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Application of Non-linear Observer with Simultaneous Perturbation Stochastic Approximation Method to Single Flexible Link SMC

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Abstract: The main object in this study concerns to vibration control of a one-link flexible arm system. The robot link manipulators are widely used in various industrial applications. A variable structure system (VSS) non-linear observer has been proposed in order to reduce the oscillation in controlling the angular. The non-linear observer parameters are optimized using a novel version of simultaneous perturbation stochastic approximation (SPSA) algorithm. The SPSA algorithm is especially useful when the number of parameters to be adjusted is large, and makes it possible to estimate them very efficiently. As for the vibration and position control , a model reference sliding mode control (MR-SMC) has been proposed. The simulations show that the vibration and position controls of a one-link flexible arm system can be achieved more easy and efficiently with a non-linear observer designed using our proposed modified SPSA algorithm.

Keywords: Non-linear Observer, Simultaneous Perturbation Method, Flexible Arm System, Fisher Information matrix, Sliding Mode Control.

1. Introduction

It is well known that the demand for increase productivity by robots can be partly met by the use of lighter robots operating at high speeds and consuming less energy. This would result in an increase in robot deflection and poor performance due to the effect of mechanical vibration in the links, and bring difficulties for control [1]. Thus, vibration control of a robotic manipulator system has been an important research area in the last decade [1]-[5]. On the other hand, the robots are highly non-linear, whose mathematical models usually consist of a set of linear or non-linear differential/difference equations derived by using some forms of approximation [2]. A flexible arm can be modeled as an infinite dimensional system. However, it is almost impossible to practically design a controller based on an infinite dimensional model. Usually, reduced order models of the flexible arm are employed to design the controller [3]. In this paper, we consider a one-link flexible arm. One end of this arm is attached to a motor and the other end carries a payload. The control of vibration and angular position of the arm is taken as our purpose. Since the feedback of only the motor angle will not be sufficient to suppress the oscillation, a variable structure system (VSS) non-linear observer is incorporated. Also, a model reference sliding mode control (MR-SMC) is established as a very efficient control method. However, there are many design parameters for the observer and SMC to be determined, so that it is difficult to design them in advance. Hence, in order to overcome the problem, the simultaneous perturbation stochastic approximation (SPSA) algorithm is used to obtain the parameters of the VSS non-linear observer and controller.

2. Dynamic Modeling of a Single Flexible Link Robot Arm

2.1 Dynamic Model

The physical configuration of a robot arm considered in this work is given in Fig.1. The mass and elastic properties are assumed to be distributed uniformly along the elastic arm. The flexible arm shown in Fig.1 is modeled as a continuous cantilever beam of length L that has a mass m, torque T that rotates the elastic arm and a mass M that is the payload at the end of the arm. We use Lagrange's equation to obtain the equations of motion [4]. The deflection $y(x,t)$ is described by an infinite series of separable modes.

$$y(x,t)=\sum_{i=1}^{n}\phi_i(x)q_i(t) \tag{1}$$

where $\phi_i(x)$ is a characteristic function and $q_i(t)$ is a mode function. The kinetic and potential energies of the arm can be determined as follows:

$$T_e = \frac{1}{2}\dot{\theta}^2 J + \frac{m}{2L}\sum_{i=1}^{n} A_i \dot{q}_i^2 + \frac{m}{2L}\dot{\theta}\sum_{i=1}^{n} A_i \dot{q}_i^2$$
$$+ \frac{m}{L}\dot{\theta}\sum_{i=1}^{n} B_i \dot{q}_i + \frac{M}{2}(L^2\dot{\theta}^2 + \sum_{i=1}^{n} C_i^2 \dot{q}_i^2 \tag{2}$$
$$+ \dot{\theta}^2 \sum_{i=1}^{n} C_i^2 q_i^2 + 2L\dot{\theta}\sum_{i=1}^{n} C_i \dot{q}_i)$$

$$V = \frac{EI}{2}\sum_{i=1}^{n} D_i q_i^2 \tag{3}$$

where θ is the angle of the joint, E is Young's modulus, and I is the area moment of inertia where

$$A_i \int_0^L \phi_i^2(x)dx \tag{4}$$

$$B_i \int_0^L x\phi_i\ (x)dx \tag{5}$$

$$C_i = \phi_i(L) \tag{6}$$

$$D_i = \int_0^L \left[d^2\phi_i(x)/dx^2\right]^2 dx. \tag{7}$$

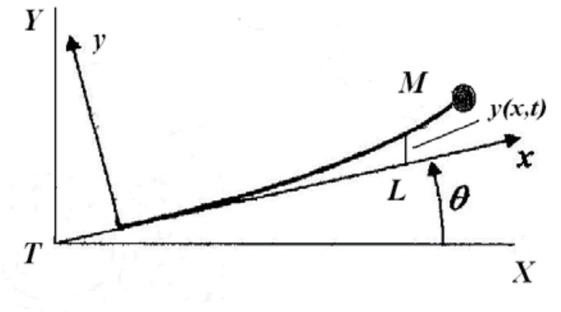

Fig. 1 One-link flexible arm.

2.2 Equation of Motion

The robot arm is assumed as a cantilever beam carrying a mass. The eigenfunctions of a cantilever beam with a constant length is given by

$$\phi = \cosh\lambda\frac{x}{l} - \cos\lambda\frac{x}{l} - \frac{\cosh\lambda + \cos\lambda}{\sinh\lambda + \sin\lambda}\left(\sinh\lambda\frac{x}{l} - \sin\lambda\frac{x}{l}\right). \tag{8}$$

Each individual mode shape function ϕ may be found by substituting the value λ determined from the following expression into (8):

$$\cosh\lambda\cos\lambda + 1 = 0 .$$

Assuming that only the first mode exists, from (2) and (3), and using Lagrange's equations as in [5][6], we obtain

$$\begin{aligned} &\frac{d}{dt}\left(\frac{\partial T_e}{\partial \dot{\theta}}\right) - \frac{\partial T_e}{\partial \theta} + \frac{\partial V}{\partial \theta} = T \\ &\frac{d}{dt}\left(\frac{\partial T_e}{\partial \dot{q}_1}\right) - \frac{\partial T_e}{\partial q_1} + \frac{\partial V}{\partial q_1} = 0 \end{aligned} \tag{9}$$

$$\begin{bmatrix} \alpha_{00} & \alpha_{01} \\ \alpha_{01} & \alpha_{11} \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{q}_1 \end{bmatrix} = \begin{bmatrix} T - 2\dot{\theta}\alpha_{11}q_1\dot{q}_1 \\ -H_1 q_1 + \alpha_{11}q_1\dot{\theta}^2 \end{bmatrix} \tag{10}$$
$$y = \theta$$

where $\alpha_{00} = J + ML^2 + \alpha_{11}q_1^2$,T is the motor's shaft torque, J is the moment of inertia, $\alpha_{01} = \omega_1 + ML\varphi_{1e}$, $\alpha_{11} = v_1 + ML\ + \varphi_{1e}^2$, $H_1 = EI\int_0^L \ddot{\varphi}_1^2 dx_1, \varphi_{1e} = \varphi_1(L), \omega_1 = \rho a\int_0^L x_1\varphi_1 dx_1$, $v_1 = \rho a\int_0^L \varphi_1^2 dx_1$, a is the area of the cross section, ρ is the density, and y is the observation of θ. Defining the state variables such that $x_1 = \theta$, $x_2 = \dot{\theta}$, $x_3 = q_1$, $x_4 = \dot{q}_1$,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ f_1(x_2, x_3, x_4) \\ x_4 \\ f_2(x_2, x_3, x_4) \end{bmatrix} + \begin{bmatrix} 0 \\ b_1 \\ 0 \\ b_2 \end{bmatrix} T \tag{11}$$

where

$$f_1(x_2, x_3, x_4) = \frac{1}{\alpha_{00}\alpha_{11} - \alpha_{01}^2}$$
$$\cdot\left[-2\alpha_{11}^2 x_2 x_2 x_4 - \alpha_{01}(-H_1 x_3 + \alpha_{11}x_3 x_2^2)\right]$$
$$f_2(x_2, x_3, x_4) = \frac{1}{\alpha_{00}\alpha_{11} - \alpha_{01}^2}$$
$$\cdot\left[2\alpha_{01}\alpha_{11}^2 x_2 x_2 x_4 - \alpha_{00}(-H_1 x_3 + \alpha_{11}x_3 x_2^2)\right]$$
$$b_1 = \frac{\alpha_{11}}{\alpha_{00}\alpha_{11} - \alpha_{01}^2}$$
$$b_2 = \frac{\alpha_{01}}{\alpha_{00}\alpha_{11} - \alpha_{01}^2} .$$

The equation of motion of the cantilever beam for free vibration is written as

$$EIL\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = 0. \tag{12}$$

The boundary conditions are

$$y(0,t) = 0 \tag{13}$$
$$\frac{dy}{dx}(0,t) = 0 \tag{14}$$

$$\frac{d^2 y}{dx^2}(L,t) = 0 \quad (15)$$

$$EI\,\frac{d^3 y}{dx^3}(L,t) = m\,\frac{d^2 y}{dt^2}(L,t). \quad (16)$$

3. Proposed Modified SPSA Algorithm

The second order of simultaneous perturbation stochastic approximation (2SPSA) algorithm provide two general recursions for the estimate $(\hat{\theta}_k)$ of a solution θ^* having a dimension *p,* this is written as [7]

$$\hat{\theta}_{k+1} = \hat{\theta}_k - \bar{a}_k \bar{\bar{H}}_k^{-1} \hat{g}_k(\hat{\theta}_k)\ \bar{\bar{H}}_k = f_k(\bar{H}_k) \quad (17a)$$

$$\bar{H}_k = \frac{k}{k+1}\bar{H}_{k-1} + \frac{1}{k+1}\hat{H}_k \quad k=0,1,2 \quad (17b)$$

where $\bar{a}_k$ is a scalar gain that satisfies stochastic approximations (SA) conditions [7], $\hat{g}_k$ is the simultaneous perturbation (SP) that estimates the loss function gradient using c_k (the perturbation vector defined in [7]), $\hat{H}_k$ is the estimates of the Hessian matrix, and f_k maps a usual non-positive-definite $\bar{H}_k$ to a positive-definite *p*x*p* matrix. Let Λ_k be a user-generated mean-zero random vector of dimension *p* with its components being independent random variables, the gradient g_k is obtained by one-side approximations (in order to limit the number of function evaluations). We suggest the following approach that eliminates the non-positive definiteness while preserving key spectral properties of $\bar{H}_k$. First, we compute the eigenvalues of $\bar{H}_k$ and sort them into descending order:

$$\Lambda_k = diag[\lambda_1, \lambda_2, \ldots \lambda_{q-1}, \lambda_q, \lambda_{q+1}, \ldots \lambda_p] \quad (18)$$

where $\lambda_q > 0$ and $\lambda_{q+1} \le 0$. Next, we assume that the negative eigenvalues will not lead to a physically meaningful solution. They are either caused by errors in $\bar{H}_k$ or are due to the fact that the iteration has not reached the neighborhood of θ^* where the loss function is locally quadratic [8]. Therefore, we replace them together with the smallest positive eigenvalue with a descending series of positive eigenvalues:

$$\hat{\lambda}_q = \varepsilon\lambda_{q-1}, \hat{\lambda}_{q+1} = \varepsilon\hat{\lambda}_q, \ldots, \hat{\lambda}_p = \varepsilon\hat{\lambda}_{p-1} \quad (19)$$

where the adjustable parameter $0 < \varepsilon < 1$ can be specified based on the existing positive eigenvalues [8]:

$$\varepsilon = (\lambda_{q-1} / \lambda_1)^{q-2} \quad (20)$$

The purpose of having the smallest positive eigenvalue (λ_q) redefined is to avoid its possible near-zero values that would make the mapping matrix $\bar{H}_k$ nearly singular. Since $\bar{H}_k$ is symmetric, it is orthogonally similar to the real diagonal matrix of its real eigenvalues [8].

$$\bar{H}_k = P_k \Lambda_k P_k^T \quad (21)$$

where, the orthogonal matrix P_k consists of all eigenvectors of $\bar{H}_k$ which are usually derived together with the eigenvalues. Now, the mapping f_k can be expressed as

$$f_k(\bar{H}_k) = P_k \hat{\Lambda}_k P_k^T \quad (22)$$

where $\hat{\Lambda}_k$ be the diagonal matrix of Λ_k, the 2SPSA algorithm based on the mapping (22) makes the procedure of eliminating the non-positive definiteness of $\bar{H}_k$ a precise one. In this part, we use the Fisher information matrix $F(n)$ instead of the Hessian matrix $\bar{H}_k$ in order to keep and guarantee the estimation matrix be positive definite [8]. This Fisher information matrix provides a summary of the amount of information in the data relative to the quantities of interest. The essence of the method is to produce a large number of efficient "almost unbiased" estimates of the Hessian matrix, and then average the negatives of these estimates to obtain an approximation of $F(n)$. In these estimates, the gain series at each iteration are determined using the 2SPSA algorithm (17a) by replacing $\hat{\Lambda}_k$ mapping f_k of (22) with $\hat{\Lambda}_k$ contains constant diagonal elements

$$\hat{\theta}_{k+1} = \hat{\theta}_k - \bar{a}_k \bar{\lambda}_k^{-1} \hat{g}_k(\hat{\theta}_k) \quad (23)$$

where $\bar{\lambda}_k$ is the geometric mean of all the eigenvalues of $F(n)$

$$\overline{\lambda}_k = (\lambda_1 \lambda_2 ... \lambda_{q-1} \hat{\lambda}_q \hat{\lambda}_{q+1} ... \hat{\lambda}_p)^{1/p}. \quad (24)$$

Recursions (23) and (17b) together with (18)–(20) and (24) form a modified version of the 2SPSA (called the 3SPSA) that takes the advantages of both the well-conditioned SPSA and the internally determined gain sequence of the 2SPSA. The unknown parameters of non-linear observer are estimated by minimizing the following maximum likelihood cost function taking into account the measurement noise [9]:

$$\min_{\theta} J_m(\theta) = \min_{\theta} \frac{1}{2} \sum_{i=1}^{N} (y_i - \hat{x}_i(\theta))^T Q_i^{-1} (y_i - \hat{x}_i(\theta))$$

where y_i, Q_i and $\hat{x}_i(\theta)$ are the measurement vector, the measurement error matrix and the state estimates obtained from the non-linear observer system, respectively [9]. N is the number of iterations for estimating the parameters. This algorithm updates the estimates using the following procedure:

(S.1) The output to be identified is observed with respect to a particular input.

(S.2) Perturbation is added to all the parameters in the estimation vector.

(S.3) The error function is calculated.

(S.4) The amount of correction is calculated and the estimation parameters are updated.

(S.5) Return to S.1.

4. Design of Non-linear Observer

Since only the motor angle x_1 is the measurable state variable, the remaining states x_2, x_3 and x_4 are predicted using intelligent state observer design [10]. For this, (10) is written as

$$\dot{x} = f(x) + g(x)T \quad (25)$$

$$y = Cx$$

$$C = [1 \quad 0 \quad 0 \quad 0]. \quad (26)$$

For this non-linear system, we consider a robust VSS observer, which predicts system states. This is defined as

$$\dot{\hat{x}} = f(\hat{x}) + g(\hat{x})T + M(\hat{y}) + K(\hat{y} - y) \quad (27)$$

$$\hat{y} = C\hat{x} \quad (28)$$

$$M(\hat{y}) = g(x) \frac{\hat{y}}{\|\overline{y}\| + \gamma} \varsigma \quad (29)$$

$$\overline{y} = \hat{y} - y = C(\hat{x} - x) \quad (30)$$

where $\hat{x}$ represents the predicted value of system state as in [10], K is the observer gain matrix, $M(\overline{y})$ is the observer non-linearity term, ς represents the gain and γ >0 is an averaging constant for removing chattering. Now defining the estimation error as

$$e = \hat{x} - x \quad (31)$$

we have

$$\dot{e} = f(\hat{x}) - f(x) + [g(\hat{x}) - g(x)]T + KC(\hat{x} - x) + M(\overline{y}). \quad (32)$$

For the evaluation of the observer gain K with x_d as the desired point, using the Taylor series expansion and its first order approximation, the error system is given as

$$\dot{e} = [f'(x_d) + g'(x_d)T + KC]e + M(\overline{y}) = A_0 e + M(\overline{y}) \quad (33)$$

where

$$A_0 = A + GT + KC \quad (34)$$

$$A = \frac{\partial f_i}{\partial x_j} \quad (35)$$

$$G = \frac{\partial g_i}{\partial x_j} \quad (i,j=1,2,3,4). \quad (36)$$

Choosing a Lyapunov function of e as

$$V = \frac{1}{2} e^2 \quad (37)$$

and integrating V with respect to e yields

$$\dot{V} = e\dot{e} = e^2 (A_0 - |g(x)| C \frac{1}{C\|e\| + \gamma} \varsigma). \quad (38)$$

If K is designed such that the eigenvalues of error system (34) are all negatives, then selection of $A_0 - g(x)\varsigma < 0$ yields $\dot{V} < 0$ and the Lyapunov's stability theory gives $e(t) \rightarrow 0$ as $t \rightarrow \infty$. In the simulation, we chose $x_d = [0.2 \quad 0 \quad 0 \quad 0]$ and computed A and G with the observer parameters determined with SPSA(see sec. 3). Therefore, as to ensure the stability of (39) minimizing the following evaluation parameters:

$$J_0 = \sum (y - \hat{y})^2 \quad (39)$$

where, K=[-339 -19002 15.2020 -10109], ς =0.012 , γ =0.013. We can get these values more easily using the SPSA algorithm, because this has less computational complexity [9].

5. Model Reference Sliding Mode Controller

In this section, the main purpose of this kind of control is to make the state converge to the sliding mode surface. Therefore, we choose the desired response based on a second order reference model given as [10]

$$\begin{bmatrix} \dot{x}_m \\ \ddot{x}_m \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\omega_n \end{bmatrix} \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} U_m \quad (40)$$

where ω_n is the eigenvalue of angular frequency and U_m is the model input.

The following sliding mode hyper-plane for (10) is defined by:

$$\sigma = s_1(x_1 - x_m) + s_2(x_2 - \dot{x}_m) + s_3 x_3 + s_4 x_4 \quad (41)$$

where all the state variables can be predicted by the observer. When the sliding mode is in operation, then

$$\sigma = 0 \quad (42)$$

$$\dot{\sigma} = 0. \quad (43)$$

The equivalent control input can be obtained by substituting (10) into (43). This gives

$$T_{eq} = 2\alpha_{11} x_2 x_3 x_4 + \frac{\alpha_{01}}{\alpha_{11}}(-H_1 x_3 + \alpha_{11} x_2^2 x_3) - \frac{\Delta}{s_2}\left[s_1(x_2 - \dot{x}_m) - s_2 \ddot{x}_m + s_3 x_4 + s_4 \dot{x}_4\right] \quad (44)$$

where it can be assumed that $\Delta = (\alpha_{00} - \alpha_{01}^2 / \alpha_{11}) > 0$.

Now, we consider the design of sliding mode controller (SMC), which in the non-linear input to make the state converging in the hyper-plane. Assuming the equivalent control input T_{eq} and non-linear control input T_ℓ [10], we have

$$T = T_{eq} + T_\ell = T_{eq} - k(x,t)\text{sat}(\sigma) \quad (45)$$

where

$$\text{sat}(\sigma) = \begin{cases} 1 & if \quad \sigma > \delta \\ \dfrac{\sigma}{\delta} & if \quad |\sigma| \le \delta \\ -1 & if \quad \sigma < -\delta \end{cases} \quad (46)$$

and $k(x,t)$ is the control input function. δ is a constant to eliminate the chattering. The condition for realization of the sliding mode is obtained from the Lyapunov function. We choose a Lyapunov function of σ to confirm $\sigma = 0$:

$$V = \frac{1}{2}\sigma^2. \quad (47)$$

$\dot{V}$ is given by

$$\dot{V} = \sigma\dot{\sigma} = \sigma\left\{ \begin{array}{l} \dfrac{s_2}{\Delta}\left[T - 2\alpha_{11}x_2x_3x_4 - \dfrac{\alpha_{01}}{\alpha_{11}}\begin{pmatrix} -H_1x_3 + \\ \alpha_{11}x_2^2x_3 \end{pmatrix} \right] \\ + s_1(x_2 - \dot{x}_m) - s_2\ddot{x}_m + s_3x_4 + s_4\dot{x}_4 \end{array} \right\} \quad (48)$$

Substituting (45) into (48), we have

$$\dot{V} = \sigma\left\{ -\frac{s_2}{\Delta} k(x,t)\text{sgn}(\sigma) \right\} = -k(x,t)\frac{s_2}{\Delta}|\sigma| < 0. \quad (49)$$

Since $\frac{s_2}{\Delta} > 0$ if we choose $k(x,t) > 0$, then the state variable x will converge in the slide mode hyper-plane. The controller gains are determined using the SPSA algorithm (see sec. 3) so as to minimize the cost function [10]

$$J_h = \sum [|L \cdot (x_1 - x_m)| + |x_3|]. \quad (50)$$

The parameters values are $s_1 = 3.4$, s_2 =2, s_3 =11.23, s_4 =-0.58 and δ = 0.43, $k(x,t)$=3.45.

6. Simulation Results

In this study, a sliding mode based controller is designed to achieve the end-point tracking of a flexible arm. These results are compared with previous simulations without the proposed algorithm [10]. The numerical values used are as follows: Δt =0.1[ms], M=0.026[kg], J=0.0013823[kg$\cdot m^2$], m=0.028[kg], $a\rho$ =0.0648[kg/m], EI=0.09088[Nm^2], L=0.4[m], x_0 =[-0.1 0 0 0]^T and x_d =[0.1 0 0 0]^T. Figure 2 shows typical responses of the system at the tip position. This position is a little over-damped with practically no overshoot.

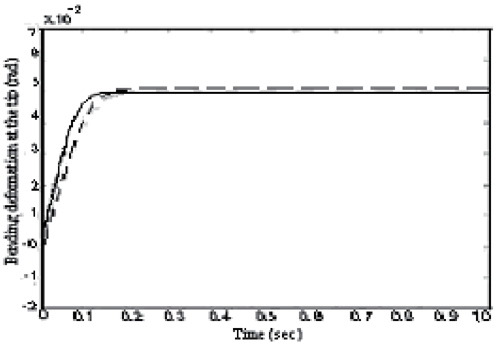

Fig.2 Bending deformation. Without SPSA algorithm (dashed line (- -)). With SPSA algorithm and non-linear observer (solid line (-)).

Fig.3 shows the tip position. The non-linear observer and the sliding mode control provides a stable operation.

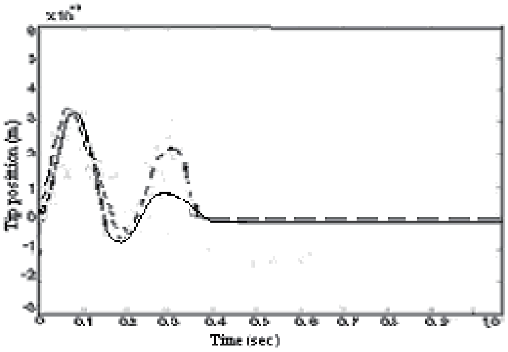

Fig.3 Tip position. Without SPSA algorithm (dashed line(- -)). With SPSA algorithm and non-linear observer (solid line (-)).

Fig.4 shows the tip velocity. The algorithm proposed reduce the magnitude of velocity to a small value.

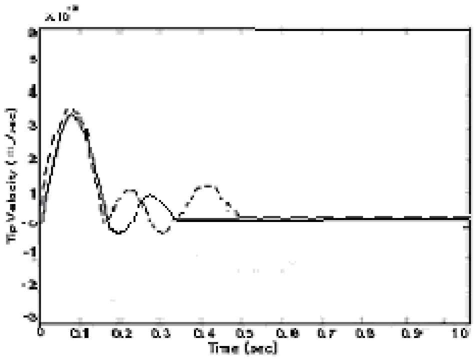

Fig.4 Tip Velocity. Without SPSA algorithm (dashed line(- -)). With SPSA algorithm non-linear observer (solid line (-)).

7. Conclusion

We have proposed a MR-SMC method using a non-linear observer for controlling the angular position of a flexible arm, suppressing its oscillation. We also have proposed the use of the SPSA algorithm for the estimation of the observer gains and the parameters in the sliding hyper-plane. The SPSA has very low computational complexity for solving difficult estimation problems in an efficient way, such as the observer gains. The non-linear observer was successful in predicting the state variables from the motor angular position. The simulation results motivate a real implementation of the proposed method with a flexible arm. The effectiveness of the proposed algorithm was verified by the simulation results and comparison with the previously algorithms under the same conditions.

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