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A Model-Free On-Off Iterative Adaptive Controller Based on Stochastic Approximation

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***Abstract*—An on-off iterative adaptive controller has been developed that is applicable to servo systems performing repeated motions under extremely strict power constraints. The motivation for this approach is the control of piezoelectric actuators in autonomous micro-robots, where power consumption in analog circuitry and/or for position sensing may be much larger than that of the actuators themselves. The control algorithm optimizes the switching instances between 'on' and 'off' inputs to the actuator using a stochastic approximation of the gradient of an objective function, namely that the system reach a specified output value at a specified time. This allows rapid convergence of system output to the desired value using just a single sensor measurement per iteration and discrete voltage inputs.**

I. INTRODUCTION

MINIATURIZATION of sensors and actuators through developments in Microelectromechanical Systems (MEMS) technology can enable very low-power, small footprint implementation of a variety of engineered systems. Piezoelectric and electrostatic actuators are two of the most common mechanisms for generating motion in MEMS devices and act as capacitive loads with very small intrinsic power consumption. For instance, a 1 nF piezoelectric actuator operating at 20 V and 20 Hz requires only about 18 μW of power. However, in many cases actuators may face a changing environment or feature nonlinear behavior requiring a servo control system, in which actuator power can be easily exceeded by the power consumption of drive and sensing circuitry.

To minimize power consumption of a complete servo control system for micro-scale piezoelectric or capacitive actuators, it is desirable to utilize switching ('on-off') control to avoid inefficiencies in driving circuits [1], and to minimize the number of times that the circuit switches during a given controlled motion. Minimizing the number of sensor measurements taken also limits system power consumption because sensing circuit power, particularly for capacitive sensors commonly used in MEMS, rises dramatically with sampling rate [2]. When the dynamics of the system to be controlled are well known sensor measurements may be omitted and open-loop input sequences used, for which optimization methods with and without capacitive charging costs are available [3], [4], [7]. On the other hand, when dynamics are unknown, difficult to model, or varying it is necessary to implement a feedback control system that is robust or can adapt the switching sequence applied to the system using feedback.

This paper examines the feedback control case for an 'on-off' switching control system in which a desired motion is to be completed many times with the ability to adjust the input sequence between movements. The motivation for this problem is the walking gait of a piezoelectrically-driven micro-robot, whose legs may need to complete the same stepping motion a large number of times, but for which power consumption is extremely constrained. A concept drawing of a bio-inspired terrestrial micro-robot design and image of a prototype piezoelectric leg joint are shown in Figure 1. The weight bearing capacity of such a robot is anticipated to be in the range of 5 to 50 milligrams [5], corresponding to an available power consumption per leg of approximately 100 to 700 microwatts based on state-of-the- art thin-film batteries or solar cells [6]. Under these conditions, it is very important to identify control strategies that could provide effective servo control with an extremely constrained power budget.

To perform low-power servo control the problem of selecting transition times of an on-off input sequence is converted to a model-free adaptive control problem with the adaptive controller based on the simultaneous perturbation stochastic approximation (SPSA) developed by Spall et al. [10], [11], [12], [13]. For power minimization in micro-robotics this controller has several benefits which include: a need for only one sensor measurement per motion, the ability to perform computation between steps to reduce processor requirements, and effectiveness in the presence of noisy sensors. A model-free approach was selected because the piezoelectric actuators targeted are highly nonlinear [19]

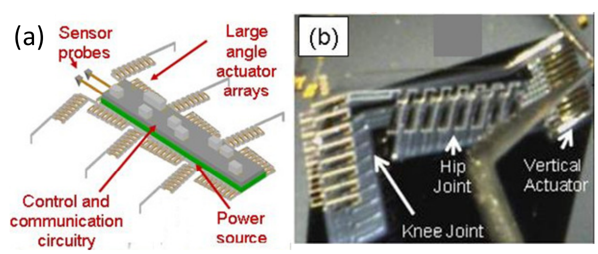

Figure 1: (a) Concept drawing of an autonomous micro-robot driven by (b) multi-degree-of-freedom piezoelectric leg joints

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and highly varying loads over time are anticipated due to the varying terrain that a micro-robot might encounter. The proposed algorithm differs from previous adaptive controllers specifically oriented towards on-off control, and most adaptive switching controllers, which have relied on model-based adaptation [20], [21]. Other, model-free, adaptive controllers have typically been organized around neural nets, which have not yet been applied to on-off control problems; these approaches could potentially perform function estimation in this context, though the assumption of an affine dependence on input in most cases prevents existing algorithms from being directly applicable [22], [23], [24].

Following this Introduction, a formal Problem Statement for the model-free, adaptive on-off controller is presented in Section II, followed by a description of the resulting control algorithm in Section III. Section IV discusses sample simulation results using the adaptive control algorithm, while Section V gives experimental results for a macro-scale piezoelectric testbed. Section VI concludes the paper.

II. Problem statement

Consider a general iterative discrete-time dynamic system governed by a corresponding sequence of input vector, u^k, and a measurement output vector, y^k, with a random noise component:

$$\begin{aligned} y^{k+1} &= \psi(y^k, u^k) \\ y_i^k &= f(y_0^k, u_{1,\ldots,i}^k) \\ y_0^k &= 0 \end{aligned} \tag{1}$$

where, k is the iterative index and i is the time index.

It is assumed that $\psi(\cdot)$ and $f(\cdot)$ are nonlinear but unknown functions for the system (1). Also it is assumed that u^k is a function of a parameter vector, $\theta^k \in R^p$, such that,

$$\theta^k = \begin{bmatrix} \theta_1^k & \theta_2^k & \cdots & \theta_p^k \end{bmatrix}^T \tag{2}$$

Now, the output measurement vector $y^k \in R^M$ may be considered to be a function of θ^k. Then the system (1) can be re-written as follows:

$$\begin{aligned} y^{k+1} &= \varphi(y^k(\theta^k)) \\ y_i^k(\theta^k) &= h(y_0^k, \theta_{1,\ldots,i}^k) \\ y_0^k &= 0 \end{aligned} \tag{3}$$

where, $\varphi(\cdot)$ and $h(\cdot)$ are unknown nonlinear functions.

Two ultimate goals for the control of system (3) are targeted in this paper. The first is to find the optimized parameter vector set, θ^k, of the input, u^k, to minimize the error between the final output measurement value, y_f^k, at the final time, t_f, and a desired target, r. And the second is to minimize the number of measurement outputs required per step to achieve the first goal.

The minimization problem with respect to θ^k and output error can be stated as follows:

$$\begin{aligned} &\min \left\| y_f^k(\theta^k) - r \right\|, \ s.t.\ y_f^k(\theta^k) = h(y_0^k, \theta_{1,\ldots,i}^k),\ y_0^k = 0 \\ &\text{subject to } \theta_i^k \geq 0, \end{aligned} \tag{4}$$

Define a cost function with quadratic form for the minimization problem (4) such that

$$J(\theta^k) = (y_f^k(\theta^k) - r)^T Q (y_f^k(\theta^k) - r) \tag{5}$$

where, Q is a weighting matrix.

To solve the problem (4) and (5), generally, a gradient based optimization method is used, using the equation

$$\begin{aligned} \theta^{k+1} &= \theta^k - \alpha^k g(\theta^k) \\ g(\theta^k) &= \left(\frac{\partial y_f^k}{\partial \theta^k} \right)^T \left(\frac{\partial J}{\partial y_f^k} \right)^T \end{aligned} \tag{6}$$

where, α^k is the step size of the algorithm and $g(\theta^k)$ is the gradient vector for the cost function $J(\cdot)$ with respect to θ^k. This approach is used by several model based controllers [3], [7], [8], [9]. In this case the system dynamics functions, $h(\cdot)$ and $J(\cdot)$, are completely known and there is no consideration of minimal use of measurement outputs.

However, the gradient in equation (6) for the cost function cannot be determined in this paper because it is assumed that equations (4) and (6) are based on an unknown function, $h(\cdot)$. Hence, any standard gradient based optimization algorithm or a direct measurement of gradient is not feasible to solve problem (4). Additionally, any consideration for the minimal use of measurement outputs cannot be stated by the standard gradient because the standard gradient is based on derivation of a complete function.

To solve this problem, the simultaneous perturbation stochastic approximation (SPSA) algorithm is used for gradient approximation in this paper. This approximation method was developed by previous researchers as a model-free approach for finding optimal control parameters [15], [16], [17], [18]. These papers have considered a fixed functional structure for a controller, such as a Neural Network or a polynomial ([15], [16], [17], [18]) and have generated analog inputs to these controllers. In contrast, an on-off controller permits only two possible input states, and this behavior must be defined in a way that is compatible with the SPSA algorithm.

SPSA is based on relating random perturbations of controller parameters with their influence on a cost function, which is in turn a function of output measurements from an unknown system with random noise. Thus, SPSA doesn't require full knowledge of the form of the system dynamics. Most importantly, since SPSA requires only one or two measurements to compute gradient approximations regardless of the dimension, such as the problem (6), it is very useful and effective for minimizing the number of measurement outputs, especially in problems with a large number of variables to be determined. The SPSA algorithms used in this paper are based on results by Spall et al. as discussed in detail in reference works [10], [11], [12], [13], [14].

III. Model-free On-off Control Algorithm

In this paper, an on-off controller is developed that can be represented using the description in Section II, such that performance is improved over repeated iterations of system movements. The final goal of this paper is to find the optimized parameter vector set, θ^k, of this iterative adaptive on-off controller that minimizes the cost function, and to reduce the number of measurement outputs as much as possible.

In order to perform these tasks, the iterative adaptive on-off controller will be applied to unmodeled nonlinear systems and the one-measurement form of SPSA will be used to estimate the gradient approximation of a cost function. The one-measurement form of SPSA can reduce the number of iterations required to reach a desired final output of the system and therefore be more efficient in terms of sensor power consumption in feedback control applications than other adaptation algorithms [12].

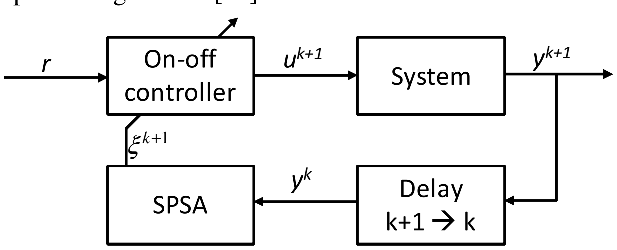

Figure 2. The block diagram of the controller

In this section we define all of the controller's parameters, a cost function, and a parameter updating rule . Additionally, a short overview and convergence conditions of the one-measurement form of the SPSA (as provided in [10] and [12]) will be presented.

Figure 2 shows the block diagram of the iterative adaptive on-off controller using the SPSA algorithm.

A. Adaptive On-off Controller

The control task is to steer the system (3) to a desired target, *r,* by finite-time on-off control using *N* switching instances and a single measurement output. Under finite-time on-off control, the input, u^k, applied to the system (3) alternates between zero and a constant maximum value, u_{max}.

Switching times are defined by the final time, t_f, and time vectors, $\tau^k \in R^{N+2}$, and $\xi^k \in R^N$ such that

$$\begin{aligned}\tau^k &= \begin{bmatrix}\tau_1^k & \tau_2^k & \cdots & \tau_{N+1}^k & t_f\end{bmatrix}^T \\ \xi^k &= \begin{bmatrix}\xi_1^k & \xi_2^k & \cdots & \xi_N^k\end{bmatrix}^T\end{aligned} \tag{7}$$

Here, ξ^k is defined by

$$\xi_j^k = \tau_{j+1}^k - \tau_j^k \text{ and } \xi_j^k \ge 0, \quad j = 1, 2, ..., N \tag{8}$$

which is the finite on-off time duration. The system (3) can be re-written as a function of ξ^k :

$$\begin{aligned}&y^{k+1} = \varphi(y^k(\xi^k)) \\ &y_f^k(\xi^k) = h(y_0^k, \xi^k_{1,...,N}) \\ &y_0^k = 0\end{aligned} \tag{9}$$

The zero initial condition is based on the assumption of a system with stable dynamics and sufficient time between motions to return to its equilibrium point.

Since only one measurement output will be required, the cost function (5) and the minimization problem (4) may be re-defined as

$$J(\xi^k) = \frac{1}{2}(y_f^k(\xi^k) - r)^2 \tag{10}$$

$$\begin{aligned}&\min\left(J(\xi^k) = \frac{1}{2}(y_f^k(\xi^k) - r)^2\right) s.t.\ y_f^k(\xi^k) = h(y_0^k, \xi^k_{1,...,i}),\ y_0^k = 0 \\ &\text{subject to 1. } \xi_i^k \ge 0\end{aligned} \tag{11}$$

By the SPSA algorithm, the problem (11) may be solved and input parameter vector ξ^k determined and updated until the cost function $J(\cdot)$ goes to zero as iterative index *k* goes to infinity.

Once the input parameter vector is determined, the input u^k may be characterized by ξ^k such that, for $r = 1, 2, ..., (N/2)$,

$$\begin{aligned}&u_{2r-1}^k = u_{\max}, \quad for\ \xi_{2r-1}^k \\ &u_{2r}^k = 0, \qquad for\ \xi_{2r}^k \\ &u_f^k = 0, \qquad for\ t_f - \sum_{i=1}^N \xi_i^k\end{aligned} \tag{12}$$

where, *r* is time index at the iteration *k.*

B. Iterative Update of ξ^k using SPSA Algorithm

Let us now briefly review the one-measurement form of the SPSA algorithm [12] for the control problem (11). Let $\hat{\xi}^k$ be the estimate of $\xi^k \in R^N$ at the iteration k. SPSA has the standard iterative form

$$\hat{\xi}^{k+1} = \hat{\xi}^k - \alpha^k \hat{g}^k(\hat{\xi}^k) \tag{13}$$

where, α^k is a sequential gain coefficient and $\hat{g}^k(\hat{\xi}^k)$ is a simultaneous perturbation (SP) approximation to the unknown gradient vector, $g(\cdot)$, for the cost function $J(\cdot)$ with respect to ξ^k at the k^{th} iteration.

The one-measurement SPSA form of the $g(\cdot)$ estimation at iteration k is

$$\hat{g}^k(\hat{\xi}^k) = \frac{Y^{(+)k}}{c^k}\begin{bmatrix} 1/\Delta_1^k & 1/\Delta_2^k & \cdots & 1/\Delta_N^k \end{bmatrix}^T \tag{14}$$

where $\left\{\Delta^k \in R^N \mid \Delta^k = \begin{bmatrix} \Delta_1^k & \Delta_2^k & \cdots & \Delta_N^k \end{bmatrix}^T\right\}$ is a vector of N independent zero-mean random variables, c^k is a sequential gain, and $Y^{(+)k}$ is the cost function value with the control parameter's random perturbation vector, Δ^k, and the noise measurement, ε^k, i.e.

$$Y^{(+)k} = J(\hat{\xi}^k + c^k\Delta^k) + \varepsilon^k \tag{15}$$

Note that only one measurement is required to form the gradient estimation at the each iteration.

C. Convergence conditions of SPSA Algorithm

The specific conditions are necessary for the convergence of the system (9). These conditions have been stated and proven in [10] and [12].

For an on-off controller driving a system with continuous, differentiable dynamics, the finite duration, t_f, and finite input magnitude, $u_{\max}$, of the on-off controller ensure that the system output measurement, y_f, and control parameters in ξ will be bounded. The source of noise in the system is taken to be white, Gaussian noise in sensor measurements independent of noise during prior steps and internal computations. Thus our system may ensure all convergence conditions stated in [10] and [12]. While this appears to be a realistic assumption for common microactuators when discontinuous forces encountered by the controlled system, such as impact forces, are not present, rigorous examination of microactuator characteristics meeting these conditions remains to be done. Simulation and experimental results to follow have shown good convergence. For the motivating problem of a piezoelectric micro-robotic leg joint, it can currently be said that at the least, the algorithm should only be implemented if the leg is either in contact or out of contact with ground for the entire controlled motion. In addition, for it to be possible to reach a minimum value of the objective function that approaches zero, the target reference displacement, r, should be within the range of feasible motions of the actuator, which does require some knowledge of system capabilities, even if a full model of dynamics is not used.

D. Implementation of an Adaptive On-off Controller with SPSA Algorithm

Iterative on-off time-optimized control for the system (9) can be determined by using the method described in section A, B, and C of II with the following implementation procedure.

(Step 1) Initialize max(k), $N, \xi^1, t_f, a, c, A, \alpha,$ and γ

(Step 2) Generate the SP vector Δ^k

(Step 3) Generate u^k by $\hat{\xi}^k + c^k\Delta^k$ and the equation (12)

(Step 4) Measure system output at final time t_f

(Step 5) Evaluate the cost function, equation (15)

(Step 6) Approximate the gradient by the equation (14)

(Step 7) Update ξ estimate and check constraint

$$\text{Constraint:} \quad \begin{matrix} \hat{\xi}^k \in [0 \ \xi^1], & \text{for } x_f^1 > r \\ \hat{\xi}^k \geq 0, & \text{for } x_f^1 \leq r \end{matrix} \tag{16}$$

(Step 8) go to (Step 2) or terminate algorithm if either ξ is smaller than a prespecified bound or if a maximum allowable number of iterations has been reached, subject to available memory.

IV. Simulation Examples

In this section, the performance of the proposed on-off controller is illustrated by considering sample nonlinear systems with some noise input. However, within the controller itself, a representative model should not be used in generation of a control input or controller and system identification. Since the proposed controller is a model-free controller, a representative model will be used only as a simulated plant to generate the output signal used in on-off decisions and parameter updating as shown in Figure 2.

A. 2^{nd} Order Nonlinear System.

Now, consider the performance of the controller operating on a sample nonlinear system. The system (17) was randomly selected to test controller performance.

The same simulation objectives are applied to the system (17). That is, the desired target, r, is 0.5; the dimensions used for vector ξ are 4 and 8; the number of iterations is 20; and the update termination index is 15; but the selected final time to simulate is 0.2 sec, due to a slower response time of the nonlinear system. Selected gain coefficient values of *a, c, A, α,* and γ are 2.95e^{-4}, 5e^{-5}, 2000, 0.602, and 0.101, respectively.

$$\begin{aligned} x_{1,i+1}^k &= x_{1,i}^k + 0.0001 x_{2,i}^k \\ x_{2,i+1}^k &= -0.02 x_{1,i}^k + 0.9997 x_{2,i}^k - 0.012 x_{1,i}^k \sin(100 x_{1,i}^k) \\ &\quad - 0.00012 x_{2,i}^k \sin^2(100 x_{2,i}^k) + 0.021 u^k \\ y_i^k &= x_{2,i}^k + \varepsilon^k \end{aligned} \tag{17}$$

where ε^k is normally distributed random noise with $N(0, 0.01^2)$, i is time index at the k^{th} iteration, and the simulation sample time is 0.0001 sec.

The result of system response and values of the cost function $J(\cdot)$ related to iteration index are shown in Figure 3 and 4, respectively.

Obtained ξ^* is $[0.0275\ 0.0497\ 0.0301\ 0.0497]^T$ for $N = 4$ and $[0.0134\ 0.0107\ 0.0106\ 0.0107\ 0.0205\ 0.0236\ 0.0106\ 0.0106]^T$ for $N = 8$.

The proposed controller provides the same solution quality to determine the minimization problem (11) as the linear system case, even if the tested system is a randomly selected nonlinear system. This performance is also independent of parameter dimension and uses only one measurement.

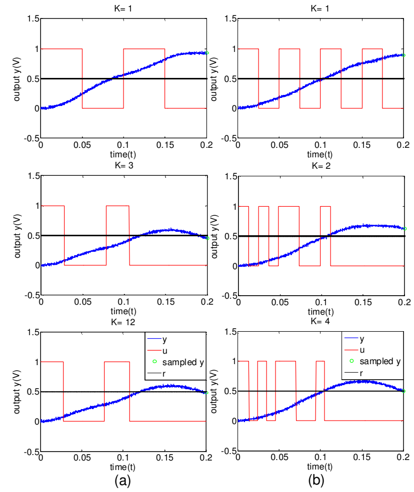

Figure 3. Output response of the system (17): (a) when $N = 4$, (b) when $N = 8$

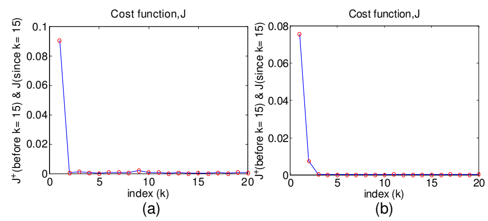

Figure 4. Cost function values for the system (17) vs. iteration index: (a) when $N = 4$, (b) when $N = 8$

V. Experimental Examples

The performance of the proposed controller was verified in an experimental test actuator.

A. Experimental Setup

A macro-scale piezoelectric actuator is used to verify behavior under the proposed controller. A strain gage was attached to the actuator to measure deflection, with the output of the system being output voltage of the strain gage sensing circuit. Mass was added to the tip of the actuator to reduce the natural frequency of the experimental apparatus to 31.97 Hz. The damping ratio of the experimental system was 0.0348, which is similar to a projected micro-robotic leg application. Also, system gain G = 0.106 is used since the input range is not 0 to 1, but rather 0 to 15 V. The full open-loop transfer function of the experimental system, modeled as a 2nd-order linear system with unmodeled nonlinear term $\phi(\cdot)$ and the approximated noise distribution, $\tilde{\varepsilon}$, has an approximate discrete-time state model of:

$$\begin{aligned} \begin{pmatrix} x_{1,i+1}^k \\ x_{2,i+1}^k \end{pmatrix} &= \begin{bmatrix} 0.9986 & -0.0158 \\ 0.0256 & 1 \end{bmatrix} \begin{pmatrix} x_{1,i}^k \\ x_{2,i}^k \end{pmatrix} + \begin{bmatrix} 0.0158 \\ 0 \end{bmatrix} u^k + \phi(x^k, u^k) \\ y_i^k &= x_{2,i}^k + \tilde{\varepsilon}^k \end{aligned} \tag{18}$$

It is assumed that the noise of the experimental environment is approximated by the ideal noise distribution of the simulated case; that is, $\tilde{\varepsilon}$ has $N(0, 0.01^2)$. This noise assumption is based on experimental measurements from the strain gage sensing circuit and was used to determine the gain coefficients of the controller.

The control signal is generated on a TMS320F28335 digital signal processor, with an H-bridge circuit acting as the on-off interface between the low-voltage DSP and a 15 V supply for the actuator.

B. Experimental Results

The desired target, r, is 0.5 where a value of 1 corresponds to the maximum static displacement of the actuator. In the experiment, dimension settings of 4 and 8 for vector ξ were used; the final time, t_f, was 0.01 sec; and the CPU operation time was 0.0001 sec. The experimental controller uses only one measured value at the final time, t_f. However, the maximum number of iterations and the update termination index were 35 and 30, which are larger than the simulation conditions because the noise and dynamic response conditions of the real system (18) may be slightly different from the sampled nonlinear system (17).

Selected gain coefficients values of a, c, A, α, and γ are 9.36e^{-5}, 5e^{-5}, 2000, 0.602, and 0.101, respectively.

Sample responses using only one measurement at the final time for two scenarios, $N = 4$ and $N = 8$, are shown in Figure 7 (a) and (b). The obtained ξ^* is $[0.0023\ 0.0025\ 0\ 0.0025]^T$ for $N = 4$ and $[0.0013\ 0\ 0.0004\ 0.0006\ 0.0002\ 0.0013\ 0\ 0.0013]^T$ for $N = 8$.

The experimental result shows a successful convergence to

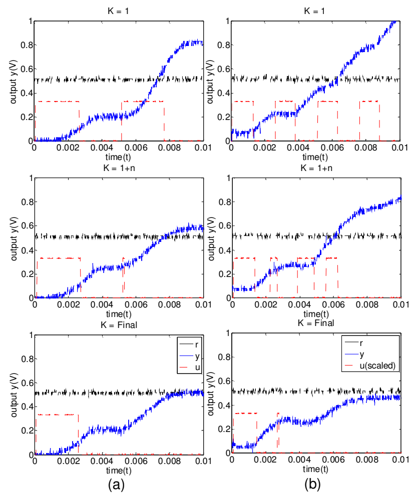

Figure 7. Experimental results for the system (20): (a) when N = 4, (b) N = 8

the target reference level.

In this results, there exist higher order mode oscillations encompassed in the unmodeled nonlinear term $\phi(\cdot)$, possibly due to weakness in the connection between the piezoelectric actuator and the added load mass. However, since the controller doesn't evaluate the entire time trajectory of the system response but responds to the final value, this phenomenon of controller behavior is both acceptable, and such unmodeled dynamics are expected to be accommodated.

VI. CONCLUSION

In this paper, we have described a method for implementing model-free adaptive on-off control through the application of simultaneous perturbation stochastic approximation to an on-off controller structure. This control technique can be very useful in control of repeated motions by systems with extremely limited power budgets, where power consumption at sensor measurements and in analog drive circuitry is to be avoided. The controller presented can be implemented with just a single sensor measurement per actuator motion and results in rapid convergence to a desired final output in both simulated and experimental tests. Such a controller may ultimately be used to help regulate leg motions in the walking gait of a piezoelectrically-actuated terrestrial micro-robot.

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