SYNTHETIC APERTURE RADAR IMAGING OF SHIP-GENERATED INTERNAL WAVES

An experiment to investigate synthetic aperture radar imaging of ship-generated internal waves was conducted in Loch Linnhe, Scotland, in September 1987. Data acquired are being used to test models of internal wave generation and radar imaging.

INTRODUCTION

Just before the turn of the century, the great Norwegian meteorologist Vilhelm W. Bjerknes advanced a tentative explanation for an interesting hydrodynamic phenomenon often observed when a slow-moving ship traverses a body of water that has a large density change just below the surface.¹ The phenomenon, known as the “dead water” effect, is commonly encountered where a shallow layer of fresh water overlies salt water and manifests itself as greatly increased drag on the ship, requiring an increase in propulsive power to maintain speed. Bjerknes suggested that when a ship moves through density-stratified water, the displacement of water by the hull can excite subsurface waves, known as internal waves, by a process analogous to the hull-generation of surface waves at the air/water interface. The work done in generating these waves either causes the vessel to lose speed or requires increased propulsive power to maintain speed. In extreme situations, the forward motion of the vessel is retarded significantly, hence the origin of the term “dead water.” His student, V. W. Ekman, subsequently demonstrated in a classic piece of experimental and theoretical research, that internal waves could indeed exist and could account for the drag.

The internal waves generated by the ship are organized into a distinct pattern along its track, in the same way that the ship-generated surface waves appear in the well-known Kelvin wave pattern. In this issue, Dysthe and Trulsen discuss details of the internal wave pattern, which manifests itself by the alterations of small ripples on the surface via a hydrodynamic interaction between near-surface currents induced by the internal waves and the ambient surface wave field. Striking examples of these patterns can be seen in the photographs published by Hughes and Grant.² Similar patterns appear in radar images because the surface ripples strongly influence the backscattering of microwave energy from a water surface. By combining oceanographic measurements with hydrodynamic models for the internal wave generation and wave–current interaction processes and electromagnetic models for radar scattering, it is possible to analyze these images and to test theories for radar imaging of a variety of ocean phenomena. Such testing was the objective of a joint United States–United Kingdom experiment conducted in Loch Linnhe, Scotland, in September 1987.

EXPERIMENT DESCRIPTION

Loch Linnhe, located in northwestern Scotland, is a large loch that begins in the Scottish Highlands and flows southwest to the Atlantic Ocean. In summer, the upper part of the loch has a shallow layer of fresh river and runoff water a few meters deep overlying salty seawater, which extends to the bottom. The UK experiment team from the Royal Aerospace Establishment (RAE) at Farnborough selected this loch as the experiment site because of its environment and the loch’s access to the sea, which allowed a Royal Marine Auxiliary Service oceangoing tug, RMAS Roystener, to serve as the vessel to generate internal waves.

Test operations were based at the Underwater Centre in Fort William, a small town at the loch’s upper end. An instrumentation site was established about 8 km down the loch, where research vessels, pontoons, and a buoy were deployed for in situ oceanographic and meteorological measurements. A CV-580 aircraft with a C-band (5.31-GHz) synthetic aperture radar (SAR) operated by the Canada Centre for Remote Sensing was used to image the internal wave pattern from the Roystener. The radar had a digital data acquisition system to record the raw radar return for subsequent ground processing. An RAE helicopter was also used to photograph the pattern.

The experiment extended over 10 days of test operations between 2 and 22 September 1987. A typical data collection sequence consisted of the Roystener running up and down the loch past the instrumentation site while the aircraft overflew the area to record radar imagery and to photograph the internal wave pattern. Test operations were directed by RAE personnel. Participants from the United States included scientific teams from APL and TRW. Members of the Scottish Marine Biological Association (SMBA) at Oban also participated.

The APL investigators provided a large instrumented spar buoy equipped with conductivity–temperature–depth (CTD) sensors and current meters to measure the internal-wave characteristics; it also had meteorological and wave sensors to measure surface data. Other CTD
measurements were made with RAE sensor strings sus-
pended from three pontoons and with sensors on the
5MBA research vessel Calanus. The TRW team supplied
a scanning laser wave-slope sensor that was operated
from a second research vessel, Loch Nevis. The sensor
measured the changes in the surface wave ripples induced
by the internal waves. Besides participating in the ex-
periment, APL personnel were responsible for process-
ing the SAR data into digital imagery for analysis by
members of the U.S. experiment team.

INTERNAL WAVE MEASUREMENTS

Depth profiles of conductivity and temperature were
taken daily to compute the ambient water column den-
sity, \( \rho(z) \), as a function of depth, \( z \), and the Brunt-
Väisälä (B-V) frequency

\[
N(z) = \sqrt{\frac{-g}{\rho(z)} \frac{d\rho(z)}{dz}},
\]

where \( g \) is the gravitational acceleration, and \( z \) is posi-
tive upward. The B-V frequency characterizes the den-
sity structure of the ambient water column and, in
particular, determines the properties of the ship-
generated internal waves. Figure 1 shows typical profiles
of the density anomaly, \( \sigma(z) \), where \( \sigma(z) = \rho(z) - \rho_0 \)
and \( \rho_0 \) is the density of fresh water, and of the B-V
frequency, \( N(z) \). Note that the maximum B-V frequen-
cy occurs at a depth of about 3 m and has a value of
about 80 cycles/h. It is precisely this shallow density gra-
dient that allowed the Roysterer, with a 6-m draft, to
be an efficient internal wave generator in Loch Linne.

The in situ measurements showed that the vertical mo-
tion induced within the water column by internal waves
was predominantly in the same direction at all depths;
that is, the internal waves were mostly mode 1 waves,
with the maximum displacement occurring near the
depth of the maximum B-V frequency. For these con-
ditions, the isopycnal (constant-density) vertical displace-
ment, \( \eta(x,z,t) \), induced by a group of internal waves can
be written as

\[
\eta(x,z,t) = \sum_c W^{(1)}(z) \xi^{(1)}(x,t),
\]

where \( x \) is a horizontal coordinate and the subscript \( c \)
refers to the phase speed of an individual wave in the
group. The mode 1 eigenfunction, \( W^{(1)}(z) \), is a par-
ticular solution of the eigenvalue equation,

\[
c^2 \frac{d^2}{dz^2} W^{(1)}(z) + N^2(z) W^{(1)}(z) = \omega^2 W^{(1)}(z),
\]

subject to the boundary conditions \( W^{(1)}(0) = W^{(1)}(D) = 0 \),
where the surface is at \( z = 0 \) and the
bottom is at \( z = -D \). The functions \( \xi^{(1)}(x,t) \) in
Equation 2 can be estimated by solving Equation 2 us-

ing the measured isopycnal displacements at each sen-
or depth, \( z_s \), and the value of \( W^{(1)}(z_s) \) computed
from Equation 3 for the appropriate wave phase speed,
\( c \), and frequency, \( \omega \). Figure 2 shows the eigenfunction,
\( W^{(1)}(z) \), for a typical internal wave phase speed of
0.4 m/s.

The colored curve in Figure 3 shows an isopycnal dis-
placement inferred from the CTD time series at a depth
of 5.3 m when part of the internal wave pattern propa-
gated past the APL spar buoy. Three prominent inter-

cal wave oscillations can be seen, with amplitudes be-

between 0.3 and 0.6 m. The black curve in Figure 3 is an
analytical representation of the measured displacement
that is used to compute the surface current induced by
this wave group. The procedure for computing the sur-
face current is straightforward; it uses the equation of
continuity for incompressible flow to compute the cur-
rent field gradients, which are then integrated to yield
the horizontal current and current gradient (or strain
rate) at the surface (Fig. 4). Here, the time dependence

\( A \)

\( B \)

Figure 1. Depth profiles of density anomaly (A) and Brunt-
Väisälä frequency (B).
of the isopycnal displacements measured at a fixed location has been transformed to the spatial pattern of the surface current using the internal-wave propagation speed. Note that the maximum currents range from about 0.015 to 0.030 m/s, with peak strain rates of approximately 0.002 s⁻¹, which are typical values for the internal waves generated by the Roysterer during the experiment. These currents modulate the ambient surface wave spectrum, thereby rendering the internal wave pattern visible to the radar.

SURFACE WAVE MODULATIONS

To estimate the effect of the internal wave pattern on the radar return, it is necessary first to compute the modifications to the surface wave spectrum. Previous experiments have demonstrated that the surface wave modulations can be obtained by solving an action-balance equation of the form

$$
\frac{d}{dt} N(k;x,t) = \beta(k) N(k;x,t) \left[ 1 - \frac{N(k;x,t)}{N_0(k)} \right], \quad (4)
$$

where $N(k;x,t)$ is the spectral density of the surface wave action for wave number $k$ at horizontal position $x$ and time $t$, $\beta(k)$ is the surface-wave relaxation rate, and $N_0(k)$ is the equilibrium (no-current) ambient action spectrum. The time dependencies of $x$ and $k$ are given by

$$
\frac{dx_i}{dt} = \frac{\partial \omega_0}{\partial k_i} + U_i, \quad (5a)
$$

$$
\frac{dk_i}{dt} = -k_j \frac{\partial U_j}{\partial x_i}, \quad (5b)
$$

where the indices $i$ and $j$ run from 1 to 2 and repeated indices are summed. The quantity $U(x,t)$ represents the surface current field induced by the internal waves. The action spectrum is related to the surface wave height spectrum, $S(k;x,t)$, by
\[ N(k;x,t) = \frac{\rho_s \omega_0}{|k|} S(k;x,t) , \]  

where \( \rho_s \) is the surface density and \( \omega_0 \) is the wave frequency in a local rest frame corresponding to wave number \( k \).

The surface wave modulations induced by internal waves can extend over a broad range of wavelengths, depending on the magnitude and spatial extent of the current pattern and on the wind speed. Centimeter-scale and shorter waves generally undergo very small modulations because they are tightly coupled to the wind, which tends to force them quickly back into equilibrium. Very long surface waves move rapidly through localized current patterns, and their modulations are limited by the available interaction time. The waves most affected by internal waves are those with an interaction time comparable to the relaxation time, \( \beta^{-1}(k) \), a function that depends also on the wind speed. For the types of internal waves generated in the Loch Linnhe experiment, surface waves with wavelengths from about 0.05 to 5 m experience the largest modulations.

To solve Equation 4, a model equilibrium wave spectrum is chosen that is appropriate for the measured wind conditions, along with an expression for \( \beta(k) \) from Hughes. Figure 5 shows the computed modulation (defined as the ratio of the spectral density in the presence of the current to its ambient value) of surface waves with wavelengths of 0.06 m (Fig. 5B) and 0.90 m (Fig. 5C), caused by the current shown in Figure 5A. A wavelength of 0.06 m corresponds to the Bragg wavelength, \( \lambda_B = \lambda_{RF}/(2 \sin \theta) \), for the C-band radar (\( \lambda_{RF} = 5.65 \) cm) at an incidence angle, \( \theta \), of 29°. A modulation of greater than unity indicates an increase in the amplitude of the particular wave component, or enhanced surface-wave roughness. Similarly, smooth areas on the surface are associated with modulations of less than unity where the wave amplitude has decreased. Note that the 0.9-m wave has a substantial modulation, indicating that the long-wave tilting of the short waves that scatter the radar energy must be taken into account when calculating the radar backscatter modulations.

**Radar Imagery**

It follows from the foregoing discussion that the pattern of internal waves generated by the ship will induce alternating regions of enhanced and diminished wave activity on the water surface. When the pattern is illuminated with microwave energy, the backscattered signal will increase or decrease depending on the changes in wave activity, thereby mapping out the pattern as intensity variations in a radar image. Figure 6 shows a radar image obtained with the C-band SAR and recorded on 17 September when the aircraft flew parallel to the ship track while the ship was traveling from left to right at 1.0 m/s. The internal waves appear as a V-shaped pattern behind the Roysterer, which is the large bright target near the right edge of the image. The other bright targets in the upper half of the pattern are the research vessels, pontoons, and buoy deployed at the instrumentation site.

The propagation speed of each wave in the pattern determines the angle between the wave and the ship's track. Because the pattern is stationary in a coordinate system moving with the ship, it follows that the half-angle, \( \phi \), of the pattern is given approximately by

\[
\sin \phi = \frac{c}{v_s} ,
\]

where \( c \) is the phase speed of the outermost wave and \( v_s \) is the ship speed. The half-angle of the outermost
wave in the pattern in Figure 6 was measured in the SAR image as about 27°, implying a phase speed of 0.45 m/s, in good agreement with the value of 0.42 m/s estimated from the in situ measurements. Note that, according to Equation 6, if the wave speed remains constant, the included angle of the pattern will decrease as the ship speed increases. This speed dependence of the pattern can be seen by comparing Figure 7 (recorded on 16 September when the ship speed was 6 m/s) with Figure 6.

An important question being addressed in the analysis of the Loch Linne data is whether the observed C-band SAR image intensity modulations can be predicted by using the same hydrodynamic and electromagnetic models previously applied to L-band (1.25-GHz) and X-band (9.35-GHz) imagery of internal waves. The hydrodynamic model for the surface wave modulation is summarized in the preceding section. These wave modulations are used as inputs to a two-scale composite scattering model to calculate the backscattered electromagnetic field. The model includes contributions from both the small-scale waves (the so-called Bragg waves) and the longer surface waves, which tilt the small waves.

Figure 8 compares the observed and calculated image-intensity modulations for a segment of the internal wave pattern shown in Figure 6. The selected image segment is located immediately to the left of the bright targets at the instrumentation site, extending about 180 m along the wave pattern. The black curve in Figure 8 represents the image intensity averaged along the wave fronts; the colored curve represents the modulations calculated from the model.

CONCLUSION

The Loch Linne experiment produced many radar images of the patterns of internal waves generated by ships and provided the first opportunity to obtain digital C-band SAR imagery of internal wave signatures for quantitative testing of radar-imaging models. The results obtained have shown that the pattern geometry is consistent with pre-test predictions, although the observed amplitudes of the internal waves have yet to be explained. The magnitudes of the radar signatures in several images from different data sessions are in good agreement with predictions from models used to interpret internal wave signatures at other radar frequencies. Additional analyses of these data and data from a second experiment conducted in August 1989 are in progress. The results are expected to yield valuable contributions to our understanding of the complex process by which oceanographic phenomena are manifested in radar imagery.

REFERENCES

SAR Imaging of Ship-Generated Internal Waves

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