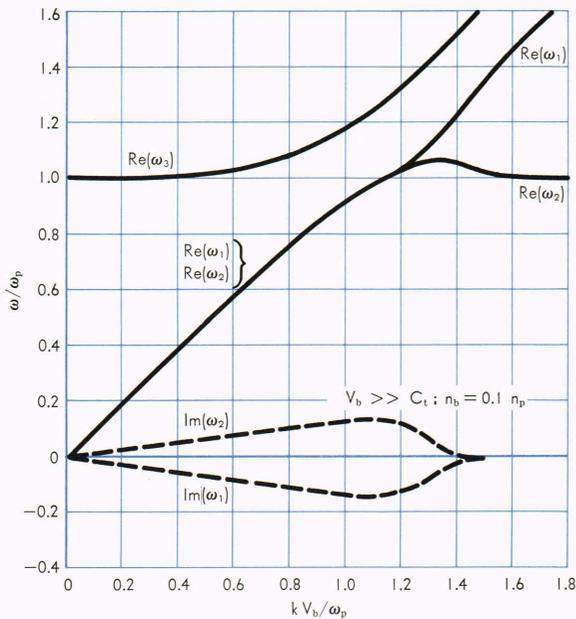
while plasma oscillations can take place only at the plasma frequency, they can have any wave number  $k$ . For an observer at rest in the stationary plasma, the doppler-shifted frequency of the beam oscillations at  $\omega_b$  becomes

$$\omega = kV_b \pm \omega_b.$$

This is the beam dispersion relation. If the wave number is such that this frequency is equal to  $\omega_p$ , then the beam oscillations will drive the oscillations in the stationary plasma, and a resonance phenomenon can be expected.

A sketch of the beam and plasma dispersion relations is shown in Fig. 3 (the latter is simply  $\omega = \pm \omega_p$ ). The interactions are expected in the two circled regions where the beam and plasma modes intersect. In these interaction regions the actual solutions of the beam-plasma dispersion relation are considerably different from the individual beam and plasma modes and are indicated by the dashed curves. Since the dispersion equation must have solutions for every value of  $k$ , the existence of a gap between these curves, with no real frequency for a finite interval in wave number, indicates the occurrence there of a complex frequency, i.e. time growth or damping, as a direct consequence of the interaction.

Solutions for the beam-plasma dispersion relation can have complex  $k$  as well as complex  $\omega$ .



**Fig. 4**—Solutions to the beam-plasma dispersion equation for the case of complex frequency. The growing wave is denoted by  $\omega_2$ , the damped wave by  $\omega_1$ . The beam density is taken as 1/10th of the plasma density, and a slight temperature effect has been included.

However, we have developed the plane-wave solutions (which led to the above dispersion relation) *only* to construct a general solution by superposition, viz.,

$$f(x, t) = \int_{-\infty}^{\infty} F(\omega) e^{i[k(\omega)x - \omega t]} d\omega,$$

or

$$f(x, t) = \int_{-\infty}^{\infty} F[\omega(k)] e^{i[kx - \omega(k)t]} \frac{d\omega(k)}{dk} dk.$$

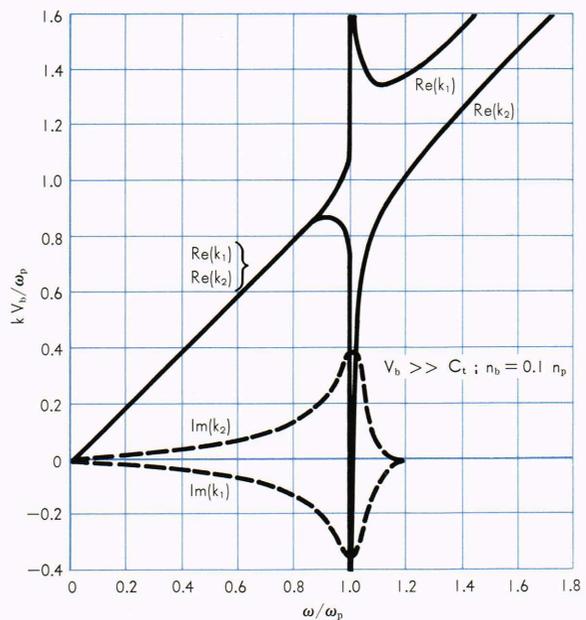
For our purpose it is therefore sufficiently general to let either  $\omega$  or  $k$  be real. Whether these two choices are equivalent depends on whether or not the two integrals can be transformed into each other. This is permissible so long as the path of integration along the real axis of the complex  $\omega$ -plane can be deformed into the path  $Im k(\omega) = 0$  without encountering any singularities. Under these circumstances either  $k$  or  $\omega$  may be assumed real. Under some experimental conditions, however, this deformation of the contour cannot be performed, and a choice of real  $\omega$  or real  $k$  is then dictated.

Sturrock<sup>3</sup> has shown, for example, that the solu-

tion for two counter-streaming plasmas is made up of modes for which  $k$  is real and  $\omega$  is complex (called an absolute or non-convective instability), while for two co-streaming plasmas  $\omega$  is real and  $k$  is complex (a convective instability). With an absolute instability the field shows time growth everywhere. With a convective instability the field at any given time keeps increasing as far as the disturbance has progressed, but eventually decays at every point in space.

Figure 4 shows solutions to the beam-plasma dispersion relation for real  $k$  and complex  $\omega$ . A small finite temperature has been introduced to resolve certain degeneracies in the calculation. Only the upper right quadrant of the  $\omega$ - $k$  plane is shown, but this is enough to indicate the existence of a time-growing wave (absolute instability) denoted by  $\omega_2$  and a damped wave denoted by  $\omega_1$ , for wave numbers less than about  $1.4 \omega_p/V_b$ . Below about 1.2 on the  $k$ -axis the two modes virtually coalesce and continue to damp or grow for even the smallest wave numbers (longest wavelengths) in this spatially unlimited system. In addition there are two other waves,  $\omega_3$  and  $\omega_4$  (the latter not shown), which neither grow nor decay and are of little interest here.

Figure 5 shows solutions to the beam-plasma dispersion relations for real  $\omega$  and complex  $k$ . This is the convective instability case, and it applies, for



**Fig. 5**—Dispersion relation for complex wave numbers. The conditions of Fig. 4 prevail. A spatially growing wave will behave as given by  $k_1$ , and the damped wave, by  $k_2$ .

<sup>3</sup> P. A. Sturrock, "Amplifying and Evanescent Waves, Convective and Nonconvective Instabilities," *Plasma Physics* (ed. J. E. Drummond, McGraw-Hill Book Co., Inc., New York 1961), chap. 5.

example, to the klystron amplifier. The  $\omega$  and  $k$  axes are now interchanged from the previous case. A region of spatially growing and decaying waves is evident for frequencies below the plasma frequency  $\omega_p$ . When the imaginary part of  $k$  is small, the solutions coincide with the previous case of complex  $\omega$ , as may be verified by rotating the graph by  $90^\circ$ , reflecting it in the vertical axis, and comparing it with Fig. 4.

Only plasmas of infinite extent have been considered here. This is a valid approximation for the one-dimensional motion of finite plasmas, if the wavelengths of the plasma oscillations as well as all other lengths characteristic of a plasma—in particular the Debye length—are small compared with the plasma dimensions. (The Debye length, or shielding distance, is the distance over which the electron concentration can differ appreciably from the ion concentration, and is therefore a measure of the minimum size necessary for a collection of electrons and ions to act as a plasma. It is given by

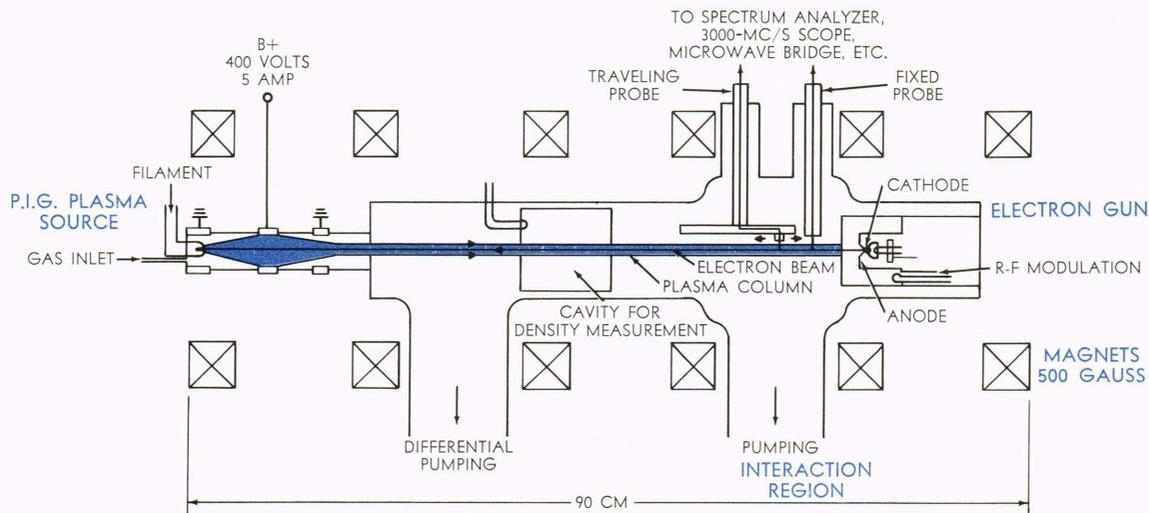
$$\lambda_D \equiv \left( \frac{kT_e}{4\pi n e^2} \right)^{1/2} = \frac{(kT_e/m)^{1/2}}{\omega_p} = \frac{c_t}{\omega_p},$$

where  $T_e$  is the electron temperature and  $c_t$  is the electron thermal velocity in the case of one-dimensional motion.) In our experiment the Debye length conditions are met. The wavelength condition is met for the stationary plasma; for the beam plasma, however, it is satisfied only for the axial dimension. A beam-plasma dispersion relation for a finite-radius beam traversing a stationary plasma of infinite extent has therefore been derived.

## Experiment

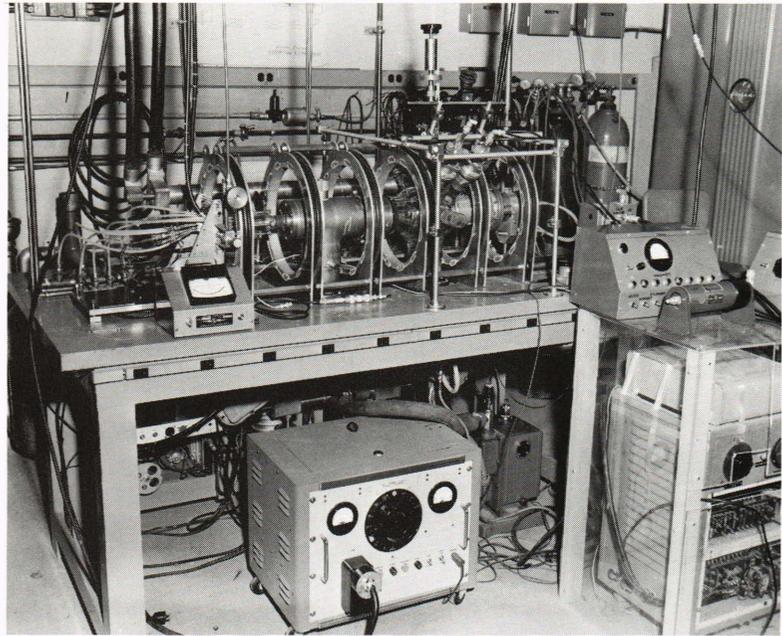
**ARRANGEMENT OF THE EXPERIMENT**—The problem of devising an experiment that meets the assumptions of the theory is twofold: (a) to provide a plasma whose properties are well defined and uniform and whose density is not so great as to preclude real-time observation of the plasma waves in it; and (b) to provide an electron beam of high density and power so as to communicate measurable amounts of energy to the plasma oscillation. The one-dimensional theory presented above is applicable only if the spatial extent of the beam and plasma is large compared with the wavelengths of any oscillations existing in them. Due to rather obvious experimental limitations, this has been achieved only in the axial coordinate. The finite-radius dispersion relations referred to above must therefore be used in the interpretation of the experiment.

The exigencies of the situation have thus dictated the arrangement of the experiment shown in Figs. 6 and 7. On the left is the plasma source and on the right is the electron gun. The former is a P.I.G. (Phillips Ionization Gauge) discharge that functions as follows: the center, hollow, anode structure is maintained at a potential of a few hundred volts positive, while the left cathode (which carries a heated tungsten filament) and the right hollow cathode are grounded. An axial magnetic field is imposed on the source. Electrons drawn from the filament by the anode voltage pass through the anode and are reflected by the potential barrier at the hollow cathode. The magnetic field constrains these particles to move in tight



**Fig. 6**—Schematic of apparatus for the beam-plasma interaction experiment. An electron beam streams to the left and the plasma to the right.

**Fig. 7—Experimental apparatus. Movable probes may be seen protruding from the glass portion. The six DC magnet coils are the prominent feature.**



circles about the field lines; thus, in the absence of perturbing forces they oscillate between cathodes. When gas is admitted, each electron makes many ionizing collisions with the gas atoms before finally reaching the anode by a random-walk process. The ions formed are accelerated away from the anode toward both cathodes, and those going to the right pass through the hollow cathode and into the main vacuum system, dragging enough electrons along with them by coulomb attraction to provide charge neutralization and thus form a plasma. The plasma is maintained in a column about 1 cm in diameter by a magnetic field of approximately 500 gauss. Most of the excess gas is removed by the first stage of differential pumping before the plasma enters the interaction region. A hydrogen plasma, when reaching this region, has a density of up to  $10^{13}$  electrons per cc, an electron temperature of over  $300,000^{\circ}\text{K}$  (30 eV), an ion temperature of about  $17,000^{\circ}\text{K}$  (1.5 eV), and is nearly 50% ionized. It appears as a fairly bright, blueish-magenta column of light.

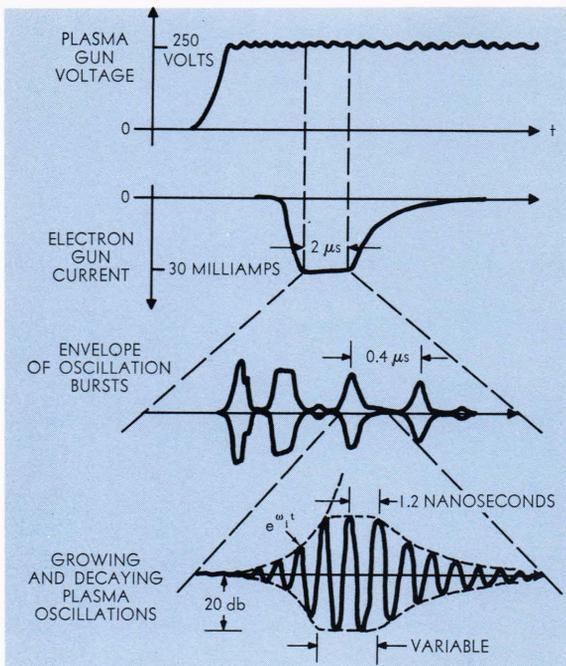
The electron gun is a Pierce design of the space-charge-limited, magnetically-focused flow type, yielding about 50 ma of beam current at 1000 volts. The beam is collimated into a pencil of 1 to 2 mm diameter, within which the electron density  $n_b$  is of the order of  $10^9$  per cc. Radio-frequency velocity modulation at frequencies between about 1000 and 2000 mc/s may be applied to the beam by a re-entrant cavity similar to that in a klystron. The beam first encounters the plasma outside the left face of the cavity, which also serves as a col-

lector for the plasma. It streams to the left coaxially with the plasma and is finally collected within the P.I.G. after traversing some 60 cm of plasma column. A second stage of pumping serves to keep the neutral pressure near  $10^{-4}$  Torr in the interaction region so that additional ionization produced by the beam will be slight.

A variety of diagnostic equipment is provided within the vacuum system. Included are movable plasma probes for density and temperature measurements, a Faraday cage, an R-F cavity for determining electron density, and an optical window for spectroscopic measurements.

**SOME EXPERIMENTAL RESULTS**—In a typical experiment designed to observe growing waves, the sequence of events is as shown in Fig. 8. The P.I.G. device is run as a steady-state plasma source, and the electron gun is pulsed on for a few microseconds without modulation. Under proper conditions of plasma density, magnetic field, and electron-beam voltage, a growing wave is excited in the plasma. This wave is detected by a Langmuir probe inserted in the edge of the plasma and is observed on an oscilloscope having a 3000-mc/s bandwidth and on an R-F spectrum analyzer.

The wave grows exponentially for several tens of cycles, reaches a limiting amplitude some 20 db above plasma noise, persists there for a variable length of time which is typically 100 periods, and then decays rather sharply. The entire wave train appears as a burst, and during one beam pulse duration several such bursts occur, separated by about  $0.5 \mu\text{sec}$ . Examples of this behavior are

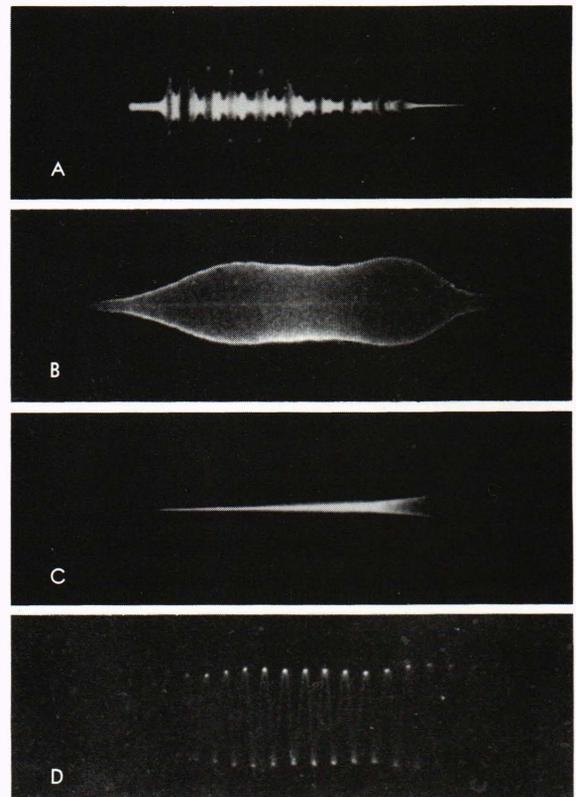


**Fig. 8—Pulse sequence during excitation of plasma oscillations. The time scales are expanded on successively lower diagrams, the last showing the interpretation of the form of the 800-mc/s oscillations shown on Fig. 9.**

shown in Fig. 9, which displays four oscilloscope traces of bursts of plasma oscillations near 800 mc/s; this frequency is somewhat below the plasma frequency of the hydrogen plasma being produced at that time. Time runs from left to right on these photos. Nearly pure exponential growth by a factor of  $e^2$  is observed; the reciprocal of the imaginary part of the frequency (i.e. the  $e$ -folding time) in this case was 25 nanoseconds. It is felt that the interaction observed was the two-stream instability discussed in the theoretical section above, since the time constant for growth agrees with theoretical estimates of the reciprocal imaginary part of the frequency to within a factor of 2 to 3, and the observed frequency of oscillation agrees with the plasma frequency to within a factor of 1.5. Conclusions based on this degree of "agreement" between theory and experiment are often valid in plasma physics, which is beset by unusual difficulties in reconciling theory and experiment, because the theory is often untractable unless drastic approximations are made. Inverse Landau damping is negligible compared with the two-stream instability in plasmas with a thermal velocity spread  $c_t$  much less than the streaming velocity of the beam,  $V_b$ . In this case the electron temperature of 30 ev and the dc beam acceleration potential of 900 volts give a ratio of  $V_b/c_t = \sqrt{30}$ .

Estimates of the power picked up by the probe lead one to believe that 5 to 10% of the dc beam power is transferred to plasma oscillations.

Another interaction that has been observed is not accounted for by the infinite-magnetic-field theory above. When the field  $B$  is finite, a transverse cyclotron wave at the electron cyclotron frequency  $\omega_c = eB/mc$  is propagated in the plasma by virtue of the helical motion of electrons around magnetic lines of force. This wave is similar to that in a backward-wave oscillator since its group and phase velocities are oppositely directed. Due to the finite beam radius, the longitudinal plasma wave on the beam at the frequency  $\omega_b$  now couples with this transverse cyclotron wave and the coupling manifests itself as a growing wave in time having a frequency slightly less than  $\omega_c$ . While the complete theory for this interaction remains to be developed, the proportionality of interaction frequency and magnetic field is well established. Figure 10 is a graph showing the dependence of the wave frequency as the confining magnetic field was varied, and illustrates that, to a good approxi-



**Fig. 9—Oscilloscope traces of 800-mc/s plasma oscillations. In (B) and (C) the individual oscillations are just resolved, and (D) shows the degree of spectral purity of the wave during 25 nanoseconds.**

mation, the observations can be represented by  $\omega/\omega_c = 0.8$ . More work, both theoretical and experimental, is in progress on the properties of this interaction.

Both of the above experiments have been concerned with processes involving complex frequencies. It can be shown that a necessary condition for growth in *time* is the existence of some type of feedback mechanism, which in the cases at hand lies in the counterstreaming of two waves. In the two-stream instability, the plasma streams slowly against the direction of the electron beam and carries bodily a density perturbation, introduced on it by the beam, back to the origin in proper phase so as to reinforce the perturbations. Similarly, in the cyclotron-wave—plasma-wave interaction, the backward wave carries energy against the direction of the electron stream and serves the regenerative function.

If the plasma is stationary, moves against the beam with a velocity less than its thermal spread, or moves with the beam but at a different speed, it is possible for the beam to excite a wave with complex wave number. This manifests itself as a signal which grows in *space*. Such a phenomenon has been observed in the present apparatus in one instance. A traveling probe was used to pick up R-F radiation during the beam pulse, and the in-

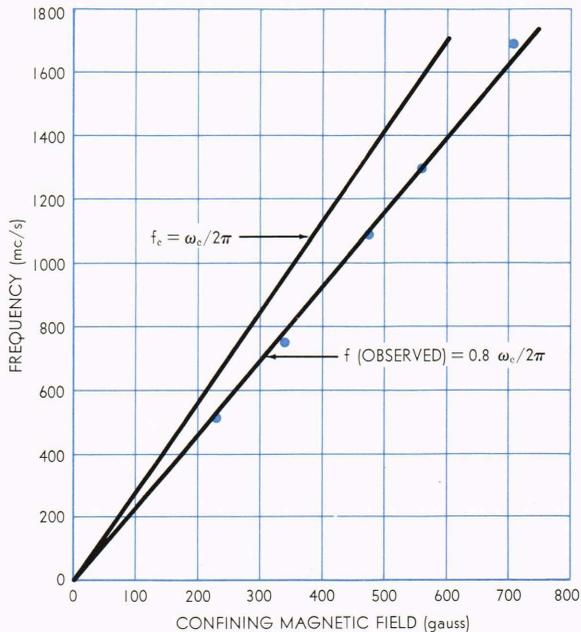


Fig. 10—The variation of interaction frequency with magnetic field in the cyclotron-wave—plasma-wave interaction. Also shown is the electron cyclotron frequency  $f_c$  versus field.

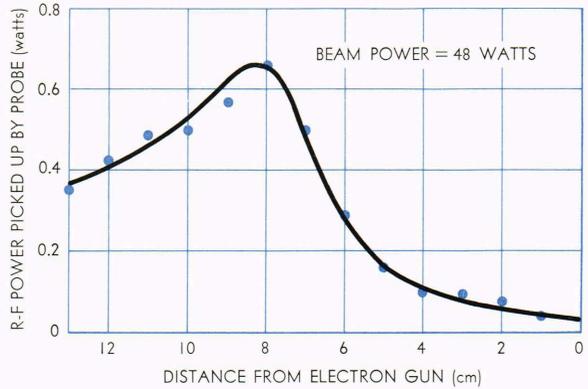


Fig. 11—Power picked up by a probe versus distance from the start of the interaction (at  $x = 0$ ). The beam power is 48 watts, of which 5 to 10% couples into the plasma.

tensity of the radiation was observed to increase to a maximum as the axial distance from the electron gun increased. Power picked up by the probe inserted to the edge of the beam is plotted in Fig. 11; the fraction of the oscillation power picked up by the probe is probably of the order of 10%, as determined from crude considerations based on the areas of beam and probe. The frequency of interaction is very broadband, varying from instant to instant between 500 and 900 mc/s. When viewed in time at a given point in space, the waves exhibit growth and decay similar to that in the previous experiments (Fig. 9).

Why this set of experiments showed growth in space while others showed only growth in time is not yet known. A tentative line of inquiry is based on the fact that an electron beam in a magnetic field may be scalloped and carries a corresponding density modulation, and that this modulation may be the driving source of the growth in space, somewhat similar to that occurring in a klystron. Further work on this problem is in progress.

## Summary

We may summarize by noting that a vast class of physical phenomena, ranging from the operation of microwave amplifiers to the acceleration of interstellar particle streams, depends on collective interactions of the sort described. Although a great deal of theoretical work has been accomplished to date, only a relatively few experimental demonstrations of wave growth have been carried out. As far as we know, the pictures shown in Fig. 9 are the first to show the actual time-resolved wave growth. The complexity of the phenomena and their physical significance make this an extremely rich vein to mine.