Incorporating Within-Day Transitions in the Simultaneous Off-line Estimation of Dynamic Origin-Destination Flows without Assignment Matrices

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ABSTRACT

An off-line methodology for simultaneously estimating dynamic origin-destination matrices without using assignment matrices that incorporates within day transition equations is presented. The proposed formulation and solution approach extend a calibration method recently developed that directly uses the output of any network loading model (such as a dynamic traffic assignment or simulation model) so that the complex relationships between OD flows and model outputs are accurately captured (as opposed to the more common method of approximate linear relationships based on the assignment matrix). The paper extends the original formulation by incorporating spatial and temporal relationships among various OD flows (transition equations). These transition equations link OD flow variables across time intervals in such a way that known structural demand patterns can be preserved in the new estimates. Such transition equations, while common in the context of real-time OD flows, complicate the off-line simultaneous estimation of OD flows and have not been used to their full potential in the past. The approach is demonstrated through a case study.

INTRODUCTION

The transportation field is seeing the widespread use of dynamic models in various planning, analysis, evaluation, operations, and management applications. Dynamic traffic assignment (DTA) and high-fidelity microscopic simulation tools are becoming popular for the analysis and evaluation of a variety of intelligent transportation systems (ITS) on large networks. While such tools are based on complex models that capture the real world accurately, they require several inputs and parameters that must be representative of the region in which the tools will be deployed.

Time-dependent origin-destination (OD) flows are one such critical input to most dynamic traffic models, and capture the spatial and temporal distribution of travel demand. However, this demand is not directly observable, and must be inferred from indirect measurements such as time-varying traffic counts.

The problem of estimating OD flows from aggregate, sensor data has received considerable attention in the literature. The basic principle is to infer multiple OD matrices $\mathbf{x}_h$, $h = 1,2,\ldots,H$, using sensor counts recorded at the end of each interval *h*. The problem has been addressed in the context of both real time and off-line applications. The majority of the approaches are based on formulations that use assignment matrices to map the OD flow variables to link counts. Each assignment matrix $\mathbf{a}_h^p$ captures the contribution of OD departures in interval *p* (denoted by the matrix $\mathbf{x}_p$) to the counts (say $\mathbf{y}_h$) measured at the end of interval *h*. These temporal linkages between many intervals are critical on large, congested networks, since the counts $\mathbf{y}_h$ contain information about several OD matrices from past departure intervals. However, due to the computationally expensive solution approach involving the inversion of very large matrices (each consisting of several large assignment matrices), the OD estimation problem has typically been solved sequentially. (*1*) shows the complexity of the simultaneous approach even for off-line calibration applications on small to medium-sized networks. In the sequential approach, the flows departing in all prior intervals $p < h$ are fixed at their best known values ($\hat{\mathbf{x}}_p$) while $\mathbf{x}_h$ is estimated. The assignment matrix based approaches have other limitations, such as the need for solving expensive fixed-point problems and being limited only to count data.

A recent off-line OD estimation methodology (*2*, *3*) provides a way of simultaneously and efficiently estimating the OD flows for many departure time intervals, without using assignment matrices. The method has been shown to be superior to previous approaches (see *2*, for example) and also scales well to large problems. Further, the methodology can be applied to any network loading model (microscopic, macroscopic or mesoscopic). However, the use of transition equations that capture known temporal and spatial relationships among the OD flows has not been used with the above approach. In general, while transition equations have proved to be an important component of the state-space formulation and solution of the sequential OD estimation problem in real time, they have not been fully explored in off-line applications.

The objective of this paper is to extend the formulation of the off-line simultaneous dynamic OD estimation problem proposed by (*2*, *3*) with the addition of transition equations and examine their role, through a case study. The paper is organized as follows. The main existing approaches for OD estimation are briefly reviewed, and a general formulation and solution approach with the incorporation of transition equations are presented. A case study examines the value of including these relationships and their impact on the accuracy of the estimated OD flows. The final section concludes the paper.

LITERATURE REVIEW

Initial attempts at OD estimation using vehicle count data focused on identifying a matrix of *static* OD flows (see for example *4-8*). These approaches estimate a matrix of average OD flows (assumed to be constant over a significant time period such as the entire morning peak), based on counts collected for the same period on a subset of network links.

Estimation of dynamic OD flows assumes that the study period of interest is divided into multiple time intervals, $h = 1,2,\ldots,H$. A matrix $\mathbf{x}_h$ of trips departing during interval h is estimated based on counts $\mathbf{y}_h$ measured during interval h and beyond. The counts $\mathbf{y}_h$ present indirect measurements of the OD flows that departed during interval h and earlier. The relationship between $\mathbf{y}_h$ and the unknown OD flows $\mathbf{x}_h$ is expressed by a function $f(\ )$ which provides the mapping between OD flows and counts. In general, $f(\ )$ is a complex function of the OD demand, driver route choice decisions and the resulting network travel times. It is typically approximated with a linear assignment matrix mapping with the following structure:

$$\mathbf{y}_h = \sum_{p=h-p'}^{h} \mathbf{a}_h^p \mathbf{x}_p + \mathbf{v}_h \qquad (1)$$

where,

- $\mathbf{y}_h$: vector of link sensor counts for interval h;
- $\mathbf{x}_p$: vector of OD flows departing their origins during interval p;
- $\mathbf{a}_h^p$: assignment matrix;
- $\mathbf{v}_h$: error vector;
- p': number of intervals spanning the longest trip.

However, the number of equations that map the OD flows to sensor counts in most practical problems is much smaller than the number of OD flows (due to the sparse coverage of sensors in real networks). Hence, the problem of estimating dynamic OD flows from sensor measurements requires additional equations. These equations may be provided by estimates of OD matrices obtained from surveys, prior days, intervals, etc. Cascetta et al. (*9*), for example, recommend a simple scheme in which the target flows for interval h are the estimates for interval h-1:

$$\mathbf{x}_h^a = \hat{\mathbf{x}}_{h-1} \qquad (2)$$

Based on the above they propose a statistical updating framework in which target OD flows are adjusted to better replicate the observed count data. An optimization step is performed, subject to non-negativity constraints on the OD flows:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \left[z_1\left(\mathbf{x}, \mathbf{x}^a\right) + z_2\left(\mathbf{y} - \mathbf{A}\mathbf{x}\right) \right] \tag{3}$$

$\mathbf{x} = \left[\mathbf{x}_1 \cdots \mathbf{x}_h \cdots \mathbf{x}_H\right]$ is the matrix of OD vectors to be estimated; $\hat{\mathbf{x}}$ are the required optimal estimates; $\mathbf{x}^a$ are target OD flows; $\mathbf{y} = \left[\mathbf{y}_1 \cdots \mathbf{y}_h \cdots \mathbf{y}_H\right]$ are traffic counts; z_1 and z_2 are goodness of fit measures (e.g. least squares); and **A** is the assignment matrix. The resulting problem was solved sequentially with a generalized least squares approach (*5, 9*).

Ashok and Ben-Akiva (*10*) develop a state-space model for the real-time estimation of OD flows. This approach uses a measurement equation similar to Equation (1) together with a transition equation that captures the evolution of system *state*:

$$\mathbf{x}_h - \mathbf{x}_h^H = \sum_{p=h-q'}^{h} \mathbf{f}_h^p \left(\mathbf{x}_p - \mathbf{x}_p^H\right) + \mathbf{w}_h \tag{4}$$

A key innovation in the above equation is the definition of state in terms of *deviations* of OD flows $\mathbf{x}_h$ from their historical values $\mathbf{x}_h^H$. The deviations help in efficiently including all information in prior intervals while estimating flows for the current interval *h*. The model is solved recursively using a Kalman Filter, with historical flows initialized through a least squares procedure.

Ashok (*11*) presents an off-line extension to the real-time estimator, in which the sequential OD estimates in a forward pass of the Kalman Filter are "smoothed" during a backtracking step. The updated OD flows for interval *h* thus reflect count information from future intervals. Kang (*12*) and Zhou and Mahmassani (*13*) use a similar state-space approach and Kalman Filter solution algorithm together with polynomial transition equations to model day-to-day structural deviations in OD flows. It should be noted that these off-line applications of dynamic OD estimation have largely been adaptations of the methods first developed for the on-line context.

Both types of approaches outlined above require knowledge of the assignment matrices $\mathbf{a}_h^p$. Assignment fractions can be calculated easily under free-flow conditions, or when travel times on all links are known (*14*). Assignment matrices may also be computed within the traffic simulation model, by tracking vehicle trajectories or by measuring the simulated travel times from each demand origin node to every sensor location (see *11* for details). However, if the network is congested, the assignment fractions become functions of prevailing travel times, which depend on the OD flows that are yet to be determined. As a result, this OD estimation formulation requires the solution of a fixed-point problem.

Cascetta and Postorino (*15*) present various algorithmic approaches to solve for a fixed point in the static context. They search for consistent OD flows and assignment fractions by iterating between the OD estimation step and a network loading method (such as the one in a DTA model). The iterative nature of the solution, while still a heuristic, also possesses high computational overhead. Ben-Akiva et. al. (*16*) use an iterative scheme to solve the fixed point problem for the real-time dynamic OD estimation case.

Assignment matrix-based dynamic OD estimation formulations have a number of shortcomings:

- The assignment matrix is a linear approximation of the complex relationships between OD flows and sensor counts, which might be valid close to the optimal OD flows. In general, though, an iterative fixed-point problem must be solved. Existing solution approaches are generally based on heuristics.
- The computational effort of simultaneously estimating OD flows for multiple time intervals has been found to be prohibitive even on medium-sized networks, since it involves the calculation, storage and inversion of a large, augmented assignment matrix (see *1*, *17-18*). Sequential OD estimation (see for *9* for example) has therefore been used as an approximation that fixes the OD flows departing in all prior intervals. However counts from future intervals are not used to refine these past estimates, which can affect the accuracy of OD estimates on large, congested networks with trips spanning many intervals.
- Relationships between general data (such as speeds and densities) and OD flows are expected to be non-linear, and approximations similar to the assignment matrix cannot be justified. The assignment matrix formulation thus restricts OD estimation to the use of count data alone, which can potentially over-fit to counts at the expense of traffic dynamics.
- The assignment matrix excludes the estimation of other related parameters such as route choice or supply models, together with OD flows. These parameters can significantly impact the calculation of the assignment matrix.

While traffic simulation has been used before for dynamic OD estimation, the problem has generally been solved sequentially. Other recent papers on simultaneous estimation have used algorithms (such as evolutionary searches) that do not scale even to medium-sized problems (*19*). Further, the use of the assignment matrix goes beyond just presentation: it is an approximation of non-linear relationships, and results in a much easier (though less accurate) linear problem. It also constrains the modeler to estimate only OD flows (supply parameters, for example, cannot be included easily) using only count data (speeds, travel times, etc are much harder to incorporate).

A recent development in the off-line estimation of OD flows that relaxes the dependence of the solution on the assignment matrix and overcomes many of the above limitations is reported in (*3, 20*). The method does not use assignment matrices, instead using the output of any network loading model (for example a dynamic traffic assignment or a microscopic traffic simulation model) to directly capture the complex relationships between OD flows and any available data (not necessarily confined to traditional loop detector counts). Confirmed incidents, if available in the archived sensor data, may be included while running the simulation model. Thus the impact of network disruptions can be explicitly modeled (and the capacity reduction effects estimated, since the methodology is not specific to just OD flows). This aspect was demonstrated in (*20*). Furthermore, the proposed solution approach, based on the SPSA algorithm (*21*-*22*), can solve for multiple intervals simultaneously. The methodology has been shown to be superior to the assignment matrix approach (*2*), and to scale well to large problems. Finally, the approach allows for other relevant effects (such as route choice and supply parameters) to be estimated simultaneously with the OD flows. Our approach is thus more general and flexible, and solves the original problem without linearizing any of the complex relationships.

However, the formulation proposed by (*3*) does not include any transition equations. Hence it does not incorporate the additional information that these equations provide. As noted earlier, transition equations are commonly used in the context of the real-time and on-line OD estimation problem, which takes place sequentially (the results from previous intervals are used as inputs to the estimation in the current interval, as in a filtering problem). They are also relatively easy to incorporate in the off-line problem when the solution approach is sequential. In this paper, we focus on extending the previous off-line, simultaneous OD estimation methodology by including transition equations.

PROBLEM FORMULATION

The problem to be addressed is one of simultaneously estimating (off-line) dynamic OD flows given aggregate sensor measurements and transition equations that capture the evolution of OD flows from one interval to the next. The available data that are used for calibration are related to the unknown OD flows that need to be estimated, via measurement equations. The measurement equations represent both direct and indirect relationships between the data and the unknown OD flows. Aggregate sensor data are *indirect measurements*:

$$\mathbf{M} = \mathcal{M} + \mathbf{u} \qquad (5)$$

M are the data (as measured by the sensors), M are the corresponding outputs from network loading model, and **u** is an error vector. They are indirect measurements because they map the unknown OD flows to the observed data (through the DTA model). Practically any type of available traffic measurements can be used to develop *indirect* measurement equations.

A priori OD estimates, if available, contain valuable structural information about the calibration variables and represent *direct measurements*:

$$\mathbf{x}^{a} = \mathbf{x} + \mathbf{v} \qquad (6)$$

x is the vector of OD flows to be calibrated; $\mathbf{x}^{a}$ are the corresponding *a priori* values, and $\mathbf{v}$ is the error vector. In the original formulation (*3*) it was assumed that $\mathbf{x}_{h}^{a} = \mathbf{x}_{h}^{H}$.

However, other relationships can be used, in particular in the form of transition equations that capture the evolution of the OD flows over time and space, providing additional information that should be exploited in the estimation of the OD flows. Such relationships have been used successfully in the context of real time OD estimation (*10, 13*)

A number of transition equations have been proposed in the literature (mainly in the context of real time OD estimation), in addition to the ones already mentioned in the previous section. Ashok (*11*) proposes several formulations to develop a-priori OD flows and extends the relationship used by (*9*) with the hypothesis that the interval-over-interval flows are stable from day to day:

$$\mathbf{x}_h^{a,d} = \left(\frac{\hat{\mathbf{x}}_h^{d-1}}{\hat{\mathbf{x}}_{h-1}^{d-1}} \right) \hat{\mathbf{x}}_{h-1}^{d} \tag{7}$$

where $\mathbf{x}_h^{a,d}$ is the desired target OD matrix for interval *h* on day *d*; $\hat{\mathbf{x}}_h^{d-1}$ and $\hat{\mathbf{x}}_{h-1}^{d-1}$ are the estimated OD matrices for departure intervals *h* and *h*-1 on the previous day *d*-1; and $\hat{\mathbf{x}}_{h-1}^{d}$ is the OD matrix estimated for interval *h*-1 on the current day *d*.

Ashok and Ben-Akiva (*10*) propose an autoregressive (AR) process to connect the deviations of OD flows from their historical (expected) values, across multiple time intervals.

$$\mathbf{x}_h^a = \mathbf{x}_h^H + \sum_{p=h-p'}^{h-1} \mathbf{f}_h^p \left(\hat{\mathbf{x}}_p - \mathbf{x}_p^H \right) \tag{8}$$

where,

$\mathbf{x}_h^a$: target OD matrix;

$\mathbf{x}_h^H$: the corresponding historical flows;

$\mathbf{f}_h^p$: a transition matrix that captures the relationship between OD deviations in interval *p* and those in interval *h*. The longest trip on the network spans *p*' intervals.

Zhou and Mahmassani (*13*) replace the AR process with a higher-order polynomial function.

The transition schemes outlined above are all in the context of sequential OD estimation. The target flows for each interval are therefore computed once, and remain constant thereafter (since the OD flows in prior departure intervals are always constant). When the OD matrices are estimated simultaneously, the target flows become functions of the estimation variables themselves. This aspect therefore makes the problem of simultaneous estimation of dynamic OD flows more complicated and computationally involved.

The formulation that follows extends the simultaneous dynamic OD estimation approach presented in (*2, 3, 23*) and allows the inclusion of within-interval transitions in simultaneous dynamic OD estimation.

Formulation

It is hypothesized that the target flows for interval *h* are a function of the flows in past intervals:

$$\mathbf{x}_h^a = \mathbf{x}_h^H + g\left[(\mathbf{x}_1 - \mathbf{x}_1^H),(\mathbf{x}_2 - \mathbf{x}_2^H),\ldots,(\mathbf{x}_{h-1} - \mathbf{x}_{h-1}^H);\boldsymbol{\alpha}\right] \tag{9}$$

$\boldsymbol{\alpha}$ is a vector of parameters assumed to be known *a priori*. Note that the above equation is based on the deviations of OD flows from their historical values (as introduced in *10*) and allows for the incorporation of all prior information available.

If historical flows are not available, then the target flows may be represented as:

$$\mathbf{x}_h^a = g\left(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{h-1};\boldsymbol{\alpha}\right) \tag{10}$$

Several linear function forms for such transition equations have been proposed in the literature in the context of on-line OD updating (*10-13*). The OD estimation problem is thus governed by Equations (5) and (6), and transition equations, for example of the form of (9) or (10). The OD estimates can be obtained as the optimal solution of the following generalized least squares (GLS) problem (assuming that Equation (10) is used):

$$\begin{aligned}\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_H = \arg\min z = \sum_{h=1}^{H} \Big[& \left(\mathbf{M}_h - \mathcal{M}_h\right)^T \mathbf{R}_h^{-1}\left(\mathbf{M}_h - \mathcal{M}_h\right) + \\ & \left(\mathbf{x}_h - g\left(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{h-1};\boldsymbol{\alpha}\right)\right)^T \mathbf{Q}_h^{-1}\left(\mathbf{x}_h - g\left(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{h-1};\boldsymbol{\alpha}\right)\right)\Big]\end{aligned} \tag{11}$$

GLS attempts to minimize the squared deviation of the estimated quantities from their observed (or target) values. It should be noted that M_h is the output of a network loading model. $\mathbf{R}_h$ and $\mathbf{Q}_h$ represent the variance-covariance matrices of the error terms $\mathbf{u}_h$ and $\mathbf{v}_h$. If they are assumed to be diagonal, the inverse of their variance terms may be interpreted as the relative weights between the various direct and indirect measurements. A high-variance measurement error, for example, would result in a low weight reflecting a lower confidence in the accuracy of the corresponding measurement. Correlations between measurements can also be captured by specifying off-diagonal terms in the matrices. A detailed analysis on the estimation of transition parameters and variance-covariance terms is presented in (*11*, *24*).

Solution algorithm

Equation (11) represents a complex, non-linear, non-analytical optimization problem, since a sophisticated simulator would typically be used to obtain the fitted measurements M_h. The high degree of non-linearity introduces an objective function with potentially many local optima. The local minimum closest to the starting solution may thus be far from a global optimum. The non-analytical nature is attributed to the lack of an explicit form for *f*( ) as a function of the calibration variables. Consequently, classical algorithms that rely on the knowledge of exact analytical gradients are not suitable for calibration. Methods that work directly with function values are therefore more appropriate in the context of the current application.

When the simulator is stochastic (as is often the case), the optimization problem must also account for the inherent noise in model outputs. Further, the problem is large in scale, with the number of OD pairs and time intervals increasing rapidly with the size of the network and the desired temporal modeling resolution. Consequently, appropriate solution algorithms must be able to perform a global search by overcoming local hills and valleys, work without analytical derivatives (which would generally be unavailable) and converge in a reasonable time frame that does not grow rapidly with problem size. The SPSA algorithm (*3, 21-22*) has been used with success to solve the problem. The algorithm iterates between the following steps until a stable objective function $z(\theta^i)$ is realized at some iteration *i* (the term $\boldsymbol{\theta}$ includes all the *n* OD flow variables being estimated):

1. An approximate estimation of the gradient $\hat{g}(\boldsymbol{\theta}^i)$ of the function *z*( ), evaluated at $\boldsymbol{\theta}^i$:

$$\hat{g}(\boldsymbol{\theta}^i) = \frac{z(\boldsymbol{\theta}^i + c^i\Delta_i) - z(\boldsymbol{\theta}^i - c^i\Delta_i)}{2c^i}\begin{bmatrix}\Delta_{i1}^{-1}\\ \Delta_{i2}^{-1}\\ \vdots\\ \Delta_{in}^{-1}\end{bmatrix} \tag{12}$$

   where $c^i = c / i^\gamma$ and Δ_i is an *n*-dimensional perturbation vector for iteration *i*. The *n* perturbation components $\Delta_{i1}, \Delta_{i2}, \ldots, \Delta_{in}$ are realized through independent draws from the Bernoulli distribution, and can assume values of +1 and -1 with equal probability. The perturbations are also independent across iterations. The terms *c* and γ are algorithmic parameters.

2. A parameter update that yields $\boldsymbol{\theta}^{i+1}$:

$$\boldsymbol{\theta}^{i+1} = \boldsymbol{\theta}^i - a^i \quad \hat{g}(\boldsymbol{\theta}^i) \tag{13}$$

where $a^i = a/(A+i)^\alpha$ is a step size (*a*, *A* and α are algorithmic parameters). While the optimization problem is solved exactly once, the solution algorithm is iterative and adopts a path search based on stochastic approximations of the search direction at each iteration. Theoretical considerations and practical guidelines for the selection of algorithmic parameters *c*, γ, *a*, *A* and α are provided in (*21*). The paper also mentions the use of step sizes (*a* and *c*) that vary with the magnitudes of the corresponding calibration parameters.

Unlike FDSA (Finite Differences Stochastic Approximation, *25*), wherein each component of the gradient is estimated by perturbing only the corresponding component of $\boldsymbol{\theta}^i$, SPSA generates the complete gradient estimate from just 2 evaluations of *z*( ), yielding an *n*-fold improvement in running times mentioned earlier. (*22*) compares the savings and computational performance of SPSA and FDSA.

It should be noted that the transition equations, whose functional form and parameters are assumed to be known *a priori*, are directly embedded in the objective function. These equations provide extra information tying various optimization variables.

Thus for a given solution, the objective function may be evaluated by simply evaluating $z(\ )$ at the current solution, along with the corresponding simulated measurements.

CASE STUDY

The objective of the case study is to demonstrate the feasibility of the proposed formulation and examine the value of transition equations in estimating more accurate OD flows. A test network is used in the case study (Figure 1) consisting of 8 nodes and 8 links. Each link was divided into 3 segments in order to better represent the spatial and temporal evolution of congestion. Demand was assumed to flow between all three feasible OD pairs (connecting the three origin nodes O1, O2 and O3 to the destination node D). Travelers making trips between O1 and D could choose from two alternative paths, while the remaining two OD pairs were captive to a single path each. Link geometries reflect varying numbers of lanes, as indicated in Figure 1.

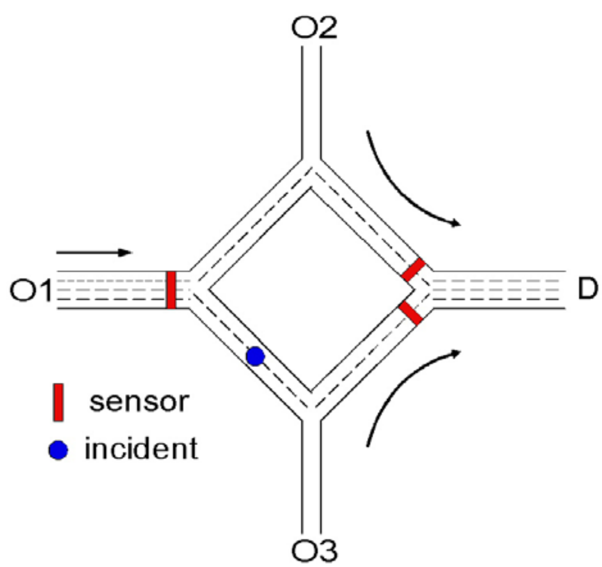

Figure 1: Test Network

The period of interest spanned 50 minutes, designated arbitrarily as 6:50 AM - 7:40 AM. This time period was further divided into uniform intervals, each of duration 5 minutes (so that the number of intervals $H = 10$). An incident was assumed to impact the available capacity on the network for a period of 10 minutes, beginning at 7:05 AM. The location of the incident is indicated in Figure 2. One out of two available lanes at the incident location was blocked while the incident was in effect. Drivers traveling between O1 and D who chose to travel through the affected link were likely to experience incident delays, depending on their departure times and prevailing traffic conditions.

MITSIMLab (*26*, *27*), a detailed microscopic traffic simulation laboratory, was used to represent the real world and the corresponding "true" state of the network. MITSIMLab simulates the movement of individual vehicles between various OD pairs on the "real" traffic network, through detailed models of driving behavior and route choice decision-making. The system also represents a variety of traffic management strategies, and captures the operations of traffic control systems and the impact of incidents on network supply.

MITSIMLab possesses several unique features (*28*) that motivate its use for the generation of synthetic sensor data for the test network. The system can accommodate a wide range of assumptions related to the generation of network demand, travel and driving behavior (and resulting traffic dynamics) and incidents. In addition, MITSIMLab replicates the operation of the surveillance (sensor) system, providing a flexible setting that can mimic traffic data under diverse demand and traffic situations.

Representative values for various demand and driving behavior parameters were selected as inputs to MITSIMLab in order to generate observed sensor measurements. These parameters, including those capturing driving behavior and vehicle mix, were originally developed from datasets collected from the Boston, MA region. The time-dependent *historical* profiles of the main OD flow (between nodes O1 and D) and the two side flows were selected so as to generate visible congestion due to the incident, while simultaneously capturing merging and weaving phenomena between the main flow and each of the minor flows. The actual demand levels realized between each OD pair on a particular day were drawn from a distribution with mean flows equal to the historical values (Figure 2). The spread of this distribution, which represents the temporal (within-day) variability in demand, was set to 25% around the mean. It was assumed that the corresponding noise was independent across time intervals.

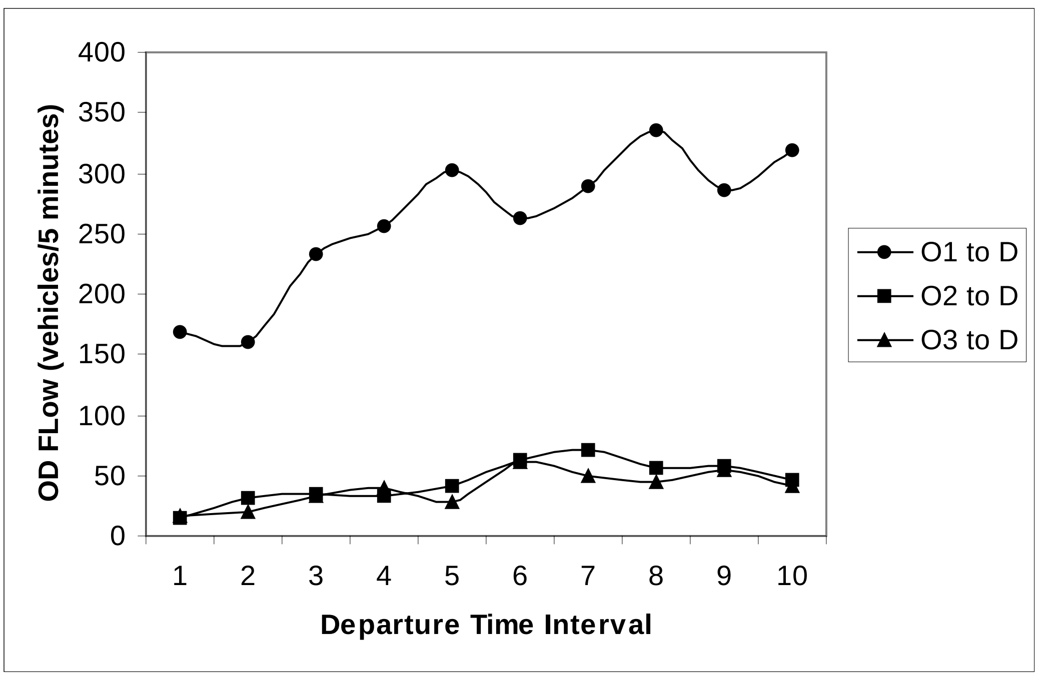

Figure 2. True OD Flows for the Three OD Pairs

Three link-wide traffic sensors (indicated in Figure 1 by rectangular boxes) provided count and speed observations from MITSIMLab. OD flows in 5-minute time intervals were estimated, yielding 30 flow variables. DynaMIT (Dynamic Network Assignment for the Management of Information to Travelers, *16*) was the mesoscopic DTA used as the network loading model, replacing M in Equation (5). DynaMIT explicitly models drivers' route and departure time choices and network supply (including capacities and traffic dynamics). When the input OD matrices are disaggregated to generate individual trips, each driver is allocated behavior characteristics (such as sensitivity to travel time) drawn from distributions. The demand and supply components interact to capture complex real-world phenomena such as queues and spillbacks. A detailed description of DynaMIT's DTA functionalities and algorithms are presented in (*16*). Since this paper focuses on OD estimation, all other parameters (such as capacities, route choice parameters and incident capacity) were obtained from earlier studies on the same dataset (*3*).

It should be pointed out that the modeling of traffic dynamics in MITSIMLab is very different than in DynaMIT. Since MITSIMLab uses detailed driving behavior models, capacities are not explicitly modeled, but are rather the result of simulated driver behavior and the geometric characteristics of the network. DynaMIT, however, uses aggregate, mesoscopic speed-density relationships, explicit capacities, and queuing models to represent traffic dynamics. On the demand side, the two systems can employ route choice models that vary in model structure, explanatory variables, and parameter values. Hence, MITSIMLab provides an unbiased laboratory to test the performance of the proposed methodology.

The experimental design involved the comparison of the following estimators:

A. No transitions, bad seed

Base case OD estimates were obtained by simultaneously estimating OD flows for all time intervals without the use of transition equations. A highly perturbed version of the true OD flows was used as target $\mathbf{x}_h^a$. Hence, the base case corresponds to the method proposed by (*2*), but applied with a "bad" historical matrix.

B. No transitions, good seed

In this case, the seed matrices serving as OD estimation targets were obtained by only slightly perturbing the true OD flows. The target flows were therefore closer to the true flows, though the errors were drawn at random.

C. Simple transition equations

The OD target for each interval was assumed to be the flows estimated for the previous interval, and is represented by Equation (2). The seed matrix for the first interval of estimator B was used for *h*=1.

D. True transition equations

Transition equations of the form of Equation (10), based on an autoregressive process of degree 3 estimated on the true OD flows, were included. Each OD flow was thus modeled as a linear function of the flows in the past three intervals. While every OD pair can have its own transition equation, a common equation was used in this case study. $\mathbf{x}_1^a$ was used as the target for the first interval (*h*=1), while all other targets were re-computed at each solution iteration. The seed matrix for the first interval of estimator B was used for h=1, and flows prior to h=1 were assumed to be zero.

The autoregressive process of degree 3 was estimated by running a linear regression on the "true" OD flows. Of course, in real-world examples, the true OD flows will be unobserved, and are actually the outputs of the OD estimation process. In such instances, (*24*) present a framework for creating and updating the historical OD flows and transition factors from several days of traffic counts. The approach is based on an OD estimation procedure that operates sequentially across the H time intervals. The paper demonstrates the estimation of transition equations similar to Equation (4). Simple transition equations such as Equations (2) and (7) are employed initially (when no estimates are available). The multiple days of data are processed one at a time to update the historical database with each new day of measurements.

The performance of the estimators was analyzed and compared using the Root Mean Square Normalized (RMSN) error statistic:

$$RMSN = \frac{\sqrt{S\sum_{i=1}^{S}\left(M_i - \mathcal{M}_i\right)^2}}{\sum_{i=1}^{S} M_i} \tag{14}$$

M_i and $\mathcal{M}_i$ are the elements of the vectors of observed and assigned measurements $\left[\mathbf{M}_1 \cdots \mathbf{M}_h \cdots \mathbf{M}_H\right]$ and $\left[\mathcal{M}_1 \cdots \mathcal{M}_h \cdots \mathcal{M}_H\right]$ respectively, and S is the size of either vector.

Table 1 summarizes the error statistics for each estimator. It is observed that the simple transition equation provides a significant improvement over the use of ad hoc target flows (base case), while more accurate seed flows result in improved OD estimates. Further, the inclusion of detailed transition equations results in the most accurate OD flows estimates, even though no seed information was used. The fit to the observed count data reflect the same trend, thereby reinforcing the value of transition equations in simultaneous dynamic OD estimation.

Table 1: Fit to True OD Flows and Counts

Estimator	**Fit to OD Flows**		**Fit to Counts**	
	RMSN	**% Improv.**	**RMSN**	**% Improv.**
A. "Bad seed", no transitions ($\mathbf{x}_h^a = \mathbf{x}_h^H$)	0.2723	--	0.1587	--
B. "Good" seed, no transitions ($\mathbf{x}_h^a = \mathbf{x}_h^H$)	0.1400	48.6	0.0818	48.5
C. Simple transition, $\mathbf{x}_h^a = \hat{\mathbf{x}}_{h-1}$	0.1423	47.7	0.0856	46.1
D. "True" transition, $\mathbf{x}_h^a = \sum f_h^p \mathbf{x}_h$	0.1227	54.9	0.0774	51.2

Figure 3 illustrates a comparison of the flows between the primary OD pair (O1 to D) obtained from various estimators. It can be seen that accurate transition equations result in a very good fit to the true OD flows. The drawback of using arbitrary target flows (estimator A) is also highlighted.

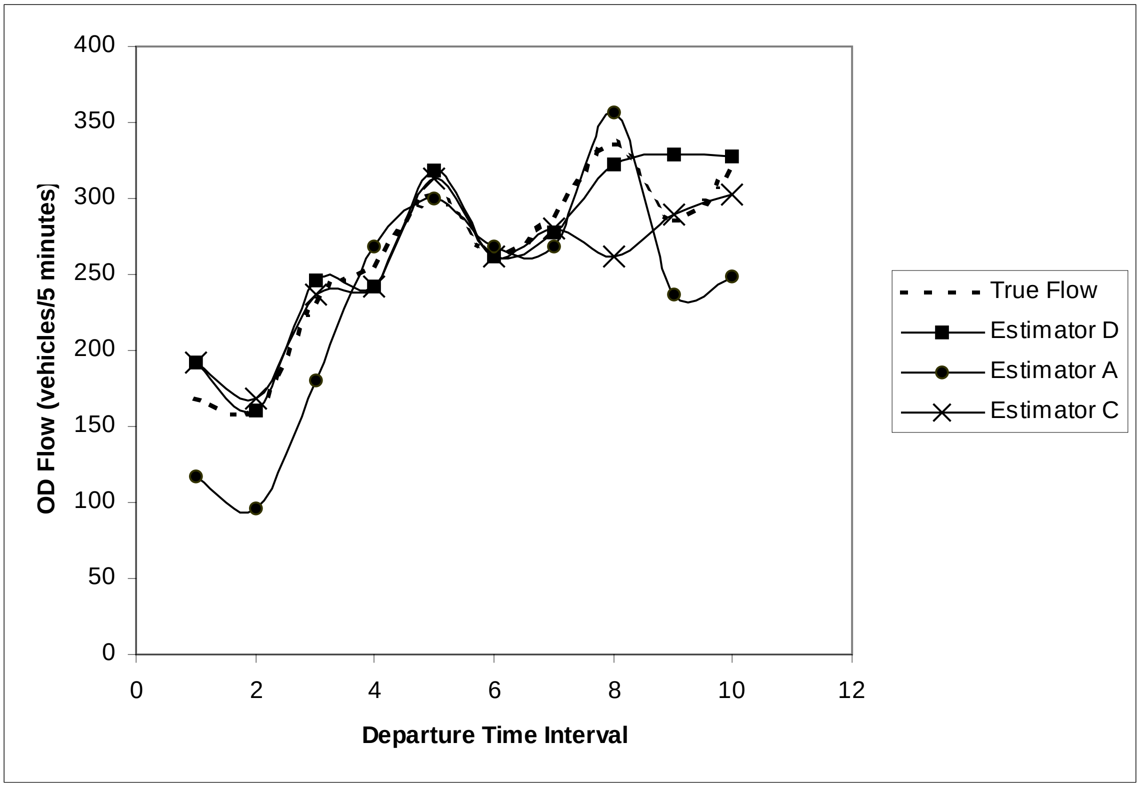

Figure 3. Comparison of Primary OD Pair

CONCLUSION

The paper extends a recent approach for the simultaneous, off-line estimation of dynamic OD flows from sensor data, by incorporating transition equations of any form. The transition equations capture spatial and temporal relationships among the OD flow variables. The method does not rely on assignment matrices, but rather represents explicitly the relationship between OD flows and sensor observations. Hence, it avoids the problem of solving expensive fixed-point problems. Furthermore it estimates OD flows simultaneously across all intervals in the time period of analysis.

A case study was used to illustrate the feasibility of the approach and compare the effectiveness of different transition equations used in simultaneous off-line OD estimation. The results are promising and highlight the importance of transition equations in improving the accuracy of the estimated OD flows. A test network was used in this paper in order to have a reference case with known OD demand values. This allowed a comparison of the accuracy not just in replicating sensor data, but also in capturing the true underlying OD flow patterns. Experiments on larger, real networks are ongoing, using data from Los Angeles, CA and Lower Westchester County, NY. The simultaneous estimation methodology has already been applied on both networks (*3*, *20*), though without the use of transition equations.

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