Blind Signal Separation Via Simultaneous Perturbation Method

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Abstract

When independent plural signals are mixed and the mixed plural signals are measured, blind signal separation technique is very interesting approach to separate these signals only based on the measured signals. This technique is applicable to many fields including communication engineering, signal processing, image processing, analysis of organs inside a body and so on [1],[2],[3].

In this paper, we propose a recursive method to obtain a separating matrix based on the mutual information, via the simultaneous perturbation optimization method [4],[5],[6]. The simultaneous perturbation method estimates a gradient of the mutual information with respect to the separating matrix, based on a kind of the finite difference. Therefore, the separating matrix is updated by only two values of the mutual information.

Some examples for image signals and audio signals are shown to confirm viability of the proposed method. In these examples, our method separated some signals from mixed ones properly. This method is applicable to on-line separation because of simplicity of the algorithm.

***Keyword** : blind signal separation ,simultaneous perturbation optimization method ,mutual information*

1. Introduction

Blind Signal Separation (BSS) is a technique that can extract original signals from their mixtures observed ones by same number of sensors. Moreover, BSS separates these signals without knowing the mixing process and the information of source signals as well.

BBS is realized on condition that the original signals are independent each other. Therefore, retrieved signals separated from the measured signals must be independent as well. Using a criterion to measure the independence of the retrieved signals, we can construct a separating matrix and estimate the source signals [1],[2].

When $\boldsymbol{a} \in R^n$ is a vector of independent source signals and $\boldsymbol{A} \in R^{n\times n}$ is a mixing process matrix, we have a vector of observation signals $\boldsymbol{x} \in \mathrm{R}^n$ which are assumed to be a linear transformation of a by non-singular matrix $\boldsymbol{A}$. We have the following relation

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{a} \tag{1}$$

On condition that mixing matrix $\boldsymbol{A}$ and source signals $\boldsymbol{a}$ are unknown, we use a criterion to measure independence of the retrieved signals $\boldsymbol{y} \in R^n$. Then we have to construct a separating matrix $\boldsymbol{W} \in R^{n\times n}$ and estimate the original source signals a.

$$\boldsymbol{y} = \boldsymbol{W}\boldsymbol{x} = \boldsymbol{W}\boldsymbol{A}\boldsymbol{a} \tag{2}$$

In BBS, we expect $\boldsymbol{W}$ to be an inverse of $\boldsymbol{A}$. However, there are two ambiguities; their order and amplitude of separating signals components, because we have no information about the mixing matrix $\boldsymbol{A}$ and the original signals $\boldsymbol{a}$. There remains indefiniteness of perturbation and dilation factors,

$$\boldsymbol{W}\boldsymbol{A} = \boldsymbol{P}\boldsymbol{D} \tag{3}$$

where $\boldsymbol{P}$ is a perturbation matrix, and $\boldsymbol{D}$ is a diagonal matrix. Problem of BSS is an identification of such $\boldsymbol{W}$.

Usually, in BSS, gradient method is used [2] to find a proper separating matrix $\boldsymbol{W}$. It updates the separating matrix using information of local gradient of a criteria defined by the independence of the separated signals.

In this paper, we apply the simultaneous perturbation optimization method to BSS problem. By using the optimization method known as the simultaneous perturbation optimization method, we can update all parameters of the separating matrix by using only two values of the evaluation function.

2. Evaluation Function

In this paper, the mutual information based on the information theory is applied to make an evaluation of independence of each separated signal. Let X and Y be two random variables, the mutual information implies a measure of the amount of information about Y contained in X [7].

The mutual information $I(\cdot)$ is defined as follows;

$$I(X,Y) = H(X) + H(Y) - H(X,Y)$$
$$= \sum r_{ij} \log \frac{r_{ij}}{p_i q_j} \tag{4}$$

Where, $H(\cdot)$ is called the entropy of two random variables X $(=x_1, x_2, \cdots, x_n)$ $Y(=y_1, y_2, \cdots, y_m)$ or of the distribution $p_i=P(X=x_i)$, $q_i=P(Y=y_j)$, $r_{ij}=P(X=x_i, Y=y_j)$.

$$H(X) = \sum_{i}^{n} -p_i \log p_i \tag{5}$$

$$H(Y) = \sum_{j}^{m} -q_j \log q_j \tag{6}$$

$$H(X,Y) = \sum_{i}^{n} \sum_{j}^{m} -r_{ij} \log r_{ij} \tag{7}$$

If the random variables X, Y are independent each other, the mutual information $I(X,Y)$ equals zero.

In this paper, we utilize the mutual information as an evaluation function.

3. Simultaneous perturbation for BSS

In this paper, we propose a recursive method to obtain the separating matrix based on the mutual information using the simultaneous perturbation optimization method. This optimization method is proposed by Spall, Maeda and others [8],[9]. It is also known that the method is a stochastic gradient method. We apply the method to a recursive learning of the blind signal separation. The algorithm proposed here is as follows;

$$\boldsymbol{W}_{t+1} = \boldsymbol{W}_t - \alpha \Delta \boldsymbol{W}_t \tag{8}$$

where, α is a positive learning coefficient to adjust a magnitude of a modifying quantity. The i-j component of the modifying matrix $\Delta \boldsymbol{W}$ is defined as follows;

$$\Delta W_t^{ij} = \frac{J(\boldsymbol{W}_t + c\boldsymbol{s}_t) - J(\boldsymbol{W}_t)}{c s_t^{ij}} \tag{9}$$

s^{ij}_t, which is an element of the sign matrix $\boldsymbol{s}_t$, is +1 or –1, and has the following properties.

$$\boldsymbol{s}_t = \left(s_t^{ij}\right) \tag{10}$$

$$E\left(s_t^{ij}\right) = 0 \tag{11}$$

$$E\left(s_t^{ij} \cdot s_t^{kl}\right) = \begin{cases} 1 & (i = k \quad and \quad j = l) \\ 0 & (else) \end{cases} \tag{12}$$

Where, $E(\cdot)$ denotes exception. That is, s^{ij}_t has zero mean and is independent with respect to the other s^{ij}_t and time t. c is a magnitude of the perturbation. $J(\cdot)$ is an evaluation function to be minimized. In this case, the evaluation is the mutual information of Eq.(4). That is,

$$J(\boldsymbol{W}) = I(X, Y \mid \boldsymbol{W}) \tag{13}$$

Next, we look into property of this optimization method. Let us consider Eq.(9). Expanding Eq.(9) at $\boldsymbol{W}_t$, There exists a matrix $\boldsymbol{W}_s$ such that

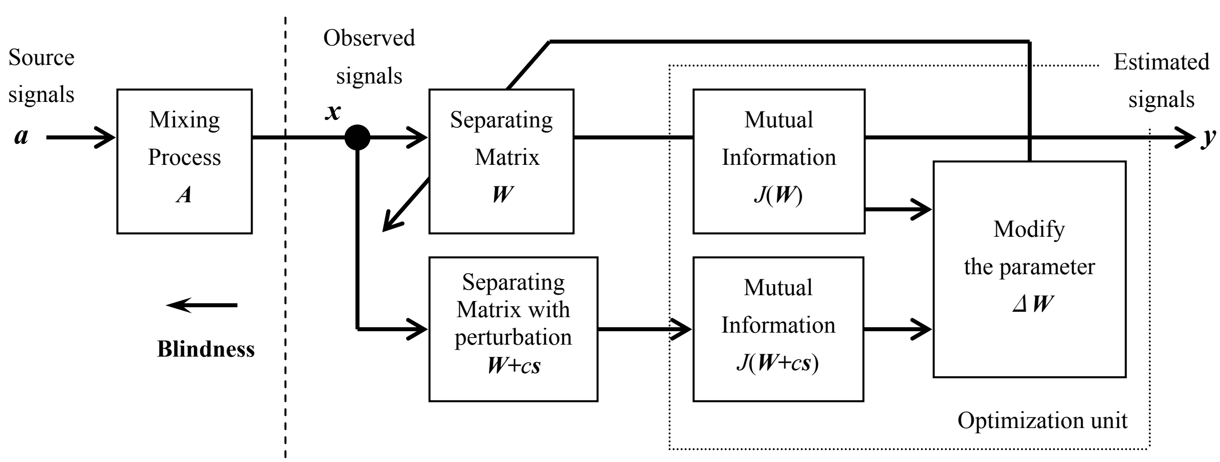


Fig.1. Blind signal separation using Simultaneous Perturbation optimization method

$$\Delta W_t^{ij} = \boldsymbol{s}_t^T \frac{\partial J(\boldsymbol{W}_t)}{\partial \boldsymbol{W}_t} s_t^{ij} + \frac{1}{2!} c\boldsymbol{s}_t^T \frac{\partial^2 J(\boldsymbol{W}_s)}{\partial \boldsymbol{W}_s^2} \boldsymbol{s}_t s_t^{ij} \tag{14}$$

Now, superscript T denotes transpose of a vector or a matrix. Then, taking expectation of above equation, from Eq.(11),(12), we have

$$E(\Delta W^{ij}) = \frac{\partial J(\boldsymbol{W}_t)}{\partial W^{ij}} \tag{15}$$

This means that the optimization method described here is a kind of stochastic gradient method.

An important point is that this optimization method requires only two values of the mutual information $J(\cdot)$, if even number of the parameters is large. Thus, in this method we need only twice calculations of the evaluation function (the mutual information) to update for all parameters of $\boldsymbol{W}$.

Total flowchart of the algorithm is shown in Fig.1.

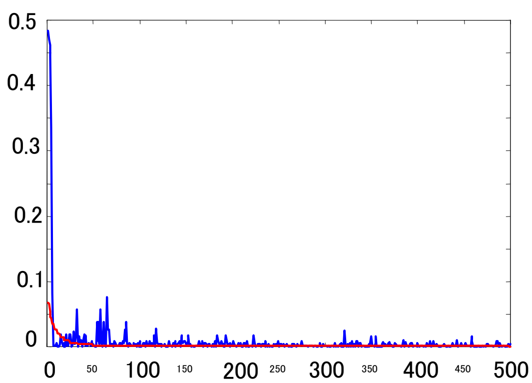
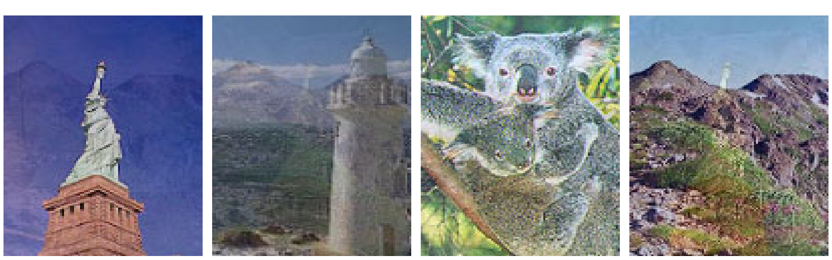


Fig.2. The mutual information

(a) Observed signals $\boldsymbol{x}_i$ (i =1~4).



(b) Estimated signals $\boldsymbol{y}_i$ (i =1~4).

Fig.3. Results of mixture and separation of four picture signals

4. Procedure of Blind Signal Separation

In this chapter, we describe details of procedure of BSS using the simultaneous perturbation method.

First, we set up the source signals $\boldsymbol{a}$ and mixing matrix $\boldsymbol{A}$. These are blind data. From these two data, observed signals $\boldsymbol{x}$ is calculated. Next, without perturbation $c\boldsymbol{s}$, we input $\boldsymbol{x}$ to separating matrix $\boldsymbol{W}$, and obtain the separated signals $\boldsymbol{y}$. Then, we calculate the mutual information $J(\boldsymbol{w})$ by Eq.(4). On the other hand, simultaneously, we add perturbation $c\boldsymbol{s}$ to all parameters of $\boldsymbol{W}$. Then, we have a value of the mutual information $J(\boldsymbol{w}+c\boldsymbol{s})$. By using these two values of the mutual information and Eq.(8),(9), we can update all parameters of the separating matrix. This method can calculate modifying qualities, by using only two values of the mutual information without perturbation $J(\boldsymbol{w})$ and with perturbation $J(\boldsymbol{w}+c\boldsymbol{s})$, by using Eq.(9), and update the separating matrix.

5. Results

We examine a viability of BSS using this optimization method for image signals and audio signals.

5.1 BSS for image data

We handle four image data. We assume that mixing matrix $\boldsymbol{A}$ is defined using random number and an initial value of separating matrix $\boldsymbol{W}_0$ is also determined by random number.

$$\boldsymbol{A} = rand \begin{pmatrix} A_{11} & \cdots & A_{14} \\ \vdots & \ddots & \vdots \\ A_{41} & \cdots & A_{44} \end{pmatrix} \tag{16}$$

$$\boldsymbol{W}_0 = rand \begin{pmatrix} W_{11} & \cdots & W_{14} \\ \vdots & \ddots & \vdots \\ W_{41} & \cdots & W_{44} \end{pmatrix} \tag{17}$$

The result shown in Fig.2 is a value of the evaluation function (the mutual information) $J(\boldsymbol{w})$ by the simultaneous perturbation method for every 500 times modification of the separating matrix parameters. Fig.3 shows a configuration of four image mixtures and a result of estimated source signals.

The perturbation c is 0.05 and the coefficient α is 0.01 in this example.

5.2 BSS for audio signals

Next, we consider mixture of two audio signals. In this case, we take time. Mixing matrix $\boldsymbol{A}$ is described as follows;

$$\boldsymbol{A} = \sum_{i=0}^{100} \boldsymbol{A}_i z^{-i} \tag{18}$$

$$\boldsymbol{A}_i = \left(0.9e^{-0.5i} + 0.1\right)\boldsymbol{A}_0 \quad \left(i = 1, \cdots, 100\right) \tag{19}$$

$$\boldsymbol{A}_0 = \begin{pmatrix} 1.0 & 0.8 \\ 0.6 & 1.0 \end{pmatrix} \tag{20}$$

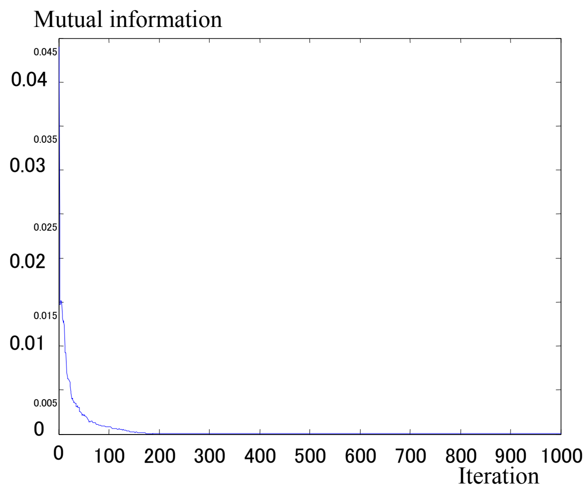


Fig.4. Mutual information for audio signals

A result with the perturbation c of 0.05 and the coefficient α of 0.01 is shown in Fig.4. Then, original signals, observed signals and separated signal are shown in Fig.5.

From Fig.4, it is seen that the mutual information is enough small after 150 iterations. In this example, presented data in Fig.5 are used repeatedly to obtain the separating matrix. However, it seems easy to implement this for on-line system, since the algorithm is very simple.

6. Conclusion

In this paper, separating mixture signals by blind signal separation using the simultaneous perturbation optimization method is described.

The presented BSS technique does not require any information, e.g. distribution or statistics of target signals. Moreover, the procedure itself is very simple, so that we can expect easy implementation for on-line system.

Handling noisy data or time-variant mixture signals will be future problem.

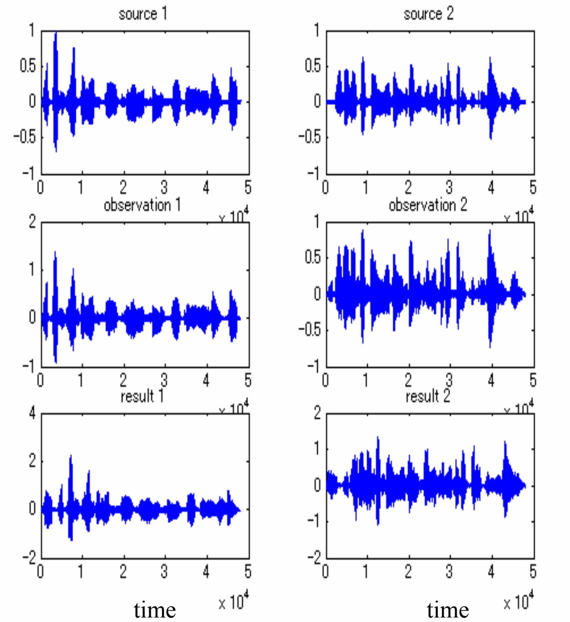


Fig.5. Results of original, mixture and separated signals

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